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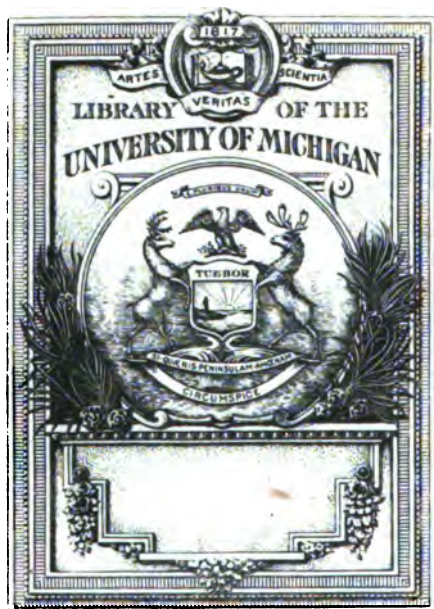
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Whittlesea
The gift of M^r Rogers

15

Edmund Price
**ARITHMETICAL
COLLECTIONS
AND
IMPROVEMENTS.**

**BEING
A COMPLETE SYSTEM
OF
PRACTICAL ARITHMETIC.**

**BY
ANTHONY and JOHN BIRKS,**

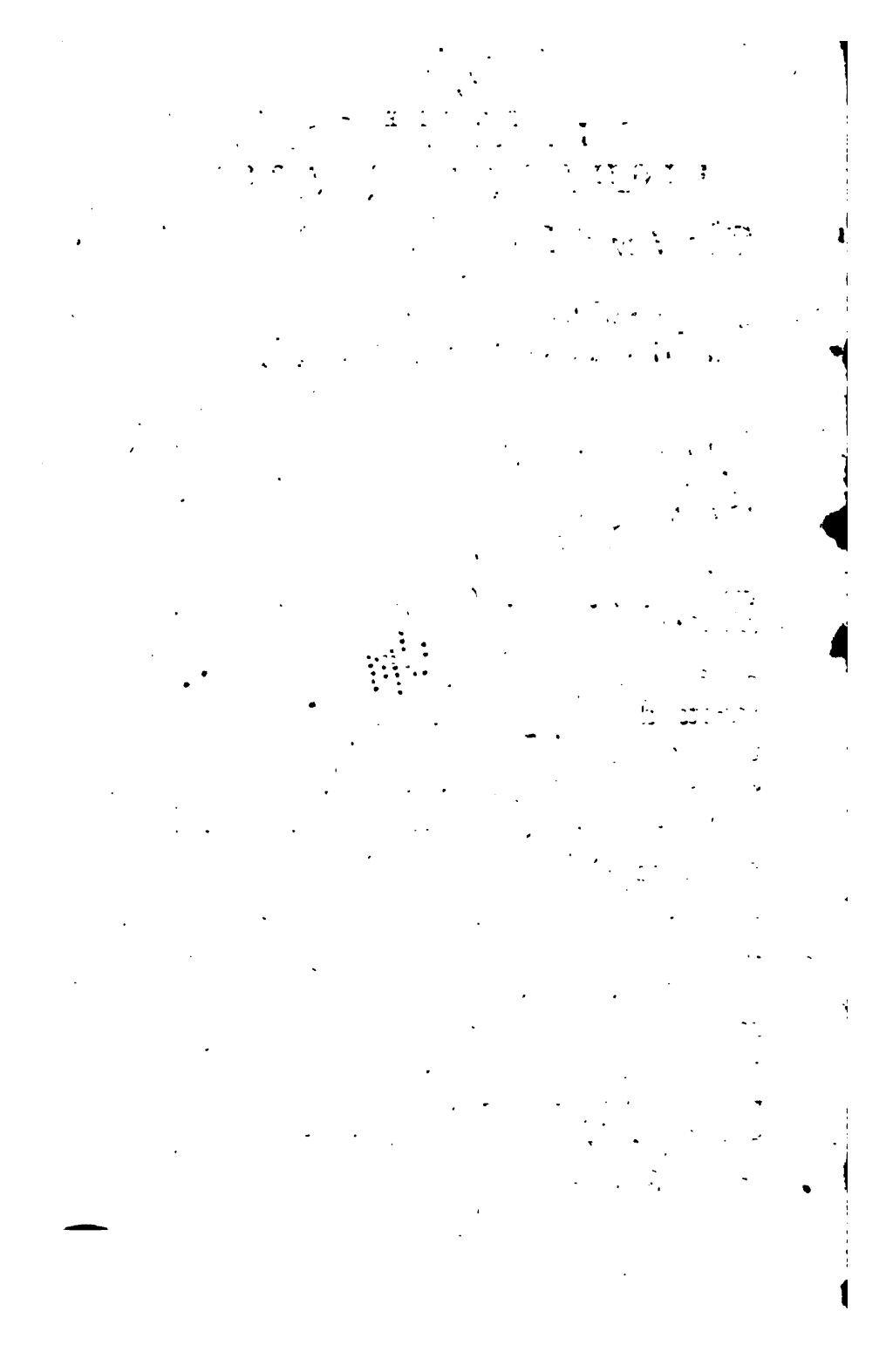
*Late Masters of a Boarding-School at Gosherton, and now
of the Free Writing-School at Danington, Lincolnshire.*



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History of Science
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TO THE

RIGHT HONOURABLE

The Lord BROWNLOW BERTIE,

One of the Representatives in Parliament
for the County of LINCOLN, &c. &c.

MY LORD,

5-15-34
THE utility of the science treated
upon in the following sheets,
must be well known to your Lordship,
who has so often been a witness of the
advantage and uncommon weight, argu-
ments drawn from arithmetical compu-
tations carry with them in that house,
whereof you are so worthy a member;
so that if this performance be found
equal to the usefulness of the subject,
it may justly be entitled to your Lord-
ship's patronage.

Your assiduity in parliament for pro-
moting the drainage of, and making
roads through a lately inundated, though
rich country, is and will be of such great
benefit, that succeeding ages, as well as
the present, must reap the advantage of

iv DEDICATION.

these salutary works, and posterity bless the time when a BERTIE graced the British senate.

That your Lordship may still succeed in promoting the good of your native country, and long live the great ornament thereof, is the hearty wish of,

My LORD,

Your Lordship's

most obedient and

most humble Servants,

THE AUTHORS.

P R E F A C E.

THE book here presented to the world is a regular system of common arithmetic, adapted to the use of the gentleman and the scholar, as well as the man of business.

B O O K I.

NUMERATION, and the next four general rules, are enriched with many compendious methods and examples; and the rules of Practice very copious and extensive.

The doctrine of Vulgar Fractions is here rendered more easy, concise, and useful, by the means of an easy axiom; Decimals are pursued through all the late improvements, in the management both of plain and circulating numbers; and the Extraction of the Roots, particularly the Cube, is done in a more easy manner than in any book of arithmetic, which has ever yet come under our inspection.

B O O K II.

In which Proportion is treated on in a scientific manner, and adapted as well to the use of the young mathematician, as together with the rules of Practice applied to all branches of business;

and the mercantile rules are exemplified and diversified with great variety of curious examples.

In Exchange are exhibited Sir Isaac Newton's tables of the assay and real and standard-weight, and value, of most of the gold and silver coins in Europe; together with those of the conformity of weights and measures, by the Sieur John Larrue, merchant, at Lyons; also the method of solving questions in the arbitration of exchange by a numerical equation.

B O O K III.

Contains the less useful, though most pleasant and delightful parts of arithmetic; viz. Alligation, medial, partial, and total; the Specific Gravity of Metals; the Rule of False, or Position; Progression both arithmetical and geometrical; also Variations, Combinations, and the method of filling Magic Squares: these, though they are done and accounted for better by Algebra, &c. yet may serve to open the mind, and excite the curiosity of youth to proceed to the most sublime and abstruse sciences.

To these are added Compound Interest, with the method of calculating the value of freehold estates at any rate of interest; also annuities in reversion, according to that late ingenious mathematician Mr. Thomas Sympson, F. R. S. from a set of tables calculated by him for that purpose. Also a collection of questions in Mensuration, with such directions as may enable any person to perform the measurement of most sorts of superficies and solids; and also some curious miscellaneous questions.

This

P R E F A C E. vii

This treatise concludes with an Appendix, by Mr. Thomas Allen, teacher of the mathematics at Spalding, containing rules and examples for finding the sum of any given number of terms in certain progressions. A collection of problems concerning the maxima and minima of quantities, with the theorems annexed. And the investigation of the sums of certain infinite series.

This work contains several hundred questions rationally solved; among which are all those in Clare's Introduction to Trade, &c. several from the Palladiums, Ladies Diaries, and other periodical books, as well as the most valuable and entertaining that could be found in other authors.

N. B. We were favoured with the Cribbage Question by Major Watson.

The algebraic demonstration of the rules here laid down are omitted for these three reasons; first, as arithmetical computations often carry their rationale along with them, the offering to prove a self evident truth renders it more obscure. Secondly, the mathematicians are already acquainted with them. And thirdly, the young student is as little benefited by them as a pure English scholar would be by an exposition of the Bible in Greek.

What oversights may have escaped the authors, either in the press (from whence their residence is more than 100 miles) or otherwise, hope their worthy subscribers will generously excuse; they having in the whole endeavoured to remove the difficulties, and render the passage easy and pleasant through this useful and delightful science.

The following algebraic signs or characters are very necessary to be understood, as being a much shorter, better, and more significant way of expression, than by words at length.

<i>Signs</i>	<i>Names</i>	<i>Significations</i>
$+$	$\left\{ \begin{array}{l} \text{Plus or} \\ \text{more.} \end{array} \right\}$	$\left\{ \begin{array}{l} \text{The sign of addition; as } 9 + 5 \\ \text{is 9 more 5, and signifies that} \\ \text{the numbers 9 and 5 are to be} \\ \text{added together.} \end{array} \right\}$
$-$	$\left\{ \begin{array}{l} \text{Minus or} \end{array} \right\}$	$\left\{ \begin{array}{l} \text{The sign of Subtraction; } 8 - 3, \\ \text{is 8 less three, and signifies that} \\ \text{3 is to be taken from 8.} \end{array} \right\}$
\times	$\left\{ \begin{array}{l} \text{Multiplied} \\ \text{into or by} \end{array} \right\}$	$\left\{ \begin{array}{l} \text{The sign of Multiplication; as} \\ \text{7 } \times \text{ 5, is 7 multiplied into, or} \\ \text{by 5.} \end{array} \right\}$
\div	$\left\{ \begin{array}{l} \text{Divided by} \end{array} \right\}$	$\left\{ \begin{array}{l} \text{The sign of Division; } 8 \div 2, \text{ is} \\ \text{8 divided by 2: also thus } \frac{8}{2} \text{ or} \\ \text{2) 8 (4, which signifies the same} \\ \text{thing.} \end{array} \right\}$
$=$	$\left\{ \begin{array}{l} \text{Equal to} \end{array} \right\}$	$\left\{ \begin{array}{l} \text{The sign of Equality; } 9 = 9, \text{ or} \\ \text{9 + 6 = 15, or 9 - 6 = 3, that} \\ \text{is 9 is equal to 9, or 9 more 6, is} \\ \text{equal to 15, and 9 less 6, is} \\ \text{equal to 3.} \end{array} \right\}$
$:$ $::$	$\left\{ \begin{array}{l} \text{Is to} \\ \text{So is} \end{array} \right\}$	$\left\{ \begin{array}{l} \text{The Signs of Proportion, or Rule} \\ \text{of Three; thus, } 2 : 8 :: 6 : 24 \\ \text{are to be read, as 2 is to 8, so is} \\ \text{6 to 24.} \end{array} \right\}$
\therefore		$\left\{ \begin{array}{l} \text{Continued Proportionals in Geo-} \\ \text{metric Progression.} \end{array} \right\}$
\odot	$\left\{ \begin{array}{l} \text{Involution.} \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Thus } 27 \odot 3 \text{ or } 27^3, \text{ signifies that} \\ \text{27 is to be involved to the third} \\ \text{power.} \end{array} \right\}$
$\sqrt{}$	$\left\{ \begin{array}{l} \text{Extraction} \\ \text{of the roots} \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Thus } \sqrt{4} = 2, \text{ signifies that the} \\ \text{square root of 4 equals 2, or} \\ \sqrt[5]{243} = 3; \text{ viz. the sursolid} \\ \text{or root, or the 5th power of} \\ \text{243 = 3.} \end{array} \right\}$

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Arithmetical Collections

AND

IMPROVEMENTS.

BOOK I.

CHAP. I. SECT. I.

NUMERATION, or NOTATION, teacheth to read or exprefs any number when wrote down; and confequently to write down any propofed number, according to its true value.

TABLE.

Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thoufands.	Tens of thoufands.	Thoufands.	Hundreds.	Tens.	Units.
3	6	7	4	2	8	7	2	5
1	3	9	1	0	6	8	5	3
1	1	8	4	6	1	3	2	7
1	1	1	3	9	2	8	7	5
1	1	1	1	6	2	4	3	8
1	1	1	1	1	4	1	3	7
1	1	1	1	1	1	7	3	5
1	1	1	1	1	1	1	6	4
1	1	1	1	1	1	1	1	7
9	8	7	6	5	4	3	2	1

All figures in the first row towards the right-hand, are units; those in the second row tens; those in the third hundreds; and those in the fourth thousands, &c.

A cypher, although by itself, it signifies nothing; yet being placed on the right-hand of any figure, augments the value of that figure ten times, by advancing it into a higher place than if the cypher had not been there. As 6 six, 60 sixty, 600 six hundred, &c.

To every three figures are orderly repeated the denominations of units, tens, hundreds; so that he that can read three figures, may, with a little more instruction, be quickly able to read any number how large soever. And to every three figures, the names units, and thousands, are alternately applied.

Likewise to every six figures from the right-hand a new general name is given. As to the first six figures, the general name of units are given; to the second six, the general name of millions; to the third six billions; to the fourth trillions; to the fifth quadrillions, &c.

The whole art of figural notation is comprehended in the following table.

Quadrillions.		Trillions.		Billions.		Millions.		Units.	
Thousf.	Units.	Thousf.	Units.	Thousf.	Units.	Thousf.	Units.	Thousf.	Units.
712	348	634	235	314	527	625	284	123	714
htu	htu	htu	htu	htu	htu	htu	htu	htu	htu

Read thus. Seven hundred twelve thousand, three hundred and forty-eight quadrillions.

Six hundred thirty-four thousand, two hundred thirty-five trillions.

Three hundred fourteen thousand, five hundred twenty-seven billions.

Six hundred twenty-five thousand, two hundred eighty-four millions.

One hundred twenty-three thousand, seven hundred and fourteen (units.)

The following numbers are also expressed in words at length, 370084, three hundred seventy thousand, and eighty-four.

418427000, four hundred eighteen million, four hundred twenty-seven thousand, nine hundred.

6210003745, six thousand two hundred ten million, three thousand seven hundred forty-five.

Chap. I. NUMERATION. 3

41027308751, forty-one thousand twenty-seven million, three hundred eight thousand, seven hundred fifty-one.

293417604712, two hundred ninety-three thousand four hundred seventeen million, six hundred four thousand, seven hundred and twelve.

618002030694713, six hundred eighteen billions, two thousand thirty millions, six hundred ninety-four thousand, seven hundred and thirteen.

47038066250433251889411, forty-seven thousand thirty-eight trillions, sixty-six thousand two hundred fifty billions, four hundred thirty-three thousand two hundred fifty-one millions, eight hundred eighty-nine thousand, four hundred and eleven.

A TABLE of NUMERICAL CHARACTERS *used by the Romans.*

I One.

V Five.

X Ten.

L Fifty.

C An hundred.

D or IO Five hundred.

M or CIO A thousand.

IOO Five thousand.

CCIOO Ten thousand.

IOOO Fifty thousand.

CCCIOOO A hundred thousand.

IOOOO Five hundred thousand.

CCCCIOOOO A million.

A line drawn over any number less than a thousand, intimates so many thousands; as LXX is 70,000; C is 100,000; and M a million.

I and X are sometimes placed before characters of greater value, namely, I before V or X, and X before L or C, in which case the value of I and X is to be subtracted from the value of the following character, as IV four, IX nine, XL forty, XC ninety.

V and L are never repeated, and none of the other characters above four times. Thus, IIII or IV; but V five, XXX thirty; but XL forty, LXXX eighty; but XC ninety, CCCC four hundred; but D five hundred.

4 NUMERATION. Book I.

In figures exprefs : a million and a half South-fea bonds. Nineſcore, and fourteen thouſand, eight hundred ſheep. Threeſcore and twelve thouſand, thirteen hundred pounds of lead. Fifteen thouſand, and fourſcore million of ſtivers. One hundred and twenty thouſand, two hundred and fix millions, ſeventy thouſand, ſeven hundred, and ſeven rials of plate. Three millions, and thirty-three thouſand, and thirty pieces of eight. Four thouſand, and forty hundred pounds, thirty-four ſhillings, and fourteen pence five farthings.

South-fea bonds	- - - - -	1500000
Sheep	- - - - -	194800
Lead	- - - - -	73300 lb.
Stivers	- - - - -	15080000000
Rials of plate	- - - - -	120206070707
Pieces of eight	- - - - -	3033030

l.	s.	d.
404001	15	3½



S E C T. II.

A D D I T I O N.

ADDITION is a rule whereby several numbers are so connected and put together, that their aggregate sum, or total amount, may be known.

Observe to place your numbers so, that each figure may stand directly underneath those of the same value; viz. units under units, tens under tens, and hundreds under hundreds, &c. Then,

R U L E,

Always begin your addition at the place of units, and add together all the figures that stand in that place; and if their sum be under ten, set it down below the line underneath its own place; but if their sum be more than ten, then you must set down only the overplus, or odd figure, above the ten (or tens) and so many tens as the sum of those units amount to, you must carry to the place of tens; adding them and all the figures that stand in the place of tens together, in the same manner as those of the units were added; then proceed in the same order to the place of hundreds, and so from place to place till all is finished.

1. In the bissextile, or leap-year, how many days and hours?

								Hours.
January	31	-	-	-	-	-	-	= 744
February	29	-	-	-	-	-	-	= 696
March	31	-	-	-	-	-	-	= 744
April	30	-	-	-	-	-	-	= 720
May	31	-	-	-	-	-	-	= 744
June	30	-	-	-	-	-	-	= 720
July	31	-	-	-	-	-	-	= 744
August	31	-	-	-	-	-	-	= 744
September	30	-	-	-	-	-	-	= 720
October	31	-	-	-	-	-	-	= 744
November	30	-	-	-	-	-	-	= 720
December	31	-	-	-	-	-	-	= 744

Answer 366 days. - - - - - = 8784

2. Find the number of chapters in the five books of Moses; after that the number of verses, and give their joint sum.

Ch.	Gen.	Ver.	Ex.	Ver.	Lev.	Ver.	Núm.	Ver.	Deu.	Ver.	Ch.
1		31		22		17		54		46	1
2		25		25		16		34		37	2
3		24		22		17		51		29	3
4		26		31		35		49		49	4
5		32		23		19		31		33	5
6		22		30		30		27		25	6
7		24		25		38		29		26	7
8		22		32		36		26		20	8
9		29		35		24		23		29	9
10		32		29		20		36		22	10
11		32		10		47		35		32	11
12		20		51		8		16		32	12
13		18		22		59		33		18	13
14		24		31		57		45		29	14
15		21		27		33		47		23	15
16		16		36		34		90		22	16
17		27		16		16		13		20	17
18		33		27		30		32		22	18
19		38		25		37		22		21	19
20		18		26		27		29		20	20
21		34		36		24		35		23	21
22		24		31		33		41		30	22
23		20		33		44		30		25	23
24		67		18		23		25		22	24
25		34		40		55		18		19	25
26		35		37		46		65		19	26
27		46		21		34		23		26	27
28		22		43				31		68	28
29		35		46		859		40		29	29
30		43		38				16		26	30
31		55		18				54		30	31
32		32		35				42		52	32
33		20		23				56		29	33
34		31		35				29		12	34
35		29		35				34			
36		43		38				13		959	
37		36		29							
38		30		31				1288			
39		23		43							
40		23		38							
41		57									
42		38		1213							
43		34									
44		34									
45		28									
46		34									
47		31									
48		22									
49		23									
50		26									
		1533									

	Chapters.	Verses.
Genesis	50	1533
Exodus	40	1213
Leviticus	27	859
Numbers	36	1288
Deuteronomy	34	959
	187	5852
		187
		Joint sum - 6039

Chap. I. ADDITION.

7

3. Decipher the following numerical letters, and find their sum; viz.

IV	- - - - -	4
VI	- - - - -	6
IX	- - - - -	9
IIX uncommon	- - - - -	8
XIII	- - - - -	13
XLV	- - - - -	45
LXXXI	- - - - -	81
XCVI	- - - - -	96
CXC	- - - - -	190
CD uncommon	- - - - -	400
DCC	- - - - -	700
MCL	- - - - -	1150
MDCXLVIII	- - - - -	1648
MCCM uncommon	- - - - -	1000800
IOIOCCCLVII	- - - - -	5857
VICCLXXXX	- - - - -	6290

Answer, 1017297

I would advise the young accomptant, in long operations in addition, to point at every 60, carrying on the overplus; and when he hath cast up the whole line, carry 6 to the next place for every point.

And to prove the work, begin at the top and cast it downwards, in the same manner as was done upwards, pointing on the other side the figures: and if the amount be the same both ways, it may be presumed that the work is right.

Or, when you have cast up the whole, divide it into two or more parts, which cast up separately; then add the sums of the said parts together, which, if like the first sum, the work may be adjudged to be right.

4. How much is A (born sixteen years ago) elder than B, who will come into the world fourteen years hence?

$$16 + 14 = 30 \text{ years, the answer.}$$

5. A person was 17 years of age 29 years since, and he will be drowned 23 years hence; pray in what year of his age will this happen?

$$17 + 29 + 23 = 69 \text{ years, the answer.}$$

B 4

S E C T.

S E C T. III.

A D D I T I O N O F E N G L I S H C O I N S.

The least piece of money used in England is a farthing.

And

Farth.

4 = 1 penny.

48 = 12 = 1 shilling.

960 = 240 = 20 = 1 pound sterling.

s. d.

And 5 : - is a crown.

6 : 8 is a noble.

10 : - is an angel.

13 : 4 is a mark.

N. B. l. } Pounds. : d. But most commonly,
 s. } ftand } Shillings. : $\frac{1}{4}$ } is wrote { 1 }
 d. } for } Pence. : $\frac{1}{2}$ } for { 2 } Farth.
 q. } Farthings. : $\frac{3}{4}$ }

P E N C E T A B L E S to be got by heart.

d.	s.	d.	s.	d.	s.	d.	d.	s.	d.
12 = 1		72 = 6		20 = 1	8	70 = 5	10		
24 = 2		84 = 7		30 = 2	6	80 = 6	8		
36 = 3		96 = 8		40 = 3	4	90 = 7	6		
48 = 4		108 = 9		50 = 4	2	100 = 8	4		
60 = 5		120 = 10		60 = 5	-	110 = 9	2		

Having placed the numbers to be added in this order, viz. pounds under pounds, shillings under shillings, and pence under pence, &c.

R U L E,

Begin with the farthings, and for every four carry one penny, setting down the overplus under the farthings; then proceed to the pence, casting up to 60, where make a prick, and so proceed to the top, setting down the odd pence

pence in their proper place; and carrying one for every 12 pence, and five for every prick (as 60 pence make five shillings;) so proceed to the units place of the shillings, setting down the overplus above 10, for each of which carry one, and six for every prick to the angels; and as two angels make one pound, carry half the number of angels to the pounds, setting down the odd one, if it so happen; then cast up the pounds, as before directed, in addition of whole numbers.

1. A nobleman going out of town, is informed by his steward that his cornchandler's bill comes to 123l. 19s. His brewer's 41l. 10s. His butcher's 212l. 6d. To his lordship's baker is owing 24l. To his tallowchandler 13l. 8s. To his taylor 137l. 9s. 9d. To his draper 74l. 13s. 6d. His coachmaker's demand was 214l. 16s. 6d. His winemerchant's 68l. 12s. His confectioner's 16l. 2s. His rent 86l. 2s. And his servants wages for half a year came to 46l. 5s. What money must he send to his banker for, in case he would carry with him 50l. to defray his expences on the road.

	l.	s.	d.
Cornchandler	123	19	—
Brewer	41	10	—
Butcher	212	—	6
Baker	24	—	—
Tallowchandler	13	8	—
Taylor	137	9	9
Draper	74	13	6
Coachmaker	214	16	6
Winemerchant	68	12	—
Confectioner	16	2	—
Rent	86	2	—
Servants wages	46	5	—
For expences	50	—	—

Answer, £ 1108 18 3

2. A collector of cash hath been out with bills, and gives an account that A paid him 13l. and half a crown; B 2l. 13s. C 14s. and a groat; D 1l. 9s. 8½d.; E 11l. 6½d.; F 17s. and a tetter; G 12s. 2d.; H a pound and half a guinea; I a moidore, and 13s.; K two broad-pieces of 23 shillings each, a Jacobus of 25s. and a shilling;

Shilling; L nine pounds and a mark; M 12l. 12s.; N a Bank note of 15 l.; and O three crown-pieces and an angel. What cash had he in charge?

								l.	s.	d.
A	13	2	6
B	2	13	-
C	-	14	4
D	1	9	8½
E	11	-	6½
F	-	17	6
G	-	12	2
H	1	10	6
I	2	-	-
K	3	12	-
L	9	13	4
M	12	12	-
N	15	-	-
O	1	5	-

£ 76 2 6½

3. A cornfactor buys seventy quarters of oats for 46l. 7s. 6d. thirty-eight quarters of beans for 100l.; twelve quarters of pease, which cost 16l. 16s. eighty-eight quarters of barley, for 73l. 8d. sixteen ditto of wheat for 56l. 9s. 10 d.; and six quarters of rye for 4l. 1s. 6d. the water carriage of all comes to 13l. 2s. 7d. his riding charges to 1l. 13s. and if he clears eighteen guineas by the bargain, what do his bill of parcels amount to?

								l.	s.	d.
Oats	-	-	-	-	-	-	-	46	7	6
Beans	-	-	-	-	-	-	-	100	-	-
Pease	-	-	-	-	-	-	-	16	16	-
Barley	-	-	-	-	-	-	-	73	-	8
Wheat	-	-	-	-	-	-	-	56	9	10
Rye	-	-	-	-	-	-	-	4	1	6
Water carriage	-	-	-	-	-	-	-	13	2	7
Riding charges	-	-	-	-	-	-	-	1	13	-
Commission	-	-	-	-	-	-	-	18	18	-

£ 330 9 1

Chap. I. ADDITION.

11

4. A of Amsterdam is debtor to B of Bristol, for mercery wares as per factory, 418 l. 2s. 6d.; for forty C weight of Cheshire cheese 52l. 18s.; for English broad-cloth fifteen pieces, 317 l. 12s. 10d.; for 19 fodder of lead 320 l.; for 12 tons of bar-iron 173l. 3d.; for eight tons of copper 1110l. 10s. 1d.; for his acceptance of a bill drawn 88 l. 14s.; for another paid for honour 50 l.; 10 dozen Morocco skins 28 l. 15s. 4d.; paid convoys, insurances, and port charges 43 l.; warehouse room, postage, sledage, boatage, and incidental charges 5 l. 5s.; the factorage of all came to 112 l. 6s. For what sum must B draw to clear the account?

	l.	s.	d.
Mercery wares - - - - -	418	2	6
Cheshire cheeses - - - - -	52	18	-
Broad-cloth - - - - -	317	12	10
Lead - - - - -	320	-	-
Bar-iron - - - - -	173	-	3
Copper - - - - -	1110	10	1
Accepted bill - - - - -	88	14	-
Ditto on honour - - - - -	50	-	-
Morocco skins - - - - -	28	15	4
Convoys, insurances, &c. - - - - -	43	-	-
Warehouse room, &c. - - - - -	5	5	-
Factorage - - - - -	112	6	-

£ 2720 4 -

5. A rate or assessment, for and towards the relief of the poor of the parish of Gosberton, &c.

	l.	s.	d.
Francis Fane, Esq. - - - - -	11	3	7
John Robinson, Gent. - - - - -	3	17	5½
Richard Calthorp, Gent. - - - - -	4	13	6
Thomas Baley, Gent. - - - - -	5	7	4½
Mr. John Terry - - - - -	3	18	6
Mr. John Turver - - - - -	1	17	4½
Henry Worley - - - - -	-	3	2
Jonathan Cheavin - - - - -	-	15	10½
William Trickett - - - - -	-	5	9
Anthony Birks - - - - -	-	6	10½

Carried over 32 9 5½

	l.	s.	d.
Brought over	32	9	5½
Mr. Robert Cole	1	2	7
John Wright	-	10	3½
Mr. John Shaw	-	19	11½
Thomas Ladd	-	5	9
John Lambson	-	3	6½
Thomas Hopper	-	2	9½
Francis Mattin	-	10	3
William Crawforth	-	15	10½
Thomas Oldgate	-	17	5½
William Wiseman	-	3	11
Mrs. Margaret Parkinson	-	11	7½
Samuel Lane	-	2	4½
Mrs. Alice Sharp	-	2	3
William Curtis	-	9	7½
John Binney	-	-	11
Mrs. Silv. Flear	-	18	9
John Pattisson	-	2	4½
John Gennils	-	19	3
Mr. John Pacy	-	18	11½
William Bilton	-	1	4
Mr. Robert Allen	-	2	12
Mrs. Eliz. Wilcox	-	2	9
John Mason	-	1	18
Mr. John Thimbleby	-	5	7
John Smith	-	3	18
Henry Ward	-	5	17
Alexander Codling	-	1	13
William Lambson	-	1	5
John Gibbons	-	17	10½
<hr/>			
	£ 69	12	7

6. The Right Honourable the Lord Bolsover Debtor
To Paul Purfeproud, upholder.

	l.	s.	d.
1758. April 19. A rich crimson damask bed, laced, } complete	75	-	-
May 5. A set of window curtains and val- } lences, ditto	16	11	8
7. A fine carpet, counterpain, and an } otter-down quilt	12	10	-

Carried over 104 1 8

Chap. I. A D D I T I O N.

13

		l.	s.	d.
	Brought over	104	1	8
June 6.	A crimson velvet easy-chair, and } two stools, ditto - - - - - }	13	7	6
13.	A wrought dimitty bed and furni- } ture, complete - - - - - }	28	18	4
Aug. 10.	A down bed, bolster, pillows, and } quilt - - - - - }	15	-	-
	Chairs 10, with two armed ditto, } walnut-tree framed - - - - - }	34	12	6
Nov. 20.	A fire-screen, bed, table, and dres- } sing-glass - - - - - }	8	14	6
	The lady Wanton's picture, in a } rich frame - - - - - }	21	-	-
		<hr/> £ 225 14 6 <hr/>		

7. A person said he had 20 children, and that it happened there was a year and half between each of their ages; his eldest was born when he was 24 years old, and the age of his youngest is now one-and-twenty: What was the father's age?

When the eldest was born	-	24	
Then 19 + 9½	- - -	=	28½
Youngest	- - -	-	21
<hr/>			
Father's age	- - - - -	-	73½
<hr/>			

8. A sheepfold was robbed three nights successively; the first night half the sheep were stolen, and half a sheep more; the second night half of the remainder were lost, and half a sheep more; the last night they took half what were left, and half a sheep more, by which time they were reduced to 20: How many were there at first?

20 left	
21 } taken the { 3d } night.	
42 } taken the { 2d }	
84 } taken the { 1st }	
<hr/>	
167	sheep in all.
<hr/>	

SECT. IV.

SUBTRACTION.

SUBTRACTION takes a lesser number from a greater, by which the excess, difference, or remainder, may be known.

In setting down numbers for work, always place the greatest number or sum uppermost, in such order, that units may stand under units, tens under tens, &c. also pounds under pounds, shillings under shillings, pence under pence, &c.

RULE.

Begin with the lowest or least denomination, (as in addition) and take or subtract the figure, or figures, in that place of the subtrahend, from the figure, or figures, that stands over them of the same denomination, setting down the remainder. But if that cannot be done, increase the upper figure, or figures, with one of the next superior denomination; and from that sum make subtraction; and so proceed to the next superior denomination, where you must pay the one borrowed, adding unity to the subtrahend in that place.

1. If a person hath 105 miles to travel, and hath gone 99, how many miles hath he yet to go?

Miles.

105
99

Answer 6

2. If a person be 49 years of age this present year 1765, what year was he born in?

1765
49

Answer 1716

3. In

Chap. I. SUBTRACTION.

15

3. In fifteen hundred nine-two there died a noble prince; how many years is that ago?

$$\begin{array}{r} 1765 \\ 1592 \\ \hline \text{Answer } 173 \end{array}$$

4. A collector of excise has received 2479 l. 12 s. 6½ d., and paid into the office, by several remittances, 1977 l. 17 s. 7½ d.; how much remains in his hands?

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ \text{Received } 2479 \quad 12 \quad 6\frac{1}{2} \\ \text{Remitted } 1977 \quad 17 \quad 7\frac{1}{2} \\ \hline \text{In hand } \pounds 501 \quad 14 \quad 10\frac{1}{2} \end{array}$$

5. Having a piece of ground 127 feet in front, let off to A 57 feet, to build on at one end; and to B at first, 27½ feet; which he afterwards, by consent, extended to 42 feet; what ground was left me in the center?

$$\begin{array}{r} \text{Feet.} \\ 127 \\ 57 + 42 = 99 \\ \hline \text{Answer } 28 \end{array}$$

6. Your grandfather, if living, is 119 years of age; your father actually 63; you are not so old as your grandfather by 83 years; what is the difference in years between your father and you?

$$\begin{array}{r} \text{Years.} \qquad \qquad \text{Years.} \\ \text{Grandfather's } 119 \quad \text{Father's } 63 \\ \quad \quad \quad - 83 \qquad \quad \quad - 36 \\ \hline \text{Your age } 36 \quad \text{Answer } 27 \end{array}$$

7. In the city of Pekin in China, is a bell weighing, it is said, 120000 pounds; at Nankin, in the same country, is another weighing 50000 pounds. The first exceeds the

the great bell at Erfurd, in Upper Saxony, by 94600 pounds ; how much then is the German bell inferior in weight to the second ?

Pekin bell 120000	Nankin bell 50000
— 94600	— 25400
Erfurd bell 25400	Answer 24600

8. Miss Kitty told her sister Charlotte, whose father had before left them twelve thousand twelve hundred pounds a piece, that their grandmother by will had raised her fortune to fifteen thousand pounds, and had made her own twenty thousand ; pray what did the old lady leave between them ?

Miss Kitty £ 20000	Miss Charlotte £ 15000
By father £ 13200	£ 13200
Grandmother £ 6800	£ 1800
+	= 8600 Ans.

9. What is the difference between the ages of A, born in the year 1693, and B that will be born 13 years hence ; the question being put in the year 1758 ?

Anno 1758
— 1693
65 + 13 = 78 the Answer.

To prove subtraction, add the remainder to the lesser number, which ought to make up the greater, if the work be right.

10. A horse in his furniture is worth 35 l. 10 s. out of it 12 l. 12 s. how much does the price of the furniture exceed that of the horse ?

	l. s.
Horse and furniture	35 10
Horse - - - -	12 12
Furniture - - -	22 18
	12 12
Answer, £	10 6

Chap. I. SUBTRACTION.

17.

11. A merchant at his outfitting in trade owed 280 l.; he had in cash, commodities, the stocks, and good debts, 11505 l. 10 s. he cleared the first year by commerce 393 l. 13 s. 1 d. What at the year's end was his net balance?

	l.	s.	d.
To cash, &c. - - -	11505	10	-
Commerce - - -	393	13	1
	<hr/>		
	11899	3	1
Debts - - - -	280	-	-
	<hr/>		
Answer £	11619	3	1

12. A trader failing, was indebted to A 71 l. 12 s. 6 d.; to B 34 l. 9 s. 9 d.; to C 16 l. 8 s. 8 d.; to D 44 l.; to E 19 l. 19 s.; to F 11 l. 2 s. 3 d.; to G 66 l. 17 s. 6 d.; to H a fine of thirty marks. At the time of this disaster he had by him in cash 31 l. 13 s. 6 d.; in commodities 23 l. 10 s.; in household furniture 13 l. 8 s. 6 d.; in plate 7 l. 18 s. 5 d.; in a tenement 56 l. 15 s.; in recoverable book debts 87 l. 13 s. 10 d. Supposing these things faithfully surrendered to his creditors, what will they then lose by him?

Debtor.				Creditor.			
	l.	s.	d.		l.	s.	d.
To A	71	12	6	By Cash - - -	3	13	6
B	34	9	9	Commodities -	23	10	-
C	16	8	8	Household furniture	13	8	6
D	44	-	-	Plate - - -	7	18	5
E	19	19	-	Tenement - -	56	15	-
F	11	2	3	Book debts - -	87	13	10
G	66	17	6		<hr/>		
H	20	-	-		£	192	19 3
	<hr/>				<hr/>		
Debtor	284	9	8				
Creditor	192	19	3				
	<hr/>						
Answer £	91	10	5				

13. You were born 34 years after me; how old shall I be when you are 17? and how old will you be, when I am 70 years of age?

$$70 - 34 = 36 \text{ You.} \quad - \quad - \quad 34 + 17 = 51 \text{ I.}$$

C

14. A

14. A made a bond for 114l. 10s. the interest came to 19l. He then paid off forty guineas, and gave a fresh bond for what was behind. By that time there was 13l. 4s. 8d. due of the second for interest. He paid off 37l. 14s. 2d. more, took up the old bond, and signed a new one still for the residue. The principal again ran on till there was 9l. 11s. 3d. more due, and then he determined to take it up: Pray what money had his creditor to receive?

	l.	s.	d.
First bond - - - - -	114	10	-
Interest - - - - -	+ 19	-	-
	<hr/>		
Paid - - - - -	£ 133	10	-
	- 42	-	-
	<hr/>		
Second bond - - - - -	91	10	-
Interest - - - - -	+ 13	4	8
	<hr/>		
Paid - - - - -	£ 104	14	8
	- 37	14	2
	<hr/>		
Third bond - - - - -	67	-	6
Interest - - - - -	+ 9	11	3
	<hr/>		
Answer - - - - -	£ 76	11	9
	<hr/>		

15. Received from my factor at Alicant, on account of sales of tin, to the value of 197l. 12s. sterling; of beeswax 71l. 7s. 6d.; of stockings 47l. 3s. 6d.; of tobacco, the net proceeds whereof were 943l. 15s. 10d.; of cotton 123l. 3s. 7d.; and of wheat to the amount of 116l. 5s. 6d. He at the same time advises, that he has per order shipped for my account, and risk, Alicant wines, to the value of 226l. 16s. 6d.; figs 157l. 11s. 3d.; fruit 90 chefts, cost 104l. 6s.; olives 136l. 10s.; oil 193l. 17s.; rafins 143l. 4d.; and Spanish wool to the value of 73l. 13s. 8d. The commission of the whole consignment came to 71l. 18s. 11d. which of us is to draw for the difference, and how much?

Debtor.

Chap. I. SUBTRACTION.

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Debtor.		Factor.			Creditor.	
		l.	s.	d.		
To	Tin - - - -	197	12	-	By	Wines - - -
	Bees-wax -	71	7	6		226 16 6
	Stockings -	47	3	6		Figs - - -
	Tobacco - -	943	15	10		157 11 3
	Cotton - - -	123	3	7		Fruit - - -
	Wheat - - -	116	5	6		104 6 -
						Olives - - -
						136 10 -
						Oil - - - -
						193 17 -
						Raisins - - -
						143 - 4
						Wool - - - -
						73 13 8
						Commission
						71 18 11
Debtor	£ 1499	7	11			
Creditor	- 1107	13	8			
Balance	£ 391	14	3			

16. A, B and C open an account with a banker, Jan. 11, 1739, and put into his hands; viz. A 17l. 17s. B 34l. 11s. 6d. C 28l. 18s. 10d. On the 21st A withdrew 9l. 10s. and C advanced 12l. and a crown. The 24th B called for 6l. 10s. The 30th C wanted 19l. 8s. 4d. On the 12th of February B deposited with him eleven broad pieces, and three moidores. On the 19th A sent for 5l. and a noble more; but on the 24th returned him 42l. On the 2d of March C paid in twenty guineas, and B drew for fix. The 14th B sent in 17l. 8s. 8d.; and the 17th A had cash 12l. 2s. 6d. On the 19th they sent for five guineas a man; and on the 24th they returned that sum, and ten marks a piece more. How much did their said banker owe them, jointly and separately, at Lady-day?

Debtor.		Banker's account current.		Creditor.		
				A		
1739.		l.	s.	d.	l. s. d.	
		1.	s.	d.	21st Jan.	- - 9 10 -
11th Jan.	- - -	17	17	-	19th Feb.	- - 5 6 8
24th Feb.	- - -	42	-	-	17th Mar.	- - 12 2 6
24th Mar.	- - -	11	18	4	19 - - -	- - 5 5 -
Debtor	-	71	15	4		
Creditor	-	32	4	2		
To A		£	39	11 2		

C 2

Debtor.

Debtor.		Banker's account current.				Creditor.	
		B				l. s. d.	
		l.	s.	d.			
11th Jan.	- -	34	11	6	24th Jan.	- -	6 10 -
12th Feb.	- -	16	14	-	2d Mar.	- -	6 6 -
14th Mar.	- -	17	8	8	19th -	- -	5 5 -
24th -	- -	11	18	4			<u>£ 18 1 -</u>
Debtor -		80	12	6			
Creditor -		18	1	-			
To B -		£ 62	11	6			

						C									
						l.	s.	d.							
11th Jan.	-	-	28	18	10	30th Jan.	-	-	19	8	4				
21st	-	-	12	5	-	19th Mar.	-	-	5	5	-				
2d Mar.	-	-	21	-	-										
24th	-	-	11	18	4				£	24	13	4			
									l.	s.	d.				
Debtor	-		74	2	2	To A	-		39	11	2				
Creditor	-		24	13	4	B	-		62	11	6				
						C	-		49	8	10				
To C	-	£	49	8	10							£	151	11	6

17. B born 161 years ago, died when C was 47 years of age; who it seems came into the world 180 years since, and outlived B 43 years. The sum of the ages of these two persons is required?

$$\begin{aligned}
 180 - 47 &= 133 \text{ years since B died.} \\
 161 - 133 &= 28 \text{ B's age.} \\
 47 + 43 &= 90 \text{ C's age.} \\
 28 + 90 &= 118, \text{ the answer.}
 \end{aligned}$$

18. Sam. was born 28 years before Toby, who died at 12, and lived 19 years after him. Rachael came to light when Sam. was 16, and died 11 years before him. Joshua (when Rachael was seven years old, being himself then 14) went abroad, where he continued nine years; and returning, survived Rachael four years. How old was each of these, and what is the sum of their ages?

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Toby - $\frac{28}{12}$ years old.
19

Sam. - 59 years old.
— 27 = 16 + 11

Rachael 32 years old.
 — $11 = 7 + 4$

Joshua 43 years old.

Then $59 + 12 + 32 + 43 = 146$.

19. A chaise, horse, and harness, were together valued at 50 l. The horse in harness was worth 38 l. 16 s. 6 d.; the chaise and harness were estimated at 13 l. 13 s. Their several valuations are required?

	l.	s.	d.		l.	s.	d.
Together - - -	50	-	-	Chaise and harness -	13	13	-
Horse and harness—	38	16	6		11	3	6

Chaise - - - £ 11 3 6 Harness alone - £ 2 9 6

	l.	s.	d.
	50	-	-
	13	13	-
	<hr/>		
Horfe alone	£	36	7 -

20. From the creation to the flood was 1656 years; thence to the building of Solomon's temple 1336 years; thence to Mahomet, who lived 622 years after Christ, 1630 years. In what year of the world was Christ then born?

From the creation to the flood	- - -	1656
To the building of Solomon's temple	-	1336
To Mahomet	- - - - -	1630

Mahomet after Christ. — ⁴⁶²²622

Answer A. M. 4000

C'3

21. A

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21. A is 13 years younger than B, and 17 years older than C, who in the year 1711 was known to be 24 years of age. How old was each of these persons in 1733?

$$\begin{array}{r} 1733 - 1711 = 22 + 24 = 46 \text{ C's} \\ \quad \quad \quad 17 + 24 = 63 \text{ A's} \\ \quad \quad \quad 13 + 63 = 76 \text{ B's} \end{array} \left. \vphantom{\begin{array}{r} 1733 - 1711 = 22 + 24 = 46 \text{ C's} \\ 17 + 24 = 63 \text{ A's} \\ 13 + 63 = 76 \text{ B's} \end{array}} \right\} \text{Age.}$$

22. W, X, Y, Z send in their money to the Bank; and draw upon it in the following manner; viz. June 4th, 1758, Z sends in 70l. 8s. Y had 116l. 14s. 10d. remaining on balance; and the 14th sent in 120l. more. W paid in 47l. 18s. 2d. in cash, and delivered in a Bank note for 200l. X paid in a bill of exchange on a good man, for 33l. 14s. 9d. and in cash made it up 100l. Y on the 16th drew for 43l. 12s. 6d. and on the 20th Z for eleven guineas. W on the 24th added 14l. 12s. 10d. and X withdrew 47l. 10s. 8d. Y on the 28th paid in 18l. 5s. and two days after drew for 88l. 13s. 4d. W sent for 63 guineas on the 30th, and in five days after for 15l. 10s. 9d. more. Z on the 7th of July demanded 12l. 8s. 3d. and X 7l. 3s. 1d. Z on the 15th remitted them 31l. 12s. 4d. and per assignment they received for him, the same time, double that sum. Y received on the 12th 81l. 19s. 8d. and W 10l. 10s. Y three days after sent in 42l. and W 52l. On the 19th X sent for 31l. 18s. 10d. and the 24th paid in 19l. 19s. The question is, how stood these gentlemen's cash severally, and what money can they jointly raise?

Debtor.	The Bank.				Creditor.		
To cash received.	W				By cash paid.		
1758.	l.	s.	d.		l.	s.	d.
4th June - -	47	18	2	30th June - -	66	3	-
	200	-	-	5th July - -	15	10	9
24th - - -	14	12	10	12th - - -	10	10	-
15th July - -	52	-	-				
Debtor -	314	11	-				
Creditor -	92	3	9				
To W -	£ 222	7	3				

Debtor.

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Debtor. The Bank. Creditor.

To cash received. X By cash paid.

1758.	l.	s.	d.		l.	s.	d.
4th June - -	100	-	-	24th June - -	47	10	8
24th July - -	19	19	-	7th July - -	7	3	1
				19th - - -	31	18	10
Debtor -	119	19	-				
Creditor -	86	12	7		£ 86	12	7
To X -	£ 33	6	5				

Y

	l.	s.	d.		l.	s.	d.
4th June - -	116	14	10	16th June - -	43	12	6
14th - - -	120	-	-	30th - - -	88	13	4
28th - - -	18	5	-	12th July - -	81	19	8
15th July - -	42	-	-		£ 214	5	6
Debtor -	296	19	10				
Creditor -	214	5	6				
To Y - -	£ 82	14	4				

Z

	l.	s.	d.		l.	s.	d.
4th June - -	70	8	-	20th June - -	11	11	-
15th July - -	94	17	-	7th July - -	12	8	3
Debtor -	165	5	-		£ 23	19	3
Creditor -	23	19	3				
To Z -	£ 141	5	9				

To W - -	222	7	3
X - -	33	6	5
Y - -	82	14	4
Z - -	141	5	9
In all -	£ 479	13	9

C 4

23. Moses

23. Moses was born anno mundi 2433. Homer 832 years after him. Julius Cæsar lived 40 years before our Saviour; and Alexander 312 years before Cæsar. Now as Christ was incarnate 4000 years after the creation, the sum of the intervals between Homer and the three great personages last mentioned is required?

Moses born A. M.	- - -	2433
		+ 832
		<hr/>
Homer born A. M.	- - -	3265
		<hr/>
Christ born A. M.	- - -	4000
		- 40
		<hr/>
Cæsar born A. M.	- - -	3960
		- 312
		<hr/>
Alexander born A. M.	- - -	3648
		<hr/>

$$3960 - 3265 = 695 \text{ from Homer to Cæsar.}$$

$$4000 - 3265 = 735 \text{ from Homer to Christ.}$$

$$3648 - 3265 = 383 \text{ from Homer to Alexander.}$$

Answer 1813

24. A merchant, taking an inventory of his capital, finds in his vault 28 pieces of brandy, which cost him 874l. 10s. 6d. Bourdeaux claret, 40 tons, which stood him in 754l. 4s.; 22 lasts, four bushels of corn in his granary, worth 675l. 17s. 3d.; with two lasts of Canary seed, worth 113l. In his warehouse were 10 casks of indigo, worth 632l. 12s. a parcel of saffron, worth 253l. 5s. W. P. of Stafford owed him 384l. 10s. In the hands of F. G. of Lynn, he had wines to the amount of 1011l. 10s. Pepper in the keeping of S. Q. of the Custom-house, value 1552l. 16s. 8d. Besides which R. O. owes him on bond 300l.; and T. M. on note 260l. 14s. He has in India bonds to the value of 450l. and the interest of those securities made 25l. 14s. 6d. He had Bank-stock to the value of 2134l. 4s. 6d. There lay in his banker's hands 1892l. 17s. 6d. He was at this time indebted to D. E. 713l. 13s. To M. F. 352l. 10s. 8d. To L. P. the foot of his account 172 guineas. To J. B. on balance 57l. 12s. 10d. To an insurance 190l. The present state of this person's fortune is required?

Stock

Chap. I. SUBTRACTION.

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Stock debtor.

	l.	s.	d.
To D. E. -	713	13	-
M. F. -	352	10	8
L. P. -	180	12	-
J. B. -	57	12	10
Insurances -	190	-	-
	<u>£ 1494</u>	<u>8</u>	<u>6</u>

Contra creditor.

	l.	s.	d.
By Brandy -	874	10	6
Claret -	754	4	-
Corn -	675	17	3
Canary feed -	113	-	-
Indigo -	632	12	-
Saffron -	253	5	-
W. P. -	384	10	-
Wines per F. G.	1011	10	-
Pepper per S. Q.	1552	16	8
A bond on R. O.	300	-	-
A note on T. M.	260	14	-
India bonds -	459	-	-
Interest -	25	14	6
Bank-stock -	2134	4	6
Banker -	1892	17	6

£ 11324 15 11
- 1494 8 6

The merchant's present worth,

£ 9830 7 5

25. Seth was born when Adam was 130 years of age, and 800 years before our said grandfire's death. Seth at the age of 105 years had Enos. He at 90 was father to Canaan, who at 70 had Mahalaleel. This man at 65 got Jared; who having lived 162 years, was father to Enoch. This patriarch at 65 years of age had Methuselah; and by the time he was 187 years of age, his son Lamech came into the world; who at 182 years old was father to Noah; and when Noah was 600 years old, the flood swept away the bulk of mankind. In what year of the world did this happen, and how long after the death of Adam?

Adam at	-	130 years had Seth.
Seth at	-	105 Enos.
Enos at	-	90 Canaan.
Canaan at	-	70 Mahalaleel.
Mahalaleel at	-	65 Jared.
Jared at	-	162 Enoch.
Enoch at	-	65 Methuselah.
Methuselah	-	187 Lamech.
Lamech at	-	182 Noah.
Noah at	-	600 entered the ark.

Year of the flood 1656
Adam 130 + 800 = 930
After his death - 726

11324-15-11
1494 8 6
12079-4-5

26. In

26. In a company S had 3l. 17 s. 2 d. more than T, who had six guineas less than R, who had within 16s. 8d. as much as W, who was known to have 100 guineas, wanting 10 marks of 13 s. 4 d. each. Pray what money had they among them?

l.	s.	d.	l.	s.	d.	l.	s.	d.
105	-	-	6	13	4 =	98	6	8 W had.
98	6	8	-	16	8 =	97	10	- R had.
97	10	-	6	6	- =	91	4	- T had.
91	4	-	+	3	17 2 =	95	1	2 S had.

£ 382 1 10 the answer.

27. If the mean distances between the earth and sun be 81 millions of miles, and between the earth and moon 240 thousand; how far are these two luminaries asunder in an eclipse of the sun, when the moon is lineally between the earth and sun? And in another of the moon, when the earth is in a line between her and him?

Distance of the sun from the earth	-	-	81000000
- - - - - of the moon	-	-	+ 240000

From each other in an eclipse of the moon 81240000

81000000
- 240000

- - - - - in an eclipse of the sun - 80760000

28. Hipparchus and Archimedes, of Syracuse, about 200 years before Christ; Ptolemy 50 years before the said grand period; and Ptolemy 140 years after it; all advanced the science of astronomy. How long did each of these persons flourish before the year of Christ 1758?

	²⁰⁰ + 1758	⁵⁰ + 1758
Hipparchus and Archimedes	1958	Ptolemy 1808
	<u>1758</u>	<u>1758</u>
	- 140	
Ptolemy	- 1618	

Chap. I. SUBTRACTION. 27

29. A grant was made by the crown, anno 1239, which was forfeited 137 years before the Revolution in 1688; how long did the same subsist?

Revolution A. D.	1688	Granted A. D.	1551
	<u>— 137</u>		<u>1239</u>
Forfeited A. D.	<u>1551</u>	Answer	<u>312</u>

30. The building of Solomon's temple was in the year of the world 3000. Troy was, by computation, built 443 years before the temple, and 260 years before London. Now Carthage was built 113 years before Rome; founded 744 years before Christ, born anno mundi 4000. Is London or Carthage the ancientest city, and how much?

Solomon's temple built	A. D.	- - -	3000
Troy before	- - - - -	- - -	<u>443</u>
	A. M.		2557
			<u>+ 260</u>
London built	- - - - -	A. M.	<u>2817</u>
Christ born	- - - - -	A. M.	4000
Rome built before	- - - - -	- - -	<u>744</u>
	A. M.	- - -	<u>3256</u>
Carthage before	- - - - -	- - -	<u>113</u>
	A. M.		3143
			<u>2817</u>
London built before Carthage	- - -	- - -	<u>326</u> years.

31. A public edifice was finished towards the close of the 10th of king John, who began his reign 134 years after the Conquest in 1066; and it stood till within 70 years of the peace of Utrecht, in 1713. Of what duration was it?

Conquest	- - A. D.	1066
	134 + 10 =	<u>+ 144</u>
Edifice finished	- A. D.	<u>1210</u>
Peace of Utrecht	A. D.	1713
	- - -	<u>70</u>
Demolition	- - A. D.	<u>1643</u>
	- - -	<u>1210</u>
Duration	- - - - -	<u>433</u> years, the answer.

32. A,

32. A, born anno 1438, died at 48 years of age. B died anno 1502, aged threecore and seventeen. C, in the year 1577, was 22 years of age, and survived that time 54 years. D, anno 1616, had lived just half his time, and died in 1648. E was 13 years old at the death of D, and fourteen years after that was father to F, who was 31 when his son G was born; who, at his grandfire's death, was seven years of age. The years of Christ, wherein those men were born; and the year wherein the first five of them died, are severally required?

A born 1438 + 48 = 1486 died.
 B died 1502 — 77 = 1425 was born.
 C in - 1577 — 22 = 1555 was born.
 And - 1577 + 54 = 1631 died.
 D died 1648 — 1616 = 32 half his age.
 And - 1648 — 64 = 1584 died.
 E in - 1648 — 13 = 1635 was born.
 And 1635 + 13 + 14 + 31 + 7 = 1700 died.
 F in - 1635 + 27 = 1662 was born.
 G in - 1662 + 31 = 1693 was born.

33. The powder-plot was discovered 88 years after the Reformation in 1517. The murder of king Charles the First was committed 43 years after that discovery. The accession of the Brunfwick family to the crown was in 1714, just 54 years after the return of king Charles the Second, who had lived in exile ever since the death of his father Charles the First. How long was that?

Reformation	- - - - -	A. D.	1517
Powder-plot discovered	- -	A. D.	+ 88
			<hr/> 1605
			+ 43
			<hr/> 1648
King Charles murdered	- -	A. D.	1648

1714 — 54 = 1660 — 1648 = 12 years, the answer.

34. Arphaxad was born to Shem two years after the Deluge, and 500 years before his father's death; but at 35 years of age he had Seth, who at 30 was father to Eber; who at 34 had Peleg, and he lived 430 years after that. The question is, whether Shem or Eber died the first; and
 at

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at nineſcore and fourteen years after the death of the longer liver, what interval might be wanting to complete the term of 1000 years after the Flood?

$$\begin{array}{r} \text{Selah } 35 + \text{Eber } 30 - - - - - = 65 \\ \text{Eber had Peleg at } 34, \text{ lived after } 430 - - = \underline{464} \end{array}$$

$$\begin{array}{r} \text{Shem died after the birth of Arphaxad} - - = 529 \\ \underline{500} \end{array}$$

$$\begin{array}{r} \text{Eber was ſurvivor} - - - - - = 29 \end{array}$$

$$1000 - 502 + 29 + 194 = 275, \text{ interval required.}$$

35. B was born 14 years after C, who came into the world 19 years before A, who was 23 years of age eight years ago. What then is the age of D, who is within 22 years of being as old as thoſe three together?

$$\begin{array}{r} 23 + 8 = 31 \text{ A } \\ 19 + 31 = 50 \text{ C } \\ 50 - 14 = 36 \text{ B } \end{array} \left. \vphantom{\begin{array}{l} 23 \\ 19 \\ 50 \end{array}} \right\} \begin{array}{r} 117 \\ - 22 \\ \hline 95 \end{array} \text{ the answer.}$$

36. Of the noble family of Cornaro, the grandfire's age was 134 years; and he was 93 years older than the ſon, at the time when the ſon and father's age together made 112 years. Diſtinguiſh their ages?

$$\begin{array}{r} \text{Grandfire's age} - - = 134 \\ - 93 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Son's} - - - - - = \underline{41} \end{array}$$

$$112 - 41 = 71, \text{ father's age.}$$

37. K is 19 years older than L, who was 27 years of age in the South-ſea year 1720. How old is M, in 1740, who in the year 1738 was within 24 years of being as old as both of them together?

$$19 + 27 = 46 \text{ K's age in } 1720.$$

$$1738 - 1720 = 18.$$

$$\begin{array}{r} 46 + 18 = 64 \text{ K's } \\ 27 + 18 = 45 \text{ L's } \end{array} \left. \vphantom{\begin{array}{l} 46 \\ 27 \end{array}} \right\} \text{ age in } 1738.$$

$$\begin{array}{r} 64 + 45 = 109 + 2 = 111. \\ 111 - 24 = 87, \text{ the answer.} \end{array}$$

38. If

38. If Sampson was born 17 years after Timothy, and Timothy 26 years before Jacob, who 28 years hence will be just 50. In what year of Christ were they severally born, the question being proposed anno 1758?

$$1758 - 50 = 1708.$$

$$1708 + 28 = 1736 \text{ Jacob.}$$

$$1736 - 26 = 1710 \text{ Timothy.}$$

$$1710 + 17 = 1727 \text{ Sampson.}$$

39. A, born anno Christi 318, lived 207 years before B, who lived 104 years after C, who was successor to D 84 years. E was also 112 years after D, but predecessor to F, by 47 years. In what year of Christ did each of those gentlemen flourish?

$$318 + 207 = 525 \text{ B}$$

$$525 - 104 = 421 \text{ C}$$

$$421 - 84 = 337 \text{ D}$$

$$337 + 112 = 449 \text{ E}$$

$$449 + 47 = 496 \text{ F}$$

} flourished.

40. A was born when B was 18 years of age. How old shall A be, when B is 41; and what will be the age of B, when A is 72?

$$41 - 18 = 23 \text{ A.} \quad - \quad - \quad - \quad 72 + 18 = 90 \text{ B.}$$

41. B, born anno 1108, lived 48 years before C, who was 113 years senior to D; and X was 114 years before Y, who was 74 years after Z, born anno 1527. In what years of Christ were these men severally born?

$$\begin{array}{rcl} \text{B born} & - & - & - & - & - & - & - & \text{A. D.} & 1108 \\ & & & & & & & & + & 48 \\ & & & & & & & & \hline \end{array}$$

$$\begin{array}{rcl} \text{C born} & - & - & - & - & - & - & - & \text{A. D.} & 1156 \\ & & & & & & & & + & 113 \\ & & & & & & & & \hline \end{array}$$

$$\begin{array}{rcl} \text{D born} & - & - & - & - & - & - & - & \text{A. D.} & 1269 \\ & & & & & & & & \hline \end{array}$$

$$\begin{array}{rcl} \text{Z born} & - & - & - & - & - & - & - & \text{A. D.} & 1527 \\ & & & & & & & & + & 74 \\ & & & & & & & & \hline \end{array}$$

$$\begin{array}{rcl} \text{Y born} & - & - & - & - & - & - & - & \text{A. D.} & 1601 \\ & & & & & & & & - & 114 \\ & & & & & & & & \hline \end{array}$$

$$\begin{array}{rcl} \text{X born} & - & - & - & - & - & - & - & \text{A. D.} & 1487 \\ & & & & & & & & \hline \end{array}$$

42. A, born 445 years before the year 1733, died anno 1362. B born 37 years ago, will die 18 years hence. C, born 256 years ago, died 197 years since. D, born anno 1578,

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1578, lived till within 75 years of the said 1733. The length of those people's lives is severally required?

$$\begin{array}{rcl}
 1733 - 445 & = & 1288 \text{ A born.} \\
 1362 - 1288 & = & 74 \text{ his age.} \\
 37 + 18 & = & 55 \text{ B's } \} \text{ age.} \\
 256 - 197 & = & 59 \text{ C's } \} \\
 1733 - 75 & = & 1658 \text{ D born.} \\
 1658 - 1578 & = & 80 \text{ his age.}
 \end{array}$$

43. If I am 42 years older than you now, what will be the difference of our ages 14 years after my decease, in case you shall then survive?

$$42 - 14 = 28, \text{ the answer.}$$

44. A, born anno 1441, lived till B was seven years of age; which was 23 years before the Reformation, in 1517. B survived this remarkable æra just 49 years. C, born nine years after the death of A, lived but till B was 36 years of age. The sum of the ages of these three persons is required?

$$\begin{array}{rcl}
 \text{Reformation} & - & - & - & - & \text{A. D.} & 1517 \\
 & & & & & & 23 \\
 \text{A died} & - & - & - & - & - & \text{A. D.} & 1494 \\
 \text{born} & - & - & - & - & - & \text{A. D.} & 1441 \\
 \text{A's age} & - & - & - & - & - & & 53 \\
 1517 + 49, \text{ B died} & - & - & - & - & - & \text{A. D.} & 1556 \\
 1494 - 7, \text{ B born} & - & - & - & - & - & \text{A. D.} & 1487 \\
 \text{B's age} & - & - & - & - & - & & 79 \\
 36 - 7 + 9 = \text{C's age} & - & - & - & - & - & & = 20 \\
 53 + 79 + 20 & = & 152, \text{ the answer.}
 \end{array}$$

45. A snail in getting up a May-pole, only 20 feet high, was observed to climb eight feet every day; but every night it came down again four feet. In what time by this method did he reach the top of the pole?

$$\begin{array}{l}
 20 - 8 + 4 = 16 \text{ to go the 2d. morning.} \\
 16 - 8 + 4 = 12 \text{ to go the 3d.} \\
 12 - 8 + 4 = 8 \text{ to go the 4th, and at night got to the top.}
 \end{array}$$

46. The semidiameter of the earth's orbit, or annual path round the sun in the center of the system, is about 81,000,000 miles; that of Venus 39,000,000. When they

are both on the same side the sun, they are in perigæo ; when on different sides, in apogæo. What is the difference of their distance in both these circumstances ?

$$81000000 - 59000000 = 22000000 \text{ miles in perigæo.}$$

$$81000000 + 59000000 = 140000000 \text{ in apogæo.}$$

Then $140000000 - 22000000 = 118000000$, the answer.

47. B was 14 years old, when C was 25. How old shall C be, when B comes to be 25 ?

$$25 - 14 = 11$$

$$25 + 11 = 36, \text{ the answer.}$$

48. A, born 17 years after C, and 13 before B, died 42 years before the late king's inauguration in 1727, aged 47 years. C died anno 1712, and B exactly eight years before him. D, born 23 years before C, died at 64. E, born 11 years after B's death, will die 12 years after the year 1733. And F, born just in the midway of the interval between the births of A and D, is not to reach the time of E's death by 14 years. What is the sum of all their ages, and which of them lived longest ?

	1727	
	<u>42</u>	
A. died	A. D. 1685	
	Aged <u>47</u>	
	Born 1638	
	<u>17</u>	
C born	- - 1621	
Died	- - <u>1712</u>	
	1621 -	
B died	- - 1704	
	<u>+ 11</u>	
	1715	
	<u>1733</u>	
	+ 12 = 1745	E died.
	<u>1745</u>	
	- 1715 =	30 E's age.
	<u>1638</u>	
	= 1598 =	40 it's half = 20.
	<u>1745</u>	
	- 14 = 1731	F died.
	<u>1638</u>	
	- 20 = 1618	born.
	<u>Aged 113</u>	
A	B	C
D	E	F
47	+ 53	+ 91
+ 30	+ 113	= 398 sum.
And 113	- 91	= 22 F, oldest.

49. Three-

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49. Three and thirty years before the Restoration in 1660, the crown granted demesnes, to certain uses, for 210 years then to come. The proprietor, in 1715, procured a reversionary grant of 99 years, to commence after the expiration of the first. In what year of Christ will the second term end?

Restoration - - - - -	A. D. 1660
Grant before - - - - -	— 33
First grant made - - - - -	A. D. 1627
Duration - - - - -	+ 210
End of the first grant - - -	A. D. 1837
Reversionary grant's continuance -	+ 99
Its expiration - - - - -	A. D. 1936

50. A young fellow owed his guardian 74l. 18s. 2d. on balance. He paid off 41l. 14s. 8d. and then declared his sister owed the gentleman half as much again as himself: on hearing this, she paid off in a pet 13l. 12s. 10d. and gives out that her uncle William was not then less in arrear than her brother and she together. The uncle hereupon pays 24l. 7s. 3d. And then the uncle's brother, who, by the bye, was not the uncle of those children, for 150l. undertakes to set them all clear, and has 35l. 15s. 5d. he says, to spare. Can that be true?

	l.	s.	d.
Brother debtor to guardian at the first - -	74	18	2
Paid - - - - -	— 41	14	8
Remains debtor - - - - -	33	3	6
	+ 16	11	9
Sister debtor at first - - - - -	49	15	3
Paid - - - - -	— 13	12	10
Sister remains debtor - - - - -	36	2	5
	+ 33	3	6
Uncle William debtor at first - - - - -	69	5	11
Paid - - - - -	— 24	7	3
Remains debtor - - - - -	44	18	8

Then 33l. 3s. 6d. + 36l. 2s. 5d. + 44l. 18s. 8d. = 114l. 4s. 7d.
 ∴ 150l. — 114l. 4s. 7d. = 35l. 15s. 5d. as was proposed.
 D 51. Five

51. Five notable discoveries were made in 215 years time; viz. 1st. The invention of the compass. 2d. Gunpowder. 3d. Printing. 4th. The discovery of America. 5th. Truth in the Reformation. The last was brought about anno 1517: the 3d 77 years before: the 2d 42 years after the 1st: and the 4th 148 years after the 2d. The question is, in what year of Christ did each of these happen to be found?

The Reformation	- - - - -	A. D. 1517
		<u>- 215</u>
Invention of the compass	- - -	A. D. 1302
		<u>+ 42</u>
Gunpowder	- - - - -	A. D. 1344
		<u>+ 148</u>
America discovered	- - - - -	A. D. 1492
		<u>1517</u>
		<u>- 77</u>
Printing invented	- - - - -	A. D. 1440
		<u>1517</u>
		<u>- 77</u>



S E C T. V.

M U L T I P L I C A T I O N.

MULTIPLICATION is a rule, by which the greater of two numbers may be speedily increased as often as there are units in the lesser, and in a concise manner performs the office of addition.

In every operation in multiplication, are two given numbers, called factors; viz.

First, The multiplicand, or number to be multiplied, which is generally the greater of the two.

Secondly, The multiplier or multiplicator, or number by which we multiply, which denotes the number of times the multiplicand is increased by, or added to itself; and from thence will arise a third number, called the product.

This

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This in geometrical operations is called the RECTANGLE, or PLAIN.

Let 7 Multiplicand } Factors.
5 Multiplier ;

By addition

35 product.

$$\begin{array}{r} 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ \hline 35 \text{ sum.} \end{array}$$

MULTIPLICATION TABLE.

2 X 2 = 4	4 X 7 = 28	7 X 9 = 63
2 X 3 = 6	4 X 8 = 32	7 X 10 = 70
2 X 4 = 8	4 X 9 = 36	7 X 11 = 77
2 X 5 = 10	4 X 10 = 40	7 X 12 = 84
2 X 6 = 12	4 X 11 = 44	
2 X 7 = 14	4 X 12 = 48	
2 X 8 = 16		8 X 8 = 64
2 X 9 = 18	5 X 5 = 25	8 X 9 = 72
2 X 10 = 20	5 X 6 = 30	8 X 10 = 80
2 X 11 = 22	5 X 7 = 35	8 X 11 = 88
2 X 12 = 24	5 X 8 = 40	8 X 12 = 96
	5 X 9 = 45	
3 X 3 = 9	5 X 10 = 50	9 X 9 = 81
3 X 4 = 12	5 X 11 = 55	9 X 10 = 90
3 X 5 = 15	5 X 12 = 60	9 X 11 = 99
3 X 6 = 18		9 X 12 = 108
3 X 7 = 21	6 X 6 = 36	
3 X 8 = 24	6 X 7 = 42	10 X 10 = 100
3 X 9 = 27	6 X 8 = 48	10 X 11 = 110
3 X 10 = 30	6 X 9 = 54	10 X 12 = 120
3 X 11 = 33	6 X 10 = 60	
3 X 12 = 36	6 X 11 = 66	11 X 11 = 121
	6 X 12 = 72	11 X 12 = 132
4 X 4 = 16		
4 X 5 = 20	7 X 7 = 49	
4 X 6 = 24	7 X 8 = 56	12 X 12 = 144

N. B. This table is to be perfectly learned by heart, so as to be readily remembered without pausing.

Then multiplication may be easily performed, observing the following

D 2

R U L E.

R U L E.

Always begin with that figure which stands in the units place of the multiplier, and with it multiply the figure which stands in the units place of the multiplicand; if their product be less than ten, set it down underneath its own place of units, and proceed to the next figure of the multiplicand. But if their product be above ten (or tens) then set down the overplus only (or odd figures, as in addition) and bear (or carry) the said ten (or tens) in mind, until you have multiplied the next figure of the multiplicand with the same figure of the multiplier; then to their product add the ten or tens beared in mind, setting down the overplus of their sum above the tens, as before; and so proceed in the very same manner, until all the figures of the multiplicand are multiplied with that figure of the multiplier.

$$\begin{array}{r} 394786 \\ \underline{7} \end{array}$$

$$\underline{2763502}$$

$$\begin{array}{r} 8643597 \\ \underline{9} \end{array}$$

$$\underline{77792373}$$

$$\begin{array}{r} 796534289 \\ \underline{11} \end{array}$$

$$\underline{8761877179}$$

2. When the multiplier is any number between 12 and 20; multiply by the figure in the units place; and as you multiply, add to the product of each single figure, that of the multiplicand, which stands next on the right-hand.

$$\begin{array}{r} 4721217 \\ \underline{15} \end{array}$$

$$\underline{70818255}$$

$$\begin{array}{r} 4713176 \\ \underline{16} \end{array}$$

$$\underline{75410816}$$

$$\begin{array}{r} 94713761 \\ \underline{18} \end{array}$$

$$\underline{1704847698}$$

$$\begin{array}{r} 12345 \\ \underline{13} \end{array}$$

$$\underline{160485}$$

$$\begin{array}{r} 72453 \\ \underline{17} \end{array}$$

$$\underline{1231701}$$

$$\begin{array}{r} 6729004 \\ \underline{19} \end{array}$$

$$\underline{127851076}$$

3. But when the multiplier consists of several figures, the multiplicand must be multiplied with every single figure of the multiplier; always placing the first figure, or cypher, of every particular product, directly underneath that figure of the multiplier you then multiply with.

$$4739284$$

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4739284 94785	6247386495 27356
23696420	37484318970
37914272	31236932475
33174988	18742159485
18957136	43731705465
42653556	12494772990
449213033940	170903504957220

4. If there be a cypher, or cyphers, intermixed with the figures, move for every figure, or cypher, one place toward the left-hand, and take care that every first figure of the several products stand directly under its respective multiplier.

630700325 6072008	50710984 4050607
5045602600	354976888
1261400650	304265904
4414902275	253554920
3784201950	202843936
3829617419002600	205410266767288

5. Cyphers placed at the end of either or both factors, are to be omitted till the last product, and then the number of cyphers as are at the end of both must be annexed to it.

42600 2200	429000 5600
852	2574
852	2145
93720000	2402400000

6. Any number given, being multiplied by 1, undergoes no alteration; but if by 10, a cypher is to be annexed; if by 100, annex two cyphers; by 1000, annex three, &c.

$$7157 \times \left\{ \begin{array}{l} - - - I = - - - - - 7157 \\ - 10 = - - - - - 71570 \\ - 100 = - - - - - 715700 \\ 1000 = - - - - - 7157000 \\ 10000 = - - - - - 71570000 \\ 100000 = - - - - - 715700000 \end{array} \right\} \text{And thus for as many cyphers as you please.}$$

7. In geometrical progressions, converging series, &c, when multiplications have been very operose, I have frequently added, subtracted, or divided; or multiplied a product by a smaller, when the former happens to be a multiple of the latter; as I shall endeavour to explain in the example following.

$$\begin{array}{r} 8496427 \\ 874359 \\ \hline 76467843 \end{array} \left\{ \begin{array}{l} \text{By subtracting the right-hand figure from a} \\ \text{cypher, and each preceding figure from} \\ \text{that following.} \end{array} \right.$$

42482135 - By dividing the multiplicand by 2.
 25489281 - By dividing the product of 9 by 3.
 33985708 - - By add. the last prod. to the multiplicand,
 59474989 - - By adding the two last products together.
 67971416 - - - By multiplying the product of 4 by 2.

7428927415293

But before the learner attempts to perform operations by this method, he ought to be acquainted with division.

8. If the multiplier be any number near 100, 1000, 10000, &c. increase the multiplicand by as many cyphers as there are figures in the multiplier; and subtract the multiplicand from itself thus increased, as often as the multiplier wants units of that by which the multiplicand was increased.

$$\begin{array}{r} \text{Let } 7943628 \times 999 \quad \text{And } 4372845 \times 9997 \\ 7943628000 \\ 7943628 \\ \hline 7935684372 \end{array} \quad \begin{array}{r} 43728450000 \\ 13118535 = \text{multiplic.} \times 3. \\ \hline 43715331465 \end{array}$$

9. If the multiplier be a repetend of the same figure, multiply by one of the repeating figures; and the figures of that

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that product added, as if they had been wrote down in as many products as the multiplier repeated the same figure, give the product required.

$$\begin{array}{r} 547856789 \\ 22222 \\ \hline 1095713578 \\ \hline 12174473565158 \end{array}$$

$$\begin{array}{r} 54018 \\ 3333 \\ \hline 162054 \\ \hline 180041994 \end{array}$$

10. When the repeating figure is a high digit, collect the product of as many ones as there are digits in the multiplier, from the multiplicand, according to the rule in the last contraction; which product being multiplied into the repetend, will give the true product.

$$\begin{array}{r} 784325634 \text{ into } 7777777. \\ \hline 871472839519374 \text{ Products collected for } \text{IIIIIIII.} \\ 7 \\ \hline 6100309876635618 \text{ Product of } 7777777. \end{array}$$

11. Find the product of the given multiplicand by the like number of nines, and divide that product by 9; the quotient multiplied by the digit which repeats in the given multiplier, will be the product required.

Ex. Let 4538769 be multiplied by 7777777.

$$\begin{array}{r} 45387690000000 \\ 4538769 \\ \hline 9) 45387685461231 \\ \hline 5043076162359 \\ \times 7 \\ \hline 35301533136513 \end{array}$$

N. B. Division must be learned before examples of this kind be attempted.

12. When the multiplier can be parted into periods, which are multiples of one another, the operation may be contracted in the following manner.

$$\begin{array}{r}
 8649347864 \\
 1325769612 \\
 \hline
 103792174368 \text{ product of } 12. \\
 830337394944 \text{ - foregoing product } \times 8. \\
 4982024369664 \text{ - - last product } \times 6. \\
 1141713918048 \text{ - - - first product } \times 11. \\
 \hline
 11467042561708308768
 \end{array}$$

13. To multiply by a factor, consisting of as many cyphers between two digits as there are places in the multiplicand, multiply by a single digit; and the product by the second figure will fall directly to the left-hand of the product by the first figure; but if the product of the first figure be less than 10, then a cypher must be put down between the two products.

$$\begin{array}{r}
 84629 \\
 7000003 \\
 \hline
 592403253887
 \end{array}$$

14. The proof of multiplication, is by making the multiplicand to be the multiplier; then if the product comes out the same as before, your work is right.

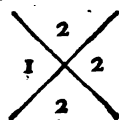
15. Or by casting away the nines, which, though not infallible, serves to confirm the other. Thus, in the last example, make a cross, and add all the figures, or digits, of the multiplicand together, as units, thus, $8 + 4 + 6 + 2 + 9 = 29$; cast away the nines, and set the remainder two on one side the cross. Do the same with the multiplier $7 + 3 = 10$; set the remainder 1 on the other side the cross. Do the like by the product, and set the remainder at top. Lastly, multiply the figures on the sides, and set the remainder at the bottom, after the nines (if any) are cast away; which must be the same with the top, if the work is right.

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QUESTIONS to exercise MULTIPLICATION.

1. A is 17, B 7; what will their ages severally be, when the elder is double the age of the younger?



A 17
B 7

10 A's age when B was born.
 $\times 2$

20 A's age, the answer.

2. Trajan's bridge over the Danube is said to have had 20 piers to support the arches, every pier being 60 feet thick, and some of them were 150 feet above the bed of the river; they were also 170 feet asunder: pray what was the width of the river in that place, and how much did it exceed the length of Westminster-bridge, which is about 1200 feet from shore to shore; and is supported by 11 piers, making the number of arches 12?

Arches - $21 \times 170 = 3570$

Piers - $20 \times 60 = 1200$

Width of the Danube 4770
of the Thames - 1200

Difference - 3570 the answer.

3. By God's blessings upon a merchant's industry, in ten years time he found himself possessed of 13000 l. It appeared from his books, that the last three years he had cleared 873 l. a year; the three preceding, but 586 l. a year; and before that, but 364 l. a year. The question is, what was the state of his fortune at every year's end that he continued in trade, and consequently what had he to begin with?

Merchant's whole stock = £ 13000

Gain per annum $364 \times 4 =$ - 1456

Ditto - $586 \times 3 =$ - 1758

Ditto - $873 \times 3 =$ - 2619

Whole gain - = £ 5833

Original stock - = £ 7167

4. What

4. What difference is there between twice eight-and-twenty, and twice twenty-eight; as also between twice five-and-fifty, and twice fifty-five?

$$\begin{array}{r} 28 \times 2 = 56 \\ 2 \times 8 + 20 = 36 \end{array}$$

Answer 20 difference

$$\begin{array}{r} \text{Also } 55 \times 2 = 110 \\ 2 \times 5 + 50 = 60 \end{array}$$

Answer 50 difference.

5. What number taken from the square of 54, will leave 19 times 46?

$$\begin{array}{r} 54 \\ \times 54 \\ \hline 216 \\ 270 \\ \hline 2916 \end{array}$$

$$\begin{array}{r} 46 \\ \times 19 \\ \hline 414 \\ 46 \\ \hline 874 \end{array}$$

874 = 2042, the answer.

6. The remainder of a division sum is 423; the quotient 423; the divisor is the sum of both, and 19 more. What then was the number to be divided?

$$\begin{array}{r} 423 \\ 423 \\ 19 \\ \hline 865 \text{ divisor} \\ \times 423 \end{array}$$

$$\begin{array}{r} 2595 \\ 1730 \\ 3460 \\ \hline \end{array}$$

$$\begin{array}{r} 365895 \\ + 423 \\ \hline \end{array}$$

366318 the answer.

7. There

Chap. I. MULTIPLICATION. 43

7. There are two numbers; the biggest of them is 73 times 109; and their difference 17 times 28; I demand their sum and product?

$$109 \times 73 = 7957 \text{ the greater number;}$$

$$28 \times 17 = 476$$

7481 lesser number.

$$7957 + 7481 = 15438 \text{ their sum.}$$

$$7957 \times 7481 = 59526317 \text{ their product.}$$

8. There are two numbers, the less is 187, the difference 34; give the square of their product, ditto of their sum and difference, and the sum of those squares.

$$187 + 34 = 221 \text{ greatest; then } 221 \times 187 = 41327 \text{ product.}$$

$$41327 \times 41327 = 1707920929 \text{ square of their product.}$$

$$221 + 187 = 408; \text{ and } 408 \times 408 = 166464 \text{ square of their sum.}$$

$$221 - 187 = 34; \text{ and } 34 \times 34 = 1156 \text{ square of their differ.}$$

$$\text{Lastly, } 1707920929 + 166464 + 1156 = 1708088549 \text{ answ.}$$

9. A person dying left his widow the use of 5000 l. To a charity he bequeathed 846 l. 10 s. To each of his three nephews 1230 l. To each of his four nieces 1050 l. To twenty poor housekeepers five guineas each; and 200 guineas to his executors. What must he have died possessed off?

	l.	s.	d.
To his widow - - -	5000	-	-
To a charity - - -	846	10	-
To nephews 1230×3 -	3690	-	-
To nieces 1050×4 -	4200	-	-
To 20 poor housekeepers	105	-	-
To executors - - -	210	-	-
	<u>14051</u>	<u>10</u>	<u>-</u>

10. In the partition of lands in an American settlement, A had 757 acres allotted to him, B had 2104 acres, C 16410, D 12881, E 11008, F 9813, H 13800, and I 8818 acres; now how many acres did the settlement contain, since the allotments made above want 416 acres of $\frac{1}{5}$ th of the whole?

$$\text{First } 757 + 2104 + 16410 + 12881 + 11008 + 9813 + 13800 + 8818 + 416 = 76007.$$

$$\text{Then } 76007 \times 5 = 380035, \text{ the answer.}$$

S E C T.

S E C T. VI.

D I V I S I O N.

DIVISION is a rule by which we speedily discover how often one *number* is contained, or may be found in another; or by which any number may be decreased; or divided into as many parts as there are units in the number you divide by.

To perform division, two numbers are always given.

I. The dividend, or number to be divided.

II. The divisor, or number by which the said dividend is to be divided.

And from thence will arise a third, called the quotient, which shews how often the divisor is contained in the dividend.

Lastly, If the divisor doth not exactly measure the dividend, a fourth number occurs, called the remainder; which is always less than the divisor, and consequently a fractional part of the quotient.

Division by a single figure, or not exceeding 12 in the divisor, is performed by the following

R U L E.

First, observe how often the divisor is contained in the first figure of the dividend (or in case the first figure of the dividend be less than the divisor, in the two first figures) and set the quotient figure under it; and if any thing remains, carry it to the next figure in the dividend, where it must be reckoned as so many tens; and so on, bearing the remainder of each figure to the next in your mind, until you have finished your operation.

Divisor	2	Dividend.			
		5738473		11	8579475321079
Quotient		2869236	1 rem.		779952301916 - 3
	5	18647279		12	2157963054731
		3729455	4		179839254894 - 3

2. But when the divisor consists of many places, distinguish by a point so many of the foremost places of the dividend

dividend towards the left-hand, as are either equal to the divisor, or else being greater, it comes nearest to it; then consider how often the divisor is contained in this first period of the dividend, and assume that number for a quotient, which multiply into the divisor; and whenever it proves greater than the dividend, strike that figure out, and put a less in the quotient: then subscribe the product of the quotient figure into the divisor, under the dividend, and draw a line under it; subtract it therefrom, subscribing the remainder under the line: then prick and bring down another figure, proceeding, as before, till your division be finished; always observing, that for every figure or cypher you bring down, you put a figure or cypher in the quotient.

EXAMPLE.

59157) 252070573915549 (4261043898
 236628.....

• 154425
 118314

• 361117
 354942

• 61753
 59157

• 259691
 236628

• 230635
 177471

• 531645
 473256

• 583894
 532413

• 514819
 473256

41563

3. Many figures may be saved, if you work by the short Italian method; that is, omit setting down your multiplications, and multiply and subtract together; always remembering to carry to the next figure as many as you borrowed.

$$\begin{array}{r}
 873469 \) \ 432756284563574 \ (\ 495445498 \\
 \underline{8336868} \\
 4756474 \\
 \underline{3891295} \\
 3974196 \\
 \underline{4803203} \\
 4358583 \\
 \underline{8647097} \\
 7858764 \\
 \hline
 871012
 \end{array}$$

4. When the divisor consists of several cyphers after a figure, or figures, cut them all off by a dash of your pen underneath them; and also cut off as many cyphers, or figures, in the dividend; but when division is finished, bring down the cyphers, or figures, cut off from the dividend, to the remainder.

$$\begin{array}{r}
 35000 \) \ 29628754963 \ (\ 846535 \\
 \underline{29628} \\
 754963 \\
 \underline{75496} \\
 3 \\
 \hline
 29963
 \end{array}$$

5. As unity, or 1, neither multiplies or divides, any number may be speedily divided by 10, 100, 1000, by only cutting off by a comma so many figures to the right-hand of the dividend, as there are cyphers in the divisor; those to the left-hand bring the quotient, and those to the right the remainder.

	Dividend.	Quotient.	Rem.
10	905672417	90567241	- 7
100	905672417	9056724	- 17
1000	905672417	905672	- 417
10000	905672417	90567	- 2417

6. To

6. To divide by any number consisting of nines; viz. 9, 99, 999, &c. This may be performed by addition, as the multiplying by those numbers was by subtraction.

R U L E.

Divide the given dividend into periods of as many places of figures as there are nines in the divisor, beginning from the left-hand; and annex as many cyphers to the right-hand of the number, as may be wanted to complete a period. Then write the figures of the left hand period under those of the second period, which is next thereto, towards the right-hand; add these two together, and place their sum under the third period; observing if the sum of the two figures in the highest place exceed nine, to place the figure that would (in common addition) be carried under the lowest place of the second period. Add the third period to those figures which stand under it, concluding the carried figure; and place them under the fourth period; and so proceed till you have placed figures under the right-hand period; and under them place such a figure as would have been there placed, had the work proceeded a period further. Then add the whole together; and beginning at the right-hand, cancel as many figures as there were cyphers annexed to the dividend; and from the figures that remain, cut off from the right-hand as many figures as the divisor contained nines; so shall the figures to the left be the quotient; and those cut off the remainder; only if the remainder be all nines, add one to the quotient.

Let 8765806137663 be divided by 9999.

The N^o with three cyphers annexed 8765,8061,3766,3000
8765,6826,0592
1. 2.

By addition ariseth this N^o - - 8766682805943,592

From which the three last figures }
being left out for the three cy- } 876668280,5943
phers annexed to the dividend }

These properly separated - - - 876668280 the quotient.
And - - 5943 remainder.

7. But if your divisor be 11,111,1111, &c. 22,222,2222, &c. or 33,333,3333, &c. divide the given dividend by the digit, which repeats in the divisor, and multiply the quotient

quotient by 9; then divide the product by 99,999,9999, and the result will be the quotient required.

Let 222671883 be divided by 777.

$$\begin{array}{r} 7 \overline{) 222671883} \\ \hline \end{array}$$

$$31810269$$

$$\times 9$$

$$\hline 286,292421$$

$$286,578$$

$$1$$

$$\hline 286579,000$$

Therefore 286579 is the quotient required.

8. If the divisor be large, and a quotient of many figures be required, as in resolving of high equations, and calculating astronomical tables, or those of interest, under the divisor set down its double; to this double add the divisor, setting down their sum against the figure 3; and proceed on by a continual addition, until there be ten times the divisor in the table; which, if true, will be the divisor itself, with a cypher to the right-hand of it.

Let it be required to divide 70251807402 by 79863.

$$\begin{array}{r|l} 1 & 79863 \\ 2 & 159726 \\ 3 & 239589 \\ 4 & 319452 \\ 5 & 399315 \\ 6 & 479178 \\ 7 & 559041 \\ 8 & 638904 \\ 9 & 718767 \\ \hline & 798630 \end{array} \quad \begin{array}{l}) 70251807402 \\ \quad 638904 \dots\dots \\ \hline \quad \quad 636140 \\ \quad \quad 559041 \\ \hline \quad \quad \quad 770997 \\ \quad \quad \quad 718767 \\ \hline \quad \quad \quad \quad 522304 \\ \quad \quad \quad \quad 479178 \\ \hline \quad \quad \quad \quad \quad 431260 \\ \quad \quad \quad \quad \quad 399315 \\ \hline \quad \quad \quad \quad \quad \quad 319452 \\ \quad \quad \quad \quad \quad \quad 319452 \\ \hline \quad \quad \quad \quad \quad \quad \quad 0 \end{array}$$

Chap. I. DIVISION.

49

9. Division and multiplication interchangeably prove each other; for in division, if you multiply the divisor by the quotient, and to the product add the remainder (if any) their sum will be the dividend. So to prove multiplication, if the product be divided by the multiplier, the quotient will be the multiplicand; or if the product be divided by the multiplicand, the quotient will be the multiplier.

10. Or cast away the nines in the divisor, and quotient, and set the remainders on the sides of a cross. Do the same with your dividend, and set the remainder at top. Multiply the figures on the sides, cast away the nines, and set the remainder at the bottom, which must be equal to the top. Note, If there be a remainder, it must be added to the product, on the sides of the cross, and the nines thrown out as before.

QUESTIONS performed by Division in conjunction with the rest of the foregoing general rules.

1. What is the difference, and what the sum of six dozen dozen, and half a dozen dozen?

$$12 \times 12 \times 6 = 864 = 6 \text{ dozen dozen.}$$

$$12 \times 12 = 144 \div 2 = 72 = \frac{1}{2} \text{ dozen dozen.}$$

936 sum.

792 difference.

2. Subtract 30079 out of fourscore and thirteen millions as often as it can be found, and say what the last remainder exceeds, or falls short of 21180?

$$30079 \text{) } 93000000 \text{ (} 3091$$

276300

53890

Rem. 25811

— 21180

4631, the answer.

E

3. What

3. What number added to the forty-third part of 4429, will make the sum 240?

$$\begin{array}{r} 43 \overline{) 4429} \text{ (} 103 \\ \text{Then } 240 - 103 = 137, \text{ the answer.} \end{array}$$

4. What number deducted from the 26th part of 2262, will leave the 87th part of the same?

$$\begin{array}{r} 26 \overline{) 2262} \text{ (} 87 \\ \underline{- 26} \\ \hline 61, \text{ the answer.} \end{array}$$

5. What number multiplied by 72084, will produce 5190048 exactly?

$$72084 \overline{) 5190048} \text{ (} 72, \text{ the answer.}$$

6. What number divided by 419844, will quote 9494, and leave just a third part of the divisor remaining?

$$\begin{array}{r} 3 \overline{) 419844} \\ \underline{139948} \\ 419844 \\ \underline{9494} \\ 1679376 \\ 3778596 \\ \underline{1679376} \\ 3778596 \\ \underline{3985998936} \\ 139948 \\ \hline 3986138884, \text{ the answer.} \end{array}$$

7. The sum of two numbers is 360; the less is 114; what is their difference, product, and larger quote?

$$\begin{array}{r} 360 \\ - 114 \\ \hline 246 \end{array}$$

$$246 - 114 = 132 \text{ difference.}$$

$$246 \times 114 = 28044 \text{ product.}$$

$$114 \overline{) 246} (= 2\frac{2}{3}, \text{ quotient; viz. } 6 \overline{) 114} (= 19\frac{2}{3}.$$

Chap. I. D I V I S I O N.

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8. I would plant 2072 elms in 14 rows, 25 feet asunder; how long will this grove be?

$$\begin{array}{r} \text{diff.} \\ 14 \) \ 2072 \ (\ 148 \text{ in each row.} \quad 148 - 1 = 147. \\ 147 \times 25 = 3675 \text{ feet} = 1225 \text{ yards.} \end{array}$$

9. A brigade of horse, consisting of 384 men, is to be formed into a square body, having 32 men in front; how many ranks will there be?

$$32 \) \ 384 \ (\ 12, \text{ the answer.}$$

10. What number is that, from which if you deduct the 25th part of 22525, and to the remainder add the 16th part of 9696, the sum will be 1440?

$$\begin{array}{r} 25 \) \ 22525 \ (\ 901 \\ 16 \) \ 9696 \ (\ 606 \\ 1440 + 901 - 606 = 1735, \text{ the answer.} \end{array}$$

11. There are two numbers, whose product is 1610; the greater is given 46: what is their sum, difference, and quotes; what the sum of their squares, and what the cube of their difference?

$$\begin{array}{l} 46 \) \ 1610 \ (\ 35 \text{ lesser.} \\ 46 + 35 = 81 \text{ their sum.} \\ 46 - 35 = 11 \text{ difference.} \\ 35 \) \ 46 \ (\ = 1\frac{1}{5} \text{ quotient.} \\ 46 \times 46 = 2116 \\ 35 \times 35 = 1225 \end{array}$$

$$\underline{3341}, \text{ sum of their squares.}$$

$$11 \times 11 \times 11 = 1331, \text{ cube of their difference.}$$

12. What number multiplied by 57, will produce just what 134 multiplied by 71 will do?

$$\begin{array}{r} 134 \times 71 = 9514 \\ 57 \) \ 9514 \ (\ 166\frac{2}{3}, \text{ the answer.} \\ \underline{381} \\ 394 \\ \underline{52} \end{array}$$

13. There are other two numbers, the greater 7050, which divided by the less, quotes 94; what is the difference of their squares, and what the square of the product of their sum and difference?

$$\begin{array}{rcl} & 94) 7050 (75 \text{ lesser.} \\ 7050 \times 7050 & = & 49702500 \text{ square of the greater.} \\ 75 \times 75 & = & \underline{5625} \text{ square of the lesser.} \end{array}$$

$$\begin{array}{l} 49696875 \text{ diff. of their squares.} \\ 7050 + 75 = 7125 \text{ sum} \dots 7050 - 75 = 6975 \text{ diff.} \\ 7125 \times 6975 = 49696875 \text{ prod. of their sum and diff.} \\ 49696875 \times 49696875 = 2469779384765625, \text{ answer.} \end{array}$$

14. Six of the female cricketers, that played lately in the Artillery ground, fetched in company strokes as follow; viz. ABCDE 207, ACDEF 213, ADEFB 189, AEBCF 234, ABCDF 222, BFDCE 250: How many did they fetch on the other side, since these six persons wanted but fourscore and thirteen notches to decide the game?

$$\begin{array}{l} 207 + 213 + 189 + 234 + 222 + 250 = 1315. \\ \text{They being each mentioned five times - 5) } 1315 (263. \\ \text{Then } 263 + 93 = 356; \\ \text{And } 356 - 1 = 355, \text{ the answer.} \end{array}$$

15. In order to raise a joint stock of 10000l. L, M and N together subscribe 8500l. and O the rest. Now M and N are known together to have set their hands to 6050l.; and N has been heard to say, that he had undertaken for 420l. more than M. What did each proprietor advance?

$$\begin{array}{rcl} \text{First } 6050 - 420 & = & 5630 \\ 8500 - 6050 & = & 2450 \text{ L's} \\ 2) 5630 & = & 2815 \text{ M's} \\ 2815 + 420 & = & 3235 \text{ N's} \\ 10000 - 8500 & = & 1500 \text{ O's} \end{array} \left. \vphantom{\begin{array}{rcl} 8500 - 6050 & = & 2450 \text{ L's} \\ 2) 5630 & = & 2815 \text{ M's} \\ 2815 + 420 & = & 3235 \text{ N's} \\ 10000 - 8500 & = & 1500 \text{ O's} \end{array}} \right\} \text{ subscription.}$$

$$\underline{\underline{\pounds 10000.}}$$

16. One of the smarts in the accomptant's office making his addressee in an old lady's family, who had five fine daughters;

daughters; she told him their father had made a whimsical will, which might not soon be settled in Chancery, and till then he must refrain his visit. The young gentleman undertook to unravel the will, which imported, That the first four of the girls fortunes were together to make 25000 l.; the four last 33000 l.; the three last, with the first, 30000 l.; the three first, with the last, were to make 28000 l.; and the two last, and two first, 32000 l. Now, sir, if you can make appear what each is to have, and as you like, seemingly, my third daughter, Charlotte, who I am sure will make you a good wife, and you are welcome; what was Miss Charlotte's fortune?

$$25000 + 33000 + 30000 + 28000 + 32000 = 148000;$$

each be mentioned times 4) 148000 (37000.

Then $37000 - 25000 = 12000$ youngest.

$37000 - 33000 = 4000$ eldest.

$37000 - 30000 = 7000$ second.

$37000 - 28000 = 9000$ fourth.

$37000 - 32000 = 5000$ Miss Charlotte.

17. A father dying worth 5460 l. left his wife with child, to whom he bequeathed, if she had a son, $\frac{1}{3}$ of his estate, and $\frac{2}{3}$ to the son; but if she had a daughter, $\frac{1}{3}$ to her, and $\frac{2}{3}$ to her mother. It happened that she had both a son and a daughter; how shall the estate be divided, to answer the father's intention?

It is plain that the father designed the son's fortune to be double the mother's, and that the mother should have double the daughter's fortune.

For every pound the daughter had, the mother must have two, and the son four.

Then $1 + 2 + 4 = 7$ divisor for the daughter's portion.

$$\begin{array}{l} 7 \overline{) 5460} \text{ (780 l. daughter's} \\ \text{Also } 780 \times 2 = 1560 \text{ mother's} \\ \text{And } 1560 \times 2 = 3120 \text{ son's} \end{array} \left. \vphantom{\begin{array}{l} 7 \overline{) 5460} \\ \text{Also } 780 \times 2 \\ \text{And } 1560 \times 2 \end{array}} \right\} \text{ part.}$$

18. Fair ladies of you I must yet enquire,
How the poll stood for the knights of our shire;
The number of votes, as I have seen,
Were five thousand, two hundred, and nineteen;
Which among four was just so divided,
As one the second, and the third exceeded,
By twenty-two, and fourscore bating seven;
The fourth by no more then sixscore and ten:

Then how many votes has each candidate?

You need not in finding much trouble your pate.

L.. Diary.

5219

22

73

130

4) 5444 (- - 1361 first

1361 — 22 = 1339 second

1361 — 73 = 1288 third

1361 — 130 = 1231 fourth

} candidate.

5219 proof.

19. A general disposing his army into a square battle, finds he has 284 soldiers over and above; but increasing each side with one soldier, he wants 25 soldiers to fill up the square: How many soldiers had he?

Since the number of soldiers exceeds the lesser square by 284, and wants 25 to fill up the greater,

$284 + 25 = 309$, and $309 + 1 = 310$.

2) 310 (155 side of the greater square.

$155 \times 155 = 24025$.

Answer $24025 - 25 = 24000$ the number of soldiers required.

20. What number is that, which multiplied by 20, and that product divided by 6, gives 140 in the quotient?

First $140 \times 6 = 840$. And $840 \div 20 = 42$, the answer.

21. A man being 100 years of age upon his birth-day, had his three sons with him at dinner, viz. William, James, and Thomas; the Father saying to them, Well, sons, I am this day just 100 years old; the youngest, William, said, Father, my brother Thomas is four times as old as I am, and my brother James is three times as old as I am, and all our ages together are just 100 years: How old was each of the three sons?

4 Thomas.

3 James.

1 William.

8) 100 ($12\frac{1}{2}$ William's

Also $12\frac{1}{2} \times 3 = 37\frac{1}{2}$ James's

And $12\frac{1}{2} \times 4 = 50$ Thomas's

} age, the answer.

22. A man dies and leaves a legacy of 900l. to be disposed of among four of his relations, viz. A, B, C, D, in this manner; B is to have twice as much as A; C twice as much as A and B; and D to have as much and half as much as C. What must each person have?

$$\begin{array}{r}
 A \ 1 \\
 B \ 2 \\
 C \ 6 \\
 D \ 9 \\
 \hline
 \end{array}
 \begin{array}{r}
 1. \\
 18 \) \ 900 \ (\ 50 = A's \\
 50 \times 2 = 100 = B's \\
 100 \times 3 = 300 = C's \\
 300 \times 1\frac{1}{2} = 450 = D's \\
 \hline
 900
 \end{array}
 \left. \vphantom{\begin{array}{r} 1. \\ 18 \) \ 900 \ (\ 50 = A's \\ 50 \times 2 = 100 = B's \\ 100 \times 3 = 300 = C's \\ 300 \times 1\frac{1}{2} = 450 = D's \\ \hline 900 \end{array}} \right\} \text{part, the answer.}$$

23. A labourer, after 40 weeks working, lays up 28 crowns — three weeks wages, and finds that he has expended 36 crowns + 11 weeks wages. What was his weekly pay?

First $11 + 3 = 14$ weeks wages wanting.

Also $40 - 14 = 26$ weeks. And $28 + 36 = 64$ crowns.

$\therefore 32 \) \ 64 \ (2 \text{ crowns, his weekly pay. Q. E. F.}$

CHAPTER II.

Containing Tables of weights, measures, and time; with Addition, Subtraction, and Reduction thereof from one denomination to another.

SECT. I. TABLES.

TROY WEIGHT,

BY this weight are weighed jewels, gold, silver, and all liquors.

Grains.

24 = 1 pennyweight.

480 = 20 = 1 ounce.

5760 = 240 = 12 = 1 pound.

The moneyers also at the Mint subdivide a grain.

$$\text{Thus } \left\{ \begin{array}{l} 24 \text{ blanks} = 1 \text{ periot.} \\ 20 \text{ periot} = 1 \text{ droite.} \\ 24 \text{ droites} = 1 \text{ mite.} \\ 20 \text{ mites} = 1 \text{ grain.} \end{array} \right.$$

The carat is a weight which goldsmiths and jewellers use to weigh precious stones and pearls; it weighs four grains, each of which is subdivided into $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, &c.

Carat, or caract, is also the name which represents what degree of fineness gold is of; as fine gold, in its purity or perfection, is 24 carats; and standard gold, of which our coin is made, is 22 carats of fine gold, and two of alloy, (or a baser metal, as copper or silver.)

Whence we may observe, that this carat is $\frac{1}{24}$ th part of any quantity or weight.

APOTHECARIES WEIGHT.

Apothecaries compound their medicines according to the following division of an ounce Troy; but buy and sell their drugs by Averdupoise weight.

Grains.

$$20 = 1 \text{ scruple.}$$

$$60 = 3 = 1 \text{ dram.}$$

$$480 = 24 = 8 = 1 \text{ ounce.}$$

AVERDUPOISE WEIGHT.

By Averdupoise weight are weighed such commodities as are either very coarse and droffy, or subject to waste; as all kind of grocery wares; and pitch, tar, rosin, wax, tallow, soap, flax, hemp, &c. copper, tin, steel, iron, lead, &c.; also flesh, butter, cheese, salt, and most other common necessities of life.

N. B. 68 $\frac{1}{2}$ grains of barley hath been found to weigh exactly one ounce Averdupoise weight; therefore a pound containeth 10896 grains.

The pound Averdupoise is greater, but the ounce less, than those of Troy weight; the pound Averdupoise being equal to 14 oz. 11 pwt. 15 $\frac{1}{2}$ grains; and one ounce equal to 18 pwt. 5 $\frac{1}{2}$ grains Troy.

Drams.

Drains.

16 = 1 ounce.
 256 = 16 = 1 pound.
 7168 = 448 = 28 = 1 quarter.
 28672 = 1792 = 112 = 4 = 1 hundred.
 573440 = 35840 = 2240 = 80 = 20 = 1 tun.

N. B. A stone of flesh meat in London is 8 lb. Average, but in most other places 14 lb.

Alfo 28lb. of wool makes a ted in Norfolk, and in the
fouthern counties; but 30lb. in Yorkfhire, and other
northern ones.

A stone, horseman's weight, is 14 lb.

A fother, or fodder, of lead, $19\frac{1}{4}$ cwt.

LIQUID MEASURE.

As the original of Troy weight was a corn of wheat taken out of the middle of the ear, and being well dried, 32 were to make a pennyweight; so eight pound Troy weight of wheat (or 61440 grains) were enacted by several statutes to make one gallon wine measure. This gallon, by which all wines, brandies, spirits, strong-waters, mead, perry, cider, vinegar, oil, &c. are measured and sold, containeth 231 cubic inches.

WINE MEASURE.

Cub. In.						Note, 31½ gallons is a
28¾ =	I pint.					wine or vinegar barrel,
231 =	8 =	1 gallon.				and 236 gallons a ton
9702 =	336 =	42 =	1 tierce.			of sweet oil.
14553 =	504 =	63 =	1½ =	1 hoghead.		
19404 =	672 =	84 =	2 =	1¼ =	1 puncheon.	
29106 =	1008 =	126 =	3 =	2 =	1½ =	1 butt or pipe.
58212 =	2016 =	252 =	6 =	4 =	3 =	2 = 1 ton.

Note, $31\frac{1}{2}$ gallons is a wine or vinegar barrel, and 236 gallons a ton of sweet oil. hoghead.

A ton of 252 gallons, at $7\frac{1}{2}$ lb. to the gallon, weighs
1890 lb. = 16 cwt. 3 qrs. 14 lb.

The beer or ale gallon (which are both one) is much larger than the wine gallon; it being probably made at first to correspond with Averdupoise weight, as the wine gallon did with Troy weight. For one pound Averdupoise being nearly equal to 14 oz. 12 pwts. Troy; and as one pound Troy is in proportion to the cubic inches in a wine gallon,

fo is one pound Averdupoise to the cubic inches in an ale gallon, viz. $12 : 14\frac{1}{2} :: 231 : 282$ nearly.

A L E M E A S U R E,

Cub. In.

$35\frac{1}{4}$	=	1 pint.
282	=	8 = 1 gallon.
2256	=	64 = 8 = 1 firkin.
4512	=	128 = 16 = 2 = 1 kilderkin.
9024	=	256 = 32 = 4 = 2 = 1 barrel.
13536	=	384 = 48 = 6 = 3 = $1\frac{1}{2}$ = 1 hoghead.

B E E R M E A S U R E.

Cub. In.

$35\frac{1}{4}$	=	1 pint.
282	=	8 = 1 gallon.
2538	=	72 = 9 = 1 firkin.
5076	=	144 = 18 = 2 = 1 kilderkin.
10152	=	288 = 36 = 4 = 2 = 1 barrel.
15228	=	432 = 54 = 6 = 3 = $1\frac{1}{2}$ = 1 hoghead.
30456	=	864 = 108 = 12 = 6 = 3 = 2 = 1 butt.

N. B. This distinction, or difference, between ale and beer measure, is only used in London; but in all other places of England the following table of beer or ale, whether it be strong or small, is to be observed according to a statute of excise made in the year 1689.

B E E R and A L E in the country.

Cub. In.

$35\frac{1}{4}$	=	1 pint.
282	=	8 = 1 gallon.
2397	=	68 = $8\frac{1}{2}$ = 1 firkin.
4794	=	136 = 17 = 2 = 1 kilderkin.
9588	=	272 = 34 = 4 = 2 = 1 barrel.
14382	=	408 = 51 = 6 = 3 = $1\frac{1}{2}$ = 1 hoghead.

D R Y M E A S U R E.

By an act of parliament, made in 1697, it was decreed, That every round bushel with a plain and even bottom, being made eighteen inches and a half wide throughout, and eight inches deep, should be esteemed a legal Winchester

buftel, according to the ftandard in his majefty's Exchequer. Now a veffel thus made, will contain 2150,42 cube inches; confequently the corn gallon doth contain 2684 cubic inches.

Cub. In.

33.6 = 1 pint.
 268.8 = 8 = 1 gallon.
 537.6 = 16 = 2 = 1 peck.
 2150.4 = 64 = 8 = 4 = 1 bushel.
 8601.6 = 256 = 32 = 16 = 4 = 1 coom.
 17203.2 = 512 = 64 = 32 = 8 = 2 = 1 quarter.
 172032 = 5120 = 640 = 320 = 80 = 20 = 10 = 1 last.

But the farmer generally delivers to the merchant 10½ quarters of oats, coleseed, and some other grain, for a last, in consideration of waste, &c. by expertation.

The miners in Derbyshire have a vessel called an ore-dish, by which they buy and sell their lead-ore.

Its length 21.3 }
breadth 6 } inches;
depth 8.4 }

depth 8.4

consequently its contents 1073.52 cubic inches, very nearly equal to two pecks, or four corn gallons.

Nine of those dishes they call a load of ore; which, if pretty good, will produce about three hundred weight of lead.

LONG MEASURE.

Inches.

12 =	1 foot.				
36 =	3 =	1 yard.			
72 =	6 =	2 =	1 fathom.		
198 =	16½ =	5½ =	2½ =	1 pole.	
7920 =	660 =	220 =	110 =	40 =	1 furlong.
63360 =	5280 =	1760 =	880 =	320 =	8 = 1 mile.

LONG MEASURE.

The navigators, or seamen, reckon 60 English miles to a degree; so that the circumference of the earth, according to them, is $360 \text{ degrees} \times 60 = 21600 \text{ miles}$.

But

But Mr. Norwood, by an experiment made between London and York, in the year 1635, found, that 367196 feet = 69 miles and 958 yards make a degree; according to whom the circumference of a great circle = 25035 miles.

And according to the Transactions of the Royal Academy of Sciences at Paris, anno 1687, 57060 toises = 365184 English feet = 69 miles 288 yards. make a degree on this terraqueous globe.

S Q U A R E M E A S U R E.

Sqr. Inches. 144 = 1 sq. foot.
 1296 = 9 = 1 sq. yard.
 39204 = 272 $\frac{1}{4}$ = 30 $\frac{1}{2}$ = 1 perch.
 1568160 = 10890 = 1210 = 40 = 1 rood.
 62728640 = 43560 = 4340 = 160 = 4 = 1 acre.
 4014489600 = 27878400 = 3097600 = 102400 = 2560 = 640 = 1 sq. mile.

This table will be useful in mensuration of superficies.

N. B. The least part of long measure was at first a barley-corn taken out of the middle of the ear; and being well dried, three of them in length were to make one inch.

CLÓTH MEASURE.

Inches.				
2 $\frac{1}{2}$	=	1	nail.	
9	=	4	=	1 quarter.
36	=	16	=	4 = 1 yard.
45	=	20	=	5 = 1 ell English.
27	=	12	=	3 = 1 ell Flemish.
54	=	24	=	6 = 1 ell French.

Note, All Scotch and Irish linens are bought and sold by the yard; but all Dutch linens are bought by the ell Flemish, and sold by the ell English.

T I M E.

Time only shews the duration, or mutation of things, a year being the standard, or integer, by which such continuation or change is computed. And a year is that space of time in which the sun (apparently) completes its revolution from any one point in the ecliptic (an imaginary circle in the heavens) to the same point again.

Seconds,

Seconds.

60 = 1 minute.

3600 = 60 = 1 hour.

86400 = 2440 = 24 = 1 day.

31556937 = 5259498 = 8766 = 365.5° 48' 57" = 1 year.



SECT. II.

ADDITION of WEIGHTS, MEASURES, &c.

RULE.

ALWAYS begin with those figures of the lowest or least denomination, and add them altogether into one sum; then consider how many of the next superior denomination are contained in that sum, so many units you must carry to the said next superior denomination, to be added together with those figures that stand there; and if any thing remain over, that overplus must be set down underneath its own denomination; but if you cannot otherways discover how many of the next superior denomination are contained in that sum, divide it by the number of units contained in one of the next denomination superior thereto, and set down the remainder, and carry the quotient. And so proceed on from one denomination to another, until all be finished.

1. A merchant buys up six bags of Canterbury hops, N^o 1. of which weighed 2 cwt. 2 qrs. 10 lb.; N^o 2. 2 cwt. 1 qr. 16 lb.; N^o 3. 2 cwt. 0 qrs. 24 lb.; N^o 4. 2 cwt. 3 qrs. only; N^o 5. 2 cwt. 1 qr. 12 lb.; N^o 6. 2 cwt 1 qr. 16 lb.; besides a couple of baskets, ditto, that weighed 58½ lb. each. How many hundreds weight has he to pay carriage for, on bringing them to town?

Bags,

62. ADDITION of WEIGHTS, &c. Book I.

						C. qrs. lb.
Bags, N ^o	1	-	-	-	-	2 2 10
	2	-	-	-	-	2 1 16
	3	-	-	-	-	2 - 24
	4	-	-	-	-	2 3 -
	5	-	-	-	-	2 1 12
	6	-	-	-	-	2 1 16
Pockets	- 1	-	-	-	-	2 2 $\frac{1}{2}$
	2	-	-	-	-	2 2 $\frac{1}{2}$

Cwt. 15 2 27, the answer.

2. In a gentleman's service of plate, there are fourteen dishes, weighing 193 oz. 13 dwt.; plates thirty-six, weighing 421 oz. 11 dwt.; four dozen of spoons, weighing 104 oz. 6 dwt.; six salts, chased, weighing 32 oz.; knives and forks, weighing 83 oz. 9 dwt.; fourteen presenters, weighing 113 oz. 4 dwt.; in mugs, tumblers, beakers, and other odd pieces, weighing 264 oz. 18 dwt.; silver tea-kettle and lamp, weighing 126 oz. 9 dwt.; and the rest of that equipage 93 oz. 2 dwt. What quantity of plate had the butler under his care?

			oz. dwt.
Fourteen dishes	- - - -	wt. 193	13
Thirty-six plates	- - - -	wt. 421	11
Four dozen of spoons	- - - -	wt. 104	6
Six salts	- - - -	wt. 32	-
Knives and forks	- - - -	wt. 83	9
Four presenters	- - - -	wt. 113	4
Mugs, tumblers, beakers, &c.	- - - -	wt. 264	18
A silver tea-kettle, &c.	- - - -	wt. 126	9
And the rest of that equipage	- - - -	wt. 93	2

oz. 1432 12, the answer.

3. The distance betwixt two places is such, that if three miles and five furlongs is taken from it, what remains is equal to eight miles, four furlongs, and 100 yards; what is the distance of those two places?

M. F. Yds.

3	5	—
8	4	100

12 1 100, the answer.

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4. In my survey of the river Glen, from Bafton-hedges to the Outfall, I found the several distances as follow:

	M.	F.	P.
To Clarke's house - - - - -	1	6	4
To the toll-bridge - - - - -	-	6	24
To the falling in of Bourn-Eau - - - - -	-	1	32
To Gutram-cot - - - - -	2	5	28
To Pinchbeck-bar - - - - -	2	2	12
To Money-bridge - - - - -	1	-	4
To Herring-bridge - - - - -	1	2	4
To New-bridge - - - - -	-	5	12
To Bondman-bridge - - - - -	-	-	36
To Stone-goat - - - - -	-	4	-
To Surfleet-bridge - - - - -	-	7	2
To the Halfpenny-toll - - - - -	1	1	28
To the Outfall - - - - -	1	1	32

From Bafton-hedges to the Outfall - - - 14 5 18

5. A father was 18 years four months old (reckoning 13 months to one year, and 28 days to one month) when his eldest child was born. Betwixt the eldest and second, were 11 months, 10 days. Betwixt the second and third, were three years, eight months. When the third is 12 years, six months, 20 days, how old is the father?

Y.	M.	D.
18	4	-
-	11	10
3	8	-
12	6	20

Years 35 4 2, the answer.



S E C T. III.

SUBTRACTION of WEIGHTS, MEASURES, &c.

I refer the young student to the general rule for subtraction, Chap. I, Sect. III.

1. Having

1. Having bought two hundred weight, and three quarters of sugar, and sold thereof one hundred, two quarters, and 14 pounds; what is yet unfold?

$$\begin{array}{r} \text{cwt. gr. lb.} \\ 2 \quad 3 \quad - \\ 1 \quad 2 \quad 14 \\ \hline \end{array}$$

Cwt. 1 - 14, the answer.

2. A father was 24 years, nine months, 10 days old, when his eldest son was born; and is now 56 years, three months, and 22 days. How old is the son?

$$\begin{array}{r} \text{Y. M. D.} \\ 56 \quad 3 \quad 22 \\ 24 \quad 9 \quad 10 \\ \hline \end{array}$$

31 7 12, the answer.

3. Received in lieu of two gold repeaters, sent to Jamaica in 1756, the five chests of indigo following; and on a like adventure in 1758, the subsequent five chests. The question is, how much indigo I had less the second time than the first?

Anno 1756.				Anno 1758.			
	cwt.	qr.	lb.		cwt.	qr.	lb.
N ^o 1	2	1	16	N ^o 1	1	3	7
	Tare 1 15				Tare 1 4		
2	2	2	11	2	1	3	17
	1 19				1 4		
3	2	-	12	3	1	2	10
	1 13				1 2		
4	2	-	19	4	1	-	13
	1 14				- 27		
5	2	3	17	5	2	-	11
	1 21				cwt. 1 6		
Gross	12	-	19	Gross	8	2	2
	1 3 26				1 1 15		
	Tare 1 3 26				Tare 1 1 15		
Net	{ 10 - 21 first }				{ 7 - 15 }		
	{ 7 - 15 second }				{ }		
	3 - 6 difference.						

4. Jacob, by contract, was to serve Laban for his two daughters 14 years; and when he had accomplished 11 years, 11 months, 11 weeks, 11 days, 11 hours, and 11 minutes; pray how long had he yet to serve?

11 Y.

Y. M. W. D. H. M.						Y. M. W. D. H. M.					
11	11	11	11	11	11	=	12	2	-	4	11
						Answer					
						1 9 3 2 12 49					

5. When the air presseth with its full weight in very fair weather, it may be demonstrated, that there press upon a human body about 302 cwt. 2 qrs. 25 lb. of that fluid matter; and in very foul weather, when the air is most light, 273 cwt. 1 qr. 20 lb. What difference of weight lies on such a body, in the two greatest alterations of the weather?

cwt. qrs. lb.		
302	2	25
273	1	20
<hr/>		
Cwt.	29	1 5, the answer.

S E C T. IV.

R E D U C T I O N.

REDUCTION alters or changes any superior denomination proposed, into any inferior or lesser denomination required; still keeping them equivalent in value. And the contrary.

R U L E.

Consider how many units of the denomination required, make one of that denomination proposed to be reduced (which is easily known by its respective table) and with that number of units multiply or divide the denomination proposed, and their product, or quotient, will be the number required.

1. Reduce 753 l. into pence.

$$\begin{array}{r}
 20 \\
 \hline
 15060 \text{ shillings.} \\
 12 \\
 \hline
 180720 \text{ pence.}
 \end{array}$$

Or, 753 l. may be reduced into pence at one operation.
Thus 240

$$\begin{array}{r}
 3012 \\
 1506 \\
 \hline
 180720 = \text{pence in } 753 \text{ l. as before.}
 \end{array}$$

But when the numbers proposed to be reduced are of several denominations, and it is required to bring them all to the lowest, you must reduce the highest or greatest denomination to the next less, adding the numbers that are of that less denomination thereto; then reduce their sum to the next lower denomination, adding thereto all the numbers that are of that denomination, and so proceed gradually on until all be finished.

2. Reduce 375 l. 17 s. 10½ d. into farthings.

$$\begin{array}{r}
 \text{l. s. d.} \\
 375 \text{ } 17 \text{ } 10\frac{1}{2} \\
 20 \\
 \hline
 7517 = \text{the shillings in } 375 \text{ l. } 17 \text{ s.} \\
 12
 \end{array}$$

$$\begin{array}{r}
 90214 = \text{the pence in } 375 \text{ l. } 17 \text{ s. } 10 \text{ d.} \\
 4
 \end{array}$$

$$\begin{array}{r}
 360859 = \text{farthings} = 375 \text{ l. } 17 \text{ s. } 10\frac{1}{2} \text{ d.}
 \end{array}$$

3. In 384627 farthings, how many pounds sterling?

$$\begin{array}{r}
 4 \overline{) 384627} \\
 12 \quad \underline{96156\frac{3}{4}} \text{ pence.} \\
 20 \quad \underline{8013} \text{ shillings } 0\frac{3}{4} \text{ d.} \\
 \hline
 \pounds 400 \text{ } 13 \text{ } 0\frac{3}{4} \text{ d.}
 \end{array}$$

Note,

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Note, The remainder is always of the same denomination with the dividend.

4. In 648385 grains, how many pennyweights, ounces, and pounds?

$$\begin{array}{r}
 20) \\
 24 \overline{) 648385} \text{) } 27016 \text{ pennyweights.} \\
 \underline{168} \\
 38 \text{ 12) } 1350 \text{ oz. 16 pwt. 1 grain.} \\
 \underline{145} \\
 112 \text{ lb. 6 oz. 16 pwt. 1 grain.} \\
 \underline{1 \text{ gr.}}
 \end{array}$$

5. In 17 cwt. 3 qrs. 14 lb., how many ounces?

$$\begin{array}{r}
 4 \text{ cwt. qr. lb.} \\
 \underline{} \\
 71 \text{ quarters.} \quad 17 \text{ 3 14} \\
 \underline{28} \quad 112 \quad 84 + 14 = 98. \\
 568 \quad 2002 \text{ lb.} \\
 \underline{142} \quad \underline{16} \\
 2002 \text{ pounds.} \quad 32032, \text{ answer, as before;} \\
 \underline{16} \\
 32032 \text{ ounces.}
 \end{array}$$

6. Reduce 93 tuns, 15 cwt. 2 qrs. 12 lb. 13 oz. 14 dr. into drams.

$$\begin{array}{r}
 \text{tuns cwt. qrs. lb. oz. dr.} \\
 93 \text{ 15 2 12 13 14} \\
 \underline{20} \\
 1875 \text{ hundreds.} \\
 \underline{22568} \quad 1875 \times 100 = 187500 \\
 210068 \text{ lb.} \quad 1875 \times 12 = 22500 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{ sum 210068.} \\
 \underline{16} \quad 56 + 12 = 68 \\
 3361101 \text{ ounces.} \\
 \underline{16} \\
 53777630 \text{ drams.}
 \end{array}$$

7. In 17 tankards, each weighing 27 oz. 14 dwt. 16 gr. how many grains?

oz. dwt. gr.
27 14 16
20
<hr/>
554 penny-weights.
24
<hr/>
2232
1108
<hr/>
13312 grains in one tankard.
17
<hr/>
226304 grains, the answer.
<hr/>

8. In 53777630 drams, how many tuns?

drams.	16) ozs.	28) lb.	4)
16) 53777630	(3361101)	210068	7502
57	16	140	1875 cwt. 2 qrs.
97	110	68	20
17	141	<hr/>	<hr/>
16	<hr/>	12	93 tun, 15 cwt.
30	13	<hr/>	<hr/>
<hr/>	<hr/>	<hr/>	<hr/>
14			
<hr/>			

Tuns cwt. qrs. lb. oz. dr.
Answer 93 15 2 12 13 14.

9. The silk mill at Derby contains 26586 wheels, and 97746 movements, which wind off, or throw 73726 yards of silk every time the great water-wheel, which gives motion to all the rest, goes about, which is three times in a minute. The question is, how many yards of silk may be thrown by this machine in a day, reckoning ten hours a day's work; and how many in the compass of a year, deducting for Sundays and great holidays, 63 days, provided no part of it stand still?

73726 yards at 1 circumvolution of the wheel.

$\times 3$

221178 yards in a minute.

$\times 60$

13270680 yards in an hour.

$\times 10$

132706800 yards in a day.

302 = 365 — 63

2654136
3981204

40077453600 yards, the answer.

10. In 4712 nails of Holland, how many yards, ells English, and ells Flemish?

4 | 4712
4 | 1178 quarters.
294½ yards.

5 | 1178
235 ells English ¾ yards.
3 | 1178
392 ells Flemish ½ yard.

Sometimes multiplication and division are both required to answer the question; as in the following.

11. In 491 barrels of beer (London measure) how many hogheads, gallons, and pints?

491 barrels.
2

3) 982 (327 hogheads, and 1 barrel = 36 gallons.

54
1308
1635
17694 gallons.
141552 pints.

F 3

12, I

12. I desire to know how many days, hours, minutes, and seconds, since our Saviour's nativity, it being accounted 1758 years?

$$\begin{array}{r}
 31556937 \text{ seconds in a year.} \\
 \underline{1758 \text{ years.}} \\
 252455496 \\
 157784685 \\
 220898559 \\
 31556937 \\
 \hline
 60 \overline{) 55477095246} \text{ seconds in 1758 years.} \\
 60 \overline{) 924618254} \text{ minutes } 6'' \\
 24 \overline{) 15410304} \text{ hours } 14' 6'' \\
 101 \overline{) 1642096} \text{ days } 14' 6'' = 1758 \text{ years.} \\
 50 \\
 230 \\
 \underline{144} \\
 0
 \end{array}$$

13. In how long time would a million of million of money be in counting, supposing 100 l. to be counted every minute, without intermission, and the year to consist of 365 days, five hours, 45 minutes?

$$100) 1000000000000 = 10000000000 \text{ minutes.}$$

Years days h. m.
In 1 year minutes 525945) 10000000000 (19013 144 5 55
4740550 the answer required.

$$\begin{array}{r}
 704500 \\
 \underline{1785550}
 \end{array}$$

$$1440) 207715 \text{ Rem. 100 pounds.}$$

$$\begin{array}{r}
 637 \\
 \underline{611}
 \end{array}$$

$$60) 355$$

55

A Geographical QUESTION.

14. There is a city in a certain island 708 miles more distant from the tropic of Cancer, than another under the same meridian is from the arctic polar circle. What cities are those, what are the distances of those cities from the equator, and what from each other; remembering the polar circle is about $23\frac{1}{2}$ degrees from the pole, as is the tropic from

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from the equator; and in this please to consider 60 geographical miles as a degree?

Tropic of Cancer $23^{\circ} 30'$ from the equator.

21 30 half the temperate zone.

60) 708 (11 $48\frac{1}{2}$ 2 = 5 54 half 708 miles.

50 54 latitude of the first city.

23° $30'$ polar circle from the pole.

21 30 half the temperate zone as before.

45 —

— 5 54 the half of 708 miles.

90 — 39 6 = 50 54 latitude of the second city.

Consequently both stand on the same spot, and answer to Chichester in Suffex, Tongeren in Germany, and Upres in Flanders.

15. I would put 60 hogsheads of London beer into 30 wine-pipes, and would know what the cask must hold that receives the difference; 231 solid inches being the gallon of wine, and 282 that of beer.

60 \times 54 = 3240 gal. of beer. 30 \times 126 = 3780 gal. wine meas.

282	231
648	378
2592	1134
648	756

913680 inches of beer. 873180

913680 — 873180 = 40500 inches difference.

282) 40500 (143 gallons, two quarts, and almost a pint.

Remains 33 inches, $35\frac{1}{4}$ th being a pint.

Chronological QUESTIONS.

16. England was conquered by William I. Oct. 4, 1066; his son William II. came to the crown, Sept. 9, 1087; and left it Aug. 2, 1100. William III. received it Feb. 3, 1689; and died March 8, 1701. How many days did each of these princes govern, respect being had to the intercalary days (added to February ever leap-year) as they rose in the course of time?

Note, Every fourth year is leap-year, or biffextile; to find which are such, divide the year of our Lord by 4, and when nothing remains, those are the leap-years, and to such you add one day more than 365.

$\frac{1066}{4}$ remains 2, so that 1068 was leap-year.

And in the reign of William I. were five intercalary days.

Between September 9, and October 4, are 23 days.

1087 — 1066 = 21 years all but 25 days, William I. reigned.

$$365 \times 21 = 7665 \text{ days.}$$

Therefore $7665 + 5 - 25 = 1645$ days, William I. reigned.

$\frac{1087}{4}$ remains 3, therefore 1088 was leap-year.

And in the reign of William Rufus 4 intercalary days.

Between Aug. 2, and Sept. 9, are 38 days.

1100 — 1087 = 13 years all but 38 days, William II. reigned

$$365 \times 13 = 4745 \text{ days.}$$

Therefore $4745 + 4 - 38 = 4711$ days, William II. reigned.

$\frac{1689}{4}$ remains 1, so that 1692 was leap-year.

And in the reign of William III. 3 intercalary days.

From Feb. 3, to March 8, are 33 days.

1731 — 1689 = 42 years and 33 days, William III. reigned.

$$365 \times 42 = 15330 \text{ days.}$$

Therefore $15330 + 33 + 3 = 15366$ days, William III. reigned.

17. Richard I. succeeded his father Henry II. July 7, 1189; John, his brother, succeeded him April 6, 1199; Richard II. succeeded Edward III. on the 21st of June, 1377, and was deposed by Henry IV. on the 30th of Sept. 1399. The third Richard caused his nephew, Edward V. and his brother, to be murdered on the 18th of June, 1483; and was slain himself on the 22d of Aug. 1585; how many days was the realm governed by the three Richards, respect being still had to the intercalary days as they happened?

$\frac{1189}{4}$ remains 1, so that 1192 was leap-year.

And in the reign of Richard I. were 2 intercalary days.

Between April 6, and July 7, are 92 days.

1199 — 1189 = 10 years all but 92 days, Richard I. reigned.

$$365 \times 10 = 3650 \text{ days.}$$

And $3650 + 2 - 92 = 3560$ days, Richard I. reigned.

$\frac{1377}{4}$ remains 1, so that 1380 was leap-year.

And in Richard II's reign were 5 intercalary days.

From June 21, till September 30, are 101 days.

1399 — 1377 = 22 years, 101 days, Richard II. reigned.

$$365 \times 22 = 8030 \text{ days.}$$

And

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And $8030 + 101 + 5 = 8136$ days, Richard II. reigned.

$\frac{1483}{4}$ remains 3, so that 1484 was leap-year.

And in Richard III's reign was 1 intercalary day.

From June 18, till August 22, are 65 days.

$1485 - 1483 = 2$ years, 65 days, Richard III. reigned

$365 \times 2 = 730$ days.

And $730 + 65 + 1 = 796$ days, Richard III. reigned.

Lastly, $3560 + 8136 + 796 = 12492$ days, the answer.

18. The first queen Mary came to the crown, July 8, 1553; she reigned five years, four months, and nine days. Her sister Elizabeth succeeded; and James I. came to her throne the 14th of March, 1602; he left it to his son Charles I. on the 27th of March, 1625, who was forced from it Jan. 30, 1648. The question is, how many days did those princes reign; and at the death of Charles I. how long had England been under an uninterrupted succession of Protestant princes (Mary I. being the last professed Papist that enjoyed the crown) not neglecting the intercalary days in February, as before?

$\frac{1533}{4}$ remains 1, so that 1556 was leap-year.

Therefore from 1553, till 1602, were 12 intercalary days, and only one leap-year in queen Mary's reign; so that in queen Elizabeth's reign were 11 intercalary days.

From July 8, to Nov. 17, being four months and nine days, are 132 days. Then $365 \times 5 = 1825$ days.

And $1825 + 132 + 1 = 1958$ days, Mary reigned.

$1602 - 1553 = 49$ years.

Betwixt March 14, and July 8, are 115 days.

Then $365 - 115 = 250$. Also $365 \times 49 = 17885$ days.

And $17885 + 250 + 12 = 18147$ days to the beginning of James's reign.

Then $18147 - 1958 = 16189$ days, Elizabeth reigned.

$\frac{1602}{4}$ remains 2, so that 1604 was leap-year.

Consequently from 1602 till 1625, were six intercalary days.

$1625 - 1602 = 23$; but as the date altered at our Lady-day, the interval was no more than 22 years, and 13 days.

Also $365 \times 22 = 8030$ days.

Therefore $8030 + 13 + 6 = 8049$ days, James I. reigned.

$\frac{1625}{4}$ remains 1, so that 1628 was leap-year.

There-

Therefore from 1625, till 1648, were six intercalary days.

1748 — 1625 = 23 whole years, Charles reigned.

From March 27, till Jan. 30, are 309 days.

Also $365 \times 23 = 8395$ days.

Therefore $8395 + 309 + 6 = 8710$ days, Charles I. reigned.

Then $16189 + 8049 + 8710 = 32948$ days, the answer.

19. A grant was made Dec. 14, in the 10th of Henry I. who began his reign Aug. 2, 1100; it was resumed Nov. 19, in the 4th of Henry III. who came to the crown Oct. 19, 1216; it was revived the 16th day of July, in the 13th of Henry VII. who ascended the throne Aug. 22, 1485: but it was a second time revoked, and finally suppressed in the 16th of his successor Henry VIII. on the 10th of May. Now as this man's father died July 21, 1509, the question is, how many days was this grant in force, and how many did it lie dormant?

Henry I. began his reign Aug. 2, 1100.

Then $1100 + 9 = 1109$, when the grant began, Dec. 14.

Henry III. began Oct. 19, $1216 + 3 = 1219$, Nov. 19.

The first continuance of this grant 110 years wanting 26 days; and in that period are 27 intercalary days.

∴ $365 \times 110 = 40150$, and $40150 + 27 - 26 = 40151$ days, first continuance.

Henry VII. began his reign Aug. 22, 1485.

$1485 + 13 = 1498$, July 16, grant reassumed.

Henry VII. died, and Henry VIII. succeeded, July 21, 1509.

$1509 + 15 = 1524$, May 20, grant ended.

$1524 - 1497 = 27$ years, wanting 57 days.

And in those 27 years, are 7 intercalary days.

$365 \times 27 = 9855 + 7 - 57 = 9805$ days, last in force.

Then $40151 + 9805 = 49956$ days, the grant was in force.

Q. E. F.

Again, Henry III. began his reign, Oct. 19, 1216.

$1216 + 3 = 1219$, Nov. 19, grant reassumed.

Henry VII. began his reign, Aug. 22, 1485.

$1485 + 13 = 1498$, July 16.

$1498 - 1219 = 279$ years, bating 126 days.

In which period are 69 intercalary days.

∴ $279 \times 365 = 101835$, and $101835 + 69 - 126 = 101778$ days, superseded. Q. E. F.

N. B. This question was taken from Clare's Introduction to Trade, &c. who makes the time of the continuance of the grant nine days less than found by the solution above; so would advise the young accountant to try which is in the right.

C H A P.

CHAPTER III. RULES of PRACTICE.

THE rules of Practice, from their great and frequent use, derive their name, and are contrived speedily and compendiously, to cast up any sort of goods or merchandize.

SECT. I. PRACTICE by MULTIPLICATION. CASE I.

To multiply by a mixed number; that is, a fraction joined with a whole number.

RULE.

When you have multiplied by the whole number, take $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, or whatever part it may be of the multiplicand, which, added to the product, will give the true answer.

1. In 57 fodder of lead, each $19\frac{1}{2}$ cwt. how many cwt.?

$$\begin{array}{r} 57 \\ 19\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 1083 \\ 28\frac{1}{2} \\ \hline \end{array}$$

$1141\frac{1}{2}$ cwt. in all.

3. In 27 hogheads of sugar, each containing $7\frac{1}{2}$ cwt. how many hundred weight?

$$\begin{array}{r} 27 \\ 7\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 189 \\ 13\frac{1}{2} = \frac{1}{2} \\ 6\frac{1}{2} = \frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 209\frac{1}{2} \\ \hline \end{array}$$

2. In 359 pieces of Norwich stuffs, each $23\frac{1}{4}$ yards, how many yards?

$$\begin{array}{r} 359 \\ 28\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 1077 \\ 718 \\ 89\frac{3}{4} \\ \hline \end{array}$$

$8346\frac{1}{4}$ yards.

4. In 354 pieces of Kersey, each $27\frac{1}{4}$ yards, how many yards?

$$\begin{array}{r} 354 \\ 27\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 2478 \\ 708 \\ 177 = \frac{1}{4} \\ 88\frac{1}{2} = \frac{1}{2} \\ 44\frac{1}{4} = \frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 9867\frac{3}{4} \\ \hline \end{array}$$

CASE

CASE II

To cast up any number of things, not exceeding 12, at any given price.

RULE.

Multiply the price by the quantity or number of things, always observing to carry from one denomination to another, as in addition of money.

1. What cost seven stone of beef, at 2s. 7d. per stone?

$$\begin{array}{r}
 \text{s. d.} \\
 2 \quad 7 \\
 \quad 7 \\
 \hline
 18 \quad 1, \text{ the answer.}
 \end{array}$$

3. What cost five sheep, at 1l. 17s. 6d. each?

$$\begin{array}{r}
 \text{l. s. d.} \\
 1 \quad 17 \quad 6 \\
 \quad \quad 5 \\
 \hline
 \pounds 9 \quad 7 \quad 6
 \end{array}$$

5. What cost 11 geese, at 1s. 7½d. each?

$$\begin{array}{r}
 \text{s. d.} \\
 1 \quad 7\frac{1}{2} \\
 \quad 11 \\
 \hline
 18 \quad 1\frac{1}{4}
 \end{array}$$

2. What cost 9 cwt. of treacle, at 1l. 17s. 4½d. per cwt?

$$\begin{array}{r}
 \text{l. s. d.} \\
 1 \quad 17 \quad 4\frac{1}{2} \text{ per cwt.} \\
 \quad 9 \\
 \hline
 16 \quad 16 \quad 2\frac{1}{2}, \text{ the answer.}
 \end{array}$$

4. What cost 10 yards of broad-cloth, at 17s. 10½d.?

$$\begin{array}{r}
 \text{s. d.} \\
 17 \quad 10\frac{1}{2} \\
 \quad 10 \\
 \hline
 \pounds 8 \quad 18 \quad 9
 \end{array}$$

6. What cost 12 cwt. of sugar, at 3l. 17s. 7½d.?

$$\begin{array}{r}
 \text{l. s. d.} \\
 3 \quad 17 \quad 7\frac{1}{2} \\
 \quad 12 \\
 \hline
 \pounds 46 \quad 11 \quad 9
 \end{array}$$

Note, If the given quantity is 13, multiply the price by 12; and as you multiply, add to it the price of one, and the result will be the answer.

What cost 13 cwt. at 4l. 13s. 7½d.?

$$\begin{array}{r}
 \text{l. s. d.} \\
 4 \quad 13 \quad 7\frac{1}{2} \\
 \quad 13 \\
 \hline
 \pounds 60 \quad 17 \quad 4\frac{1}{2}
 \end{array}$$

Thus

Thus performed, $3 \times 12 = 36 + 3 = 39$ farthings.

3 and carry 9.

Then $7 \times 12 = 84 + 9 + 7 = 100$ d. - 4 and carry 8.

Also $3 \times 12 = 36 + 8 + 3 = 47$ s. - 7 and carry 4.

And $1 \times 12 = 12 + 4 + 1 = 17$ angels, 1 and carry 8.

Lastly $4 \times 12 = 48 + 8 + 4 = 60$.

CASE III.

When the quantity exceeds twelve.

RULE.

Find two numbers in the multiplication table, which being multiplied together, will make the quantity; then multiply the price by one of those numbers (it matters not which you multiply first by) and that product by the other number, and the last product will be the answer.

1. What cost 15 cwt. of treacle, at 11. 7s. 9d. per cwt.?

$$\begin{array}{r} 3 \\ \hline 4 \ 3 \ 3 \\ \hline 5 \\ \hline \pounds \ 20 \ 16 \ 3 \end{array}$$

3. What cost 56 lb. of Hyson tea, at 15s. 9½d. per lb.?

$$\begin{array}{r} 8 \\ \hline 6 \ 6 \ 4 \\ \hline 7 \\ \hline \pounds \ 44 \ 4 \ 4 \end{array}$$

5. What cost 108 lb. of nutmegs, at 12s. 3½d. per lb.?

$$\begin{array}{r} 12 \\ \hline 7 \ 7 \ 9 \\ \hline 9 \\ \hline \pounds \ 66 \ 9 \ 9 \end{array}$$

2. What cost 27 ounces of silver, at 5s. 9½d. per oz.?

$$\begin{array}{r} 3 \\ \hline 17 \ 9\frac{1}{2} \\ \hline 9 \\ \hline \pounds \ 7 \ 16 \ 11\frac{1}{2} \end{array}$$

4. What cost 77 cwt. of madder, at 3l. 15s. 6d. p. cwt.?

$$\begin{array}{r} 7 \\ \hline 26 \ 8 \ 6 \\ \hline 11 \\ \hline \pounds \ 290 \ 13 \ 6 \end{array}$$

6. What cost 132 gallons of wine, at 5s. 4d. per gallon?

$$\begin{array}{r} 12 \\ \hline 3 \ 4 \ - \\ \hline 11 \\ \hline \pounds \ 35 \ 4 \ - \end{array}$$

To

To find the price of a gross; first find the price of a dozen, by multiplying by 12; which product multiplied also by 12, gives the price of a gross.

What cost seven gross of buckles, at 1 s. 11 $\frac{1}{4}$ d. per pair?

$$\begin{array}{r}
 \text{s. d.} \\
 1 \ 11\frac{1}{4} \\
 \underline{12} \\
 13 \ 3 \ 3 \text{ price of 1 dozen pair.} \\
 \underline{12} \\
 13 \ 19 \text{ - price of a gross.} \\
 \underline{7} \\
 97 \ 13 \text{ -, the answer.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

And to find the price of 1 cwt. at so much per pound, multiply by four; which product multiplied by seven, gives the price of a quarter; then four times the last product will be the answer.

What cost 3 $\frac{1}{4}$ cwt. of tea, at 4 5 $\frac{1}{4}$ per lb.?

$$\begin{array}{r}
 \text{s. d.} \\
 4 \ 5\frac{1}{4} \\
 \underline{4} \\
 17 \ 9 \\
 \underline{7} \\
 6 \ 4 \ 3 \text{ price of a quarter,} \\
 \underline{4} \\
 24 \ 17 \text{ - price of a cwt.} \\
 \underline{3} \\
 \pounds 74 \ 11 \text{ -, the answer.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

CASE IV.

When the quantity is a prime number, viz. such an one as no two numbers in the multiplication table can be found to answer it,

RULE,

Multiply by such numbers as come nearest the quantity; and for what is wanting, multiply the price by that number, and

Chap. III. PRACTICE.

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and add to it the other product, and the total will be the answer.

1. What cost 29 lb. of Bohea tea, at 6s. 9d. per lb.?

$$\begin{array}{r}
 \text{s. d.} \\
 6 \ 9 \\
 4 \\
 \hline
 1 \ 7 \ 0 \\
 7 \\
 \hline
 9 \ 9 \ - \\
 6 \ 9 \\
 \hline
 \pounds 9 \ 15 \ 9
 \end{array}$$

3. What cost 68 yards of Holland, at 5s. 4½d. per yard?

$$\begin{array}{r}
 6 \\
 \hline
 1 \ 12 \ 3 \\
 11 \\
 \hline
 17 \ 14 \ 9 \\
 10 \ 9 = \text{price} \times 2. \\
 \hline
 \pounds 18 \ 5 \ 6
 \end{array}$$

5. What cost 117 sheep, at 1 L. 7s. 6d.?

$$\begin{array}{r}
 \text{l. s. d.} \\
 1 \ 7 \ 6 \\
 10 \\
 \hline
 13 \ 15 \ - \\
 11 \\
 \hline
 151 \ 5 \ - \\
 9 \ 12 \ 6 = \text{price} \times 7. \\
 \hline
 \pounds 160 \ 17 \ 6
 \end{array}$$

2. What cost 38 lb. of leather, at 11¼d. per pound?

$$\begin{array}{r}
 \text{d.} \\
 11\frac{1}{4} \\
 4 \\
 \hline
 3 \ 11 \\
 9 \\
 \hline
 1 \ 15 \ 3 \\
 1 \ 11\frac{1}{2} = \text{price} \times 2. \\
 \hline
 \pounds 1 \ 17 \ 2\frac{1}{2}
 \end{array}$$

4. What cost 76 quarter of wheat, at 1l. 17s. 9d. per qr.?

$$\begin{array}{r}
 \text{l. s. d.} \\
 1 \ 17 \ 9 \\
 12 \\
 \hline
 22 \ 13 \ - \\
 6 \\
 \hline
 135 \ 18 \ - \\
 7 \ 11 = \text{price} \times 4. \\
 \hline
 \pounds 143 \ 9 \ -
 \end{array}$$

6. What cost 135 yards of broad-cloth, at 17s. 7½d.?

$$\begin{array}{r}
 \text{s. d.} \\
 17 \ 7\frac{1}{2} \\
 12 \\
 \hline
 10 \ 11 \ 9 \\
 11 \\
 \hline
 116 \ 9 \ 3 \\
 2 \ 12 \ 11\frac{1}{4} = \text{price} \times 3. \\
 \hline
 119 \ 2 \ 2\frac{1}{4}
 \end{array}$$

To

To find the amount per annum of officers salaries at so much per diem, multiply the salary or wages per day by 10, and that product by 9: this last product multiplied by 4 gives the salary for 360 days; to this add one day's salary multiplied by 5, which give the answer.

If an officer's salary be 17 s. $10\frac{1}{4}$ d. a day, what is that a year?

$$\begin{array}{r}
 \text{s. d.} \\
 17 \ 10\frac{1}{4} \\
 \hline
 8 \ 18 \ 11\frac{1}{2} \\
 \hline
 9 \\
 \hline
 80 \ 10 \ 7\frac{1}{2} \\
 \hline
 4 \\
 \hline
 322 \ 2 \ 6 \\
 4 \ 9 \ 5\frac{3}{4} \\
 \hline
 \pounds \ 326 \ 11 \ 11\frac{1}{4}, \text{ the answer.}
 \end{array}$$

CASE V.

When the quantity is 1, 2, 3, 4, 5, 6, 7, or more hundreds.

RULE.

Multiply the price by 10, and that product by 10 also, which gives the value of one hundred; then multiply the product by the number of hundreds; then multiply that product, which gave the price of 10, by 2, 3, 4, or 5, as the tens happen, which place under the last product without drawing a line; and for the units always multiply the price by them, and set that down under the former products; so you will have three lines, the sum of which will be the answer.

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1. What cost 795 yards of brocade, at 1l. 7s. 10 $\frac{1}{4}$ d. per yard?

$$\begin{array}{r}
 \text{l. s. d.} \\
 1 \quad 7 \quad 10\frac{1}{4} \\
 \hline
 10 \\
 13 \quad 18 \quad 6\frac{1}{2} \\
 \hline
 10 \\
 139 \quad 5 \quad 5 \\
 \hline
 7 \\
 974 \quad 17 \quad 11 \\
 125 \quad 6 \quad 10\frac{1}{2} \\
 \hline
 6 \quad 19 \quad 3\frac{1}{2} \\
 \hline
 \text{£ } 1107 \quad 4 \quad 0\frac{3}{4}
 \end{array}$$

2. What cost 368 lambs, at 17s. 9d. each?

$$\begin{array}{r}
 \text{s. d.} \\
 17 \quad 9 \\
 \hline
 10 \\
 8 \quad 17 \quad 6 \\
 \hline
 10 \\
 88 \quad 15 \quad - \\
 \hline
 3 \\
 266 \quad 5 \quad - \\
 53 \quad 5 \quad - \\
 \hline
 7 \quad 2 \quad - \\
 \hline
 \text{£ } 326 \quad 12 \quad -
 \end{array}$$

CASE VI.

To multiply weights and measures.

RULE.

Place the multiplier under the lowest denomination of the multiplicand; then multiply the lowest denomination by the multiplier, and divide the product by as many of the lowest denomination as make one of the next superior; setting down underneath the remainder, if any, and carry the quotient to the product of the multiplier and the next superior denomination, and so proceed till all is finished.

1. In 11 pieces of kersey, each 17 yards, three quarters, three nails, how many yards in all?

$$\begin{array}{r}
 \text{Y. qrs. n.} \\
 17 \quad 3 \quad 3 \\
 \hline
 11 \\
 197 \quad 1 \quad 1, \text{ answer.}
 \end{array}$$

3. In 38 pieces of tapestry, each 37 ells Flemish, 2 qrs. 3 nails, how many ells Flemish?

$$\begin{array}{r}
 \text{E. F. qrs. n.} \\
 37 \quad 2 \quad 3 \\
 \hline
 6 \\
 227 \quad 1 \quad 2 \\
 \hline
 6 \\
 1365 \quad - \quad - \\
 75 \quad 2 \quad 2 = 1 \text{ piece} \times 2. \\
 \hline
 1440 \quad 2 \quad 2, \text{ answer.}
 \end{array}$$

2. In 42 pieces of Holland, each 27 ells English, two quarters, three nails, how many ells in all?

$$\begin{array}{r}
 \text{El. E. qrs. n.} \\
 27 \quad 2 \quad 3 \\
 \hline
 6 \\
 165 \quad 1 \quad 2 \\
 \hline
 7 \\
 1157 \quad - \quad 2
 \end{array}$$

As the next superior denomination contains only 3, 4, or 5 of the inferior ones, questions of this kind may be performed without division.

G

4. What

4. What is the weight of seven tankards, each weighing 11 oz. 16 dwt. 21 grs.?

oz. dts. gr.
11 16 21

$$\begin{array}{r} 7 \\ 82 \ 18 \ 3 \\ \hline 24 \ 147 \ 6 \end{array}$$

5. What is the net weight of 39 hogheads of sugar, each weighing 7 cwt. 3 qrs. 17 lb.?

Cwt. qr. lb.

7 3 17
6

47 1 18
6

284 1 24
23 2 23

308 - 19, answer.

$$\begin{array}{r} 28 \ 102 \ 3 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 28 \ 108 \ 3 \\ \hline 24 \end{array}$$

$$\begin{array}{r} 28 \ 51 \ 1 \\ \hline 23 \end{array}$$

As 20 pennyweights make one ounce, we carry as in multiplication of shillings; and if pounds troy be used, we carry the same as in pence.

6. What is the weight of 29 parcels of tea, each 25 lb. 7 oz. 13 drams?

lb. oz. dr.

25 7 13
7

Cwt. 1 2 10 6 11
4

6 1 13 10 12
25 7 13

6 2 11 2 9

$$\begin{array}{r} 26 \ 91 \ 5 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 26 \ 54 \ 3 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 28 \ 178 \ 6 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 26 \ 44 \ 2 \\ \hline 12 \end{array}$$

All the needful divisions are here put down.

7. What is the weight of 37 small parcels of tea, each weighing 13 ounces 12 drams?

oz. dr.
13 12
12

10 5 -

1 2 15 -
13 12

1 3 12 12

$$\begin{array}{r} 16 \ 144 \ 9 \\ \hline 0 \end{array}$$

8. What is the weight of 105 ingots of silver, each weighing 21 oz. 17 dwt. 19 gr.?

oz. dwt. gr.
21 17 19
10

218 17 22
10

2188 19 4
109 8 23

Ozs. 2298 8 3

$$\begin{array}{r} 24 \ 190 \ 7 \\ \hline 22 \end{array}$$

$$\begin{array}{r} 24 \ 95 \ 5 \\ \hline 23 \end{array}$$

$$\begin{array}{r} 24 \ 220 \ 9 \\ \hline 4 \end{array}$$

QUESTIONS to exercise the foregoing RULES:

1. A person dying left his widow 1780 l. and 1250 l. to each of his four children; 30 guineas apiece to 15 of his poor relations, and 150 l. to charities: he had been 25½ years in trade, and at an average had cleared 126 l. a year: What had he to begin with?

$$\begin{array}{rcl}
 \text{To widow} & - & - \text{£ } 1780 \quad - \\
 \text{£ } 1250 \times 4 \text{ children} & - & - \quad 5000 \quad - \\
 \text{£ } 30 \times 15 \text{ poor relations} & - & - \quad 472 \quad 10 \\
 \text{To charity} & - & - \quad 150 \quad - \\
 \hline
 \text{Worth} & - & - \text{£ } 7402 \quad 10 \\
 \text{£ } 126 \text{ cleared yearly} \times 25\frac{1}{2} & - & - \quad 3213 \quad - \\
 \hline
 & & \text{£ } 4189 \quad 10, \text{ answer.}
 \end{array}$$

2. Supposing that for a quarter's rent I paid in money 7 l. 7 s. 6 d. and was allowed for a small repair 18 s. 9 d. and for the king's tax 8 s. 9 d. What did my tenement go at a year?

$$\begin{array}{rcl}
 \text{£ } 7 \quad 7 \quad 6 \text{ quarter's rent.} & & \\
 \quad 18 \quad 9 \text{ repairs.} & & \\
 \quad 8 \quad 9 \text{ tax.} & & \\
 \hline
 8 \quad 15 \quad - & & \\
 \quad \quad \quad \downarrow & & \\
 \hline
 35 \quad - \quad - & & \text{the answer.}
 \end{array}$$

3. At Leicester, and other places, they weigh their coals by a machine, in the nature of a steelyard, waggon and all: three of these draughts together amount to 117 cwt. 2 qrs. 10 lb.; and the tare of the waggon was 13½ cwt. How many coals had the customer to pay for?

$$\begin{array}{rcl}
 & & \text{Cwt. qr. lb.} \\
 \text{Cwt. qr. lb.} & & 117 \quad 2 \quad 10 \\
 \text{£ } 3 \quad 1 \times 3 = & & 39 \quad 3 \quad - \\
 \hline
 & & 77 \quad 3 \quad 10, \text{ the answer.}
 \end{array}$$

4. A gentleman hath 536 l. per annum, and his expences are, one day with another, 18 s. 10½ d. I desire to know how much he layeth up at the year's end?

G 2

18 s.

$$\begin{array}{r} \text{l. s. d.} \\ 18 \ 10\frac{1}{2} \\ \hline 4 \end{array}$$

$$4 \times 13 \times 7 = 364.$$

$$\begin{array}{r} 3 \ 15 \ 7 \\ \hline 13 \\ \hline 49 \ 2 \ 7 \\ \hline 7 \end{array}$$

$$\begin{array}{r} \text{l. s. d.} \\ \text{Year's rent} - \ 536 \ - \ - \\ \text{Expended} - - \ 344 \ 16 \ 11\frac{1}{2} \\ \hline \end{array}$$

$$\text{Laid up} - - - \underline{191 \ 3 \ -\frac{1}{2}}, \text{ the answer.}$$

$$\begin{array}{r} 343 \ 18 \ 1 \\ \hline 18 \ 10\frac{1}{2} \end{array}$$

$$\underline{344 \ 16 \ 11\frac{1}{2}}, \text{ expended in one year.}$$

5. A gentleman expendeth daily 1 l. 7 s. 10½ d. and at the year's end layeth up 340 l. I demand how much is his yearly income?

$$\text{l. s. d.}$$

$$1 \ 7 \ 10\frac{1}{2} \text{ expences per day.}$$

$$\begin{array}{r} \times 4 \\ \hline 5 \ 11 \ 6 \\ \times 13 \\ \hline \end{array}$$

$$\begin{array}{r} 72 \ 9 \ 6 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 507 \ 6 \ 6 \\ 1 \ 7 \ 10\frac{1}{2} \\ \hline \end{array}$$

$$\underline{508 \ 14 \ 4\frac{1}{2}}, \text{ expended in one year.}$$

$$\begin{array}{r} \text{l. s. d.} \\ \text{Year's expences} \ 508 \ 14 \ 4\frac{1}{2} \\ \text{To lay up} - - \ 340 \ - \ - \\ \hline \end{array}$$

$$\text{Answer } \pounds \underline{848 \ 14 \ 4\frac{1}{2}}$$

Here trace the Dutcheſs of PLANTIMON,

1758.
October

3. A chimney-glaſs, and a pair of ſconces - - - - -
 10. A pair of pier-glaſſes, 72 inches, in gilt frames - - - - -
 10. A pair of Indian cabinets, at 43 l. 10 s. each - - - - -
 A fine Indian four-leaved ſcreen, and a fire-ſcreen - - - - -
 November 18. A book-caſe, with glaſs-doors, and a corner-cupboard, ditto - - - - -
 30. A walnut-tree table, and a ſet of dreſſing-boxes, japanned - - - - -
 December 7. A tea-table and ſtand, plated, weight 103 ounces, at 8 s. 4 d. per ounce - - - - -
 30. A dozen and half of fine matted chairs, at 18 s. 6 d. each - - - - -
 January 24. Twelve elbow-chairs, ſtuffed with hair, at 1 l. 15 s. 6 d. each - - - - -
 A dreſſing-table, with implements for writing - - - - -

Chap. III. PRACTICE.

Debtor	l.	s.	d.
	5	18	-
	60	-	-
	87	-	-
	17	10	-
	21	-	-
	3	4	10
	42	18	4
	16	13	-
	21	6	-
	6	16	6
	<hr/>		
	£	282	6 8

Q	l.	s.	d.	s.	d.	l.	s.	d.
	43	10	-	18	6	1	15	6
	$\times 2$		9	$\times 6$		$\times 12$		-
	<hr/>		1	<hr/>		<hr/>		-
	£	87	-	1	-	5	11	-
	<hr/>		3	<hr/>		<hr/>		21
	$\times 11$		-	$\times 3$		<hr/>		6
	<hr/>		42	<hr/>		<hr/>		-
	5	-	-	16	13	<hr/>		-
	$\times 2 =$		1	<hr/>		<hr/>		-
	<hr/>		42	<hr/>		<hr/>		-
	£	42	18 4					

Messieurs DRAKE and COMPTON,

Debtors

To RICHARDS and JACOB BROUGHTON, Dyers,

1762. For dyeing the goods following, delivered per order, to Edward Evelin, Packer. l. s. d.
 May 7. Exeter stuffs, yellow - - - 70 pieces - - at 14 s.-d. per piece - - - 49 - -
 13. Norwich ditto, blue - - - 30 - - - at 11 6 - - - 17 5 - -
 20. Tamies, black - - - 42 - - - at 12 8 - - - 26 12 - -
 July 1. Colchester bays, green - - - 28 - - - at 9 - - - 12 12 - -
 28. Camblets, orange - - - 21 - - - at 15 - - - 15 15 - -

121 4 -

s.	d.	s.	d.	s.	d.	s.	d.	s.	d.
14	-	12	8	9	-	15	-	15	-
X 30		X 6		X 4		X 3		X 3	
1.		1.		1.		1.		1.	
7	-	3 16	-	1 16	-	2 5	-	2 5	-
X 7		X 7		X 7		X 7		X 7	
£ 49	-	£ 26 12	-	£ 12 12	-	£ 15 15	-	£ 15 15	-

Q 4

Mr. ROWLAND, Upholder,

To WILLIAM WAREHOUSE and Company.

Debtor

1762.

October 7. Superfine blankets

9. Medium ditto

Harateen blue

November 6. Cheney ditto

18. Green Linseys

December 9. Printed quilts $\frac{1}{2}$ Callico

l. s. d.

20 5 2

8 15 6

70 15 8

38 - -

14 - -

12 10 -

£ 164 6 4

PRACTICE.

Book I.

Sir

l. s. d.

1 3 10

× 2

2 7 8

× 8

19 1 4

1 3 10

£ 20 5 2

s. d.

13 6

× 13

£ 8 15 6

l. s. d.

2 5 8

× 5

11 8 4

× 6

68 10 0

2 5 8

£ 70 15 8

l. s. d.

1 18 -

× 4

7 12 -

× 5

£ 38 - -

l. s. d.

1 8 -

× 10

£ 14 - -

l. s. d.

16 8 -

× 3

2 10 -

× 5

£ 12 10 -

EDWARD CORNWALL,

To BERNARD, Bricklayer,

For work and materials in his house on Tower-hill, London.

Debtor

1.	s.	d.
19	9	7
10	13	7
22	5	8
3	12	10
13	10	1
7	10	1
11	5	1
<hr/>		
£	88	6 8

1762.	March	28.	Bricks	-	-	-	at 15s.	7d.	per thousand	-	-
		30.	Tiles	-	-	-	at 19	5	-	-	-
	April	1.	Lime	-	-	-	at 15	11	per hundred	-	-
		12.	Sand	-	-	-	at 3	10	per load	-	-
	June	28.	Work for myself	-	-	-	at 3	-	per diem	-	-
			Ditto for a labourer	-	-	-	at 1	8	-	-	-
			Ditto for my man	-	-	-	at 2	6	-	-	-

s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
15 7	19 5	15 11	3 10	3	1 8	2 6	2 6
× 5	× 11	× 4	× 2	× 10,	× 9	× 9	× 9
1.		1.		1.		1.	
3 17 11	£ 10 13 7	3 3 8	7 8	1 10	15	1 2 6	1 2 6
× 5		× 7	× 9	× 9	× 10	× 10	× 10
	£ 22 5 8		1.				
£ 19 9 7			3 9	£ 13 10	£ 7 10	£ 11 5	£ 11 5
			3 10				

S E C T. II.

PRACTICE *by* DIVISION.

C A S E I.

HAVING the price of any number of things not exceeding 12, to find the price of one.

R U L E.

Place the divisor under the highest denomination (or to the left-hand) of the dividend. Then divide the highest denomination of the dividend by the divisor, and bring the remainder, if any, in your mind only, into the next inferior denomination, adding thereto what is in the dividend; divide this number as above, and so proceed till the whole is finished.

1. If 3 yards of velvet cost 5 l. 3 s. 11 $\frac{1}{4}$ d. what cost 1 yard?

$$\begin{array}{r} \text{l. s. d.} \\ 5 \quad 3 \quad 11\frac{1}{4} \\ \div 3 \\ \hline \text{£} \quad 1 \quad 14 \quad 7\frac{3}{4}, \text{ answer.} \end{array}$$

3. If 7 cwt. of lead cost 6 l. 5 s. 10 $\frac{1}{2}$ d. what cost 1 cwt.?

$$\begin{array}{r} \text{l. s. d.} \\ 6 \quad 5 \quad 10\frac{1}{2} \\ \div 7 \\ \hline \text{£} - 17 \quad 11\frac{1}{2}, \text{ answer.} \end{array}$$

5. 11 geese cost 18 s. 1 $\frac{1}{2}$ d. what cost one?

$$\begin{array}{r} \text{s. d.} \\ 18 \quad 1\frac{1}{2} \\ \div 11 \\ \hline 1 \quad 7\frac{3}{4}, \text{ answer.} \end{array}$$

2. If 4 yards of silk cost 1 l. 11 s. what cost 1 yard?

$$\begin{array}{r} \text{l. s. d.} \\ 1 \quad 11 \\ \div 4 \\ \hline \text{£} - 17 \quad 9, \text{ answer.} \end{array}$$

4. If 9 ells of linen cost 1 l. 8 s. 8 $\frac{1}{2}$ d. what cost 1 ell.?

$$\begin{array}{r} \text{l. s. d.} \\ 1 \quad 8 \quad 8\frac{1}{2} \\ \div 9 \\ \hline \text{£} - 3 \quad 2\frac{1}{2}, \text{ answer.} \end{array}$$

6. If 12 cwt. of loaf-sugar cost 46 l. 12 s. 9 d. what cost one?

$$\begin{array}{r} \text{l. s. d.} \\ 46 \quad 12 \quad 9 \\ \div 12 \\ \hline \text{£} \quad 3 \quad 17 \quad 8\frac{1}{2}, \text{ answer.} \end{array}$$

C A S E

CASE II.

When the divisor exceeds 12.

RULE.

Find two or more numbers whose product is equal to the given quantity ; then divide the price by one of them, and that quotient by the other ; the last quotient will be the answer.

1. If 21 fat heifers cost 164 l. 11 s. $3\frac{1}{2}$ d. what cost one?

$$\begin{array}{r} \text{l. s. d.} \\ 164 \ 11 \ 3\frac{1}{2} \\ \div 7 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \ 10 \ 2\frac{1}{2} \\ \div 3 \\ \hline \end{array}$$

£ 7 16 $8\frac{1}{2}$, answer.

3. If 35 dollars are worth 7 l. 17 s. 6 d. what is the worth of one?

$$\begin{array}{r} \text{l. s. d.} \\ 7 \ 17 \ 6 \\ \div 5 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \ 11 \ 6 \\ \div 7 \\ \hline \end{array}$$

£ - 4 6, answer.

5. If 72 sheep cost 82 l. 10 s. what cost one?

$$\begin{array}{r} \text{l. s.} \\ 82 \ 10 \\ \div 8 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \ 6 \ 3 \\ \div 9 \\ \hline \end{array}$$

£ 1 2 11, answer.

2. If 33 lb. of butter cost 15 s. $9\frac{1}{2}$ d. what cost one pound?

$$\begin{array}{r} \text{s. d.} \\ 15 \ 9\frac{1}{2} \\ \div 11 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \ 5\frac{1}{2} \\ \div 3 \\ \hline \end{array}$$

- $5\frac{1}{2}$, answer.

4. If 54 cwt. of treacle cost 101 l. 14 s. what per cwt.?

$$\begin{array}{r} \text{l. s.} \\ 101 \ 14 \\ \div 6 \\ \hline \end{array}$$

$$\begin{array}{r} 16 \ 19 \\ \div 9 \\ \hline \end{array}$$

£ 1 17 8, answer.

6. If 132 lb. of tobacco cost 5 l. 18 s. 3 d. what per lb.?

$$\begin{array}{r} \text{l. s. d.} \\ 5 \ 18 \ 3 \\ \div 12 \\ \hline \end{array}$$

$$\begin{array}{r} - \ 9 \ 10\frac{1}{2} \\ \div 11 \\ \hline \end{array}$$

£ - - $10\frac{1}{2}$, answer.

I. If

I. If there be a remainder in the first operation, and none in the last, place it over the whole divisor for a fractional part; but if there be two remainders, multiply the last remainder by the first divisor, and to the product add the first remainder, which will be the true remainder or numerator of the fraction, under which place the whole divisor for a denominator.

7. If 28 tod of wool cost 25 l. what cost one tod?

$$\begin{array}{r} \text{l. s. d.} \\ 25 \quad - \quad - \\ \div 7 \\ \hline 3 \quad 11 \quad 5\frac{1}{7} \\ \div 4 \end{array}$$

$$\underline{\text{£} - 17 \quad 10\frac{1}{4} + \frac{1}{14}\text{d. or } \frac{1}{7}\text{farth.}}$$

8. If 77 lb. of tea cost 26 l. what per pound?

$$\begin{array}{r} \text{l. s. d.} \\ 26 \quad - \quad - \\ \div 7 \\ \hline 3 \quad 14 \quad 2\frac{1}{4}, \frac{1}{7}\text{farth.} \\ \div 11 \end{array}$$

$$\begin{array}{l} 7 \times 10 + 3 = 73 \\ \text{Answer } 6\text{s. } 8\frac{1}{4}\text{d.} + \frac{1}{7}\frac{1}{7}\text{farth.} \end{array}$$

II. If you have the price of any number of grofs, to find the price of one, divide the whole price by the number of grofs, which gives the price of one grofs; this quotient divided by 12 gives the value of 1 dozen; which last quotient, divided by 12 also, gives the value of one.

9. If 7 grofs pair of buckles cost 59 l. 17 s. what cost one pair?

$$\begin{array}{r} \text{l. s. d.} \\ 59 \quad 17 \quad - \\ \div 7 \\ \hline 8 \quad 11 \quad - \text{ per grofs.} \\ \div 12 \\ \hline - 14 \quad 3 \text{ per dozen.} \\ \div 12 \\ \hline - 1 \quad 2\frac{1}{4} \text{ per pair.} \end{array}$$

10. If 9 grofs pair of gloves cost 91 l. 16 s. what cost one pair?

$$\begin{array}{r} \text{l. s. d.} \\ 91 \quad 16 \quad - \\ \div 9 \\ \hline 10 \quad 4 \quad - \text{ per grofs.} \\ \div 12 \\ \hline - 17 \quad - \text{ per dozen.} \\ \div 12 \\ \hline - 1 \quad 5 \text{ per pair.} \end{array}$$

III. Also having the value of a last of oats, cole-feed, or other grain, customary measure, to find the value of one bushel; divide by 3, and that quotient by 7, which gives the value of a coom, this last quotient divided by 4, gives the value of a bushel.

II. If

11. If a last of cole-feed cost 16 l. what will one bushel cost at that rate?

$$\begin{array}{r}
 \text{l. s. d.} \\
 16 \quad - \quad - \\
 \div 3 \\
 \hline
 5 \quad 6 \quad 8 \\
 \div 7 \\
 \hline
 - 15 \quad 2 \frac{1}{4}, \frac{1}{7} \text{ per coom.} \\
 \div 4 \\
 \hline
 - 3 \quad 9 \frac{1}{2}, \frac{3}{4}
 \end{array}$$

$7 \times 3 + 3 = 24$ remainder.
 Anf. 3s. 9½ d. + ¾, or ⅞ far.
 or 3 9½, very near.

12. If a last of oats cost 5 guineas, what cost one bushel at that rate?

$$\begin{array}{r}
 \text{l. s. d.} \\
 5 \quad 5 \quad - \\
 \div 3 \\
 \hline
 1 \quad 15 \quad - \\
 \div 7 \\
 \hline
 - 5 \quad - \text{ per coom.} \\
 \div 4 \\
 \hline
 \text{£} - 1 \quad 3, \text{ the answer.}
 \end{array}$$

IV. Having the price of an hundred weight, to find the price of one pound, divide the price of the hundred by 4, which gives the value of ¼ of a cwt. which divide again by 4, gives the value of 7l. ; this last quotient divided by 7 gives the answer.

13. If 1 cwt. of hops cost 5l. 9s. 8d. what cost 1 lb. at that rate?

$$\begin{array}{r}
 \text{l. s. d.} \\
 5 \quad 9 \quad 8 \\
 \div 4 \\
 \hline
 1 \quad 7 \quad 5 \\
 \div 4 \\
 \hline
 - 6 \quad 10 \frac{1}{4} \\
 \div 7 \\
 \hline
 - - 11 \frac{1}{4}, \text{ the answer.}
 \end{array}$$

14. Sugar at 3l. 17s. per cwt. what per lb.?

$$\begin{array}{r}
 \text{l. s. d.} \\
 3 \quad 17 \quad - \\
 \div 4 \\
 \hline
 - 19 \quad 3 \\
 \div 4 \\
 \hline
 - 4 \quad 9 \frac{3}{4} \\
 \div 7 \\
 \hline
 - - 8 \frac{1}{4}, \text{ the answer.}
 \end{array}$$

V. Having the price of 1 c. great weight, viz. 120 lb. per cwt. to find the price of 1 stone, or of 1 pound, divide by 8, which gives the price per stone ; which quotient divided by 3, and that quotient again by 5, will give the price of 1 pound.

15. Cheese at 1 l. 15 s. per cwt. what per stone, and per pound?

	l.	s.	d.
8	1	15	—
3		4	4½ per stone.
5		1	9½
			3½ per pound.

16. Ditto at 1 l. 5 s. per cwt. what per stone, and per pound?

	l.	s.	d.
8	1	5	—
3		3	1½ per stone.
5		1	—½ per pound.
			2½ per pound.

VI. Or by considering, that as 120 pence make 10 s. every 10 s. per cwt. gives 1 penny; and every 2 s. 6 d. gives 1 farthing a pound.

So that 1 l. 10 s. gives 3 d. and 5 s. gives ½ d. viz. by inspection, 3½ d. per pound.

In the question above, 1 l. gives 2 d. and 5 s. gives ½ d. = 2½ d. per pound.

17. Ditto at 1 l. 7 s. 6 d. per cwt. what per pound?

Here 1 l. gives 2 d. and 7 s. 6 d. gives ¾ d.

Or 2¾ d. = per pound.

18. Ditto at 1 l. 12 s. 6 d. per cwt. what per pound?

Here 1 l. 10 s. gives 3 d. and 2 s. 6 d. = ¾ d.

Or 3¾ d. per pound.

VII. Having the price of a ton, to find the price of a hundred, a quarter, or a pound, divide by 5 and by 4; which last quotient is the price of an hundred; then proceed as per remark 4th.

19. Carriage at 7 l. per ton, what per pound?

	l.	s.	d.
4	7	—	—
5	1	15	0
4	—	7	— per cwt.
7	—	1	9 per qr.
4	—	—	3
			¾ d. per lb.

20. Carriage at 2 l. 6 s. 8 d. per ton, what per pound?

	l.	s.	d.
4	2	6	8
5	—	11	8
4	—	2	4 per cwt.
7	—	—	7 per qr.
4	—	—	1
			¼ d. per lb.

VIII. As 252 gallons of wine, &c. make a tun, to find the price of a hoghead or gallon, divide the price of the tun by 4, which gives the price of an hoghead; then divide the

the price of a hoghead by 7, and that quotient divided by 9, gives the price of a gallon?

21. Port wine at 60 l. per ton, what per gallon?	22. Madeira wine at 95 l. per ton, what per gallon?
l. s. d.	l. s. d.
4 60 - - per ton.	4 95 - - per ton.
9 15 - - per hoghead.	9 23 15 - per hoghead.
7 1 13 4	7 2 12 9 $\frac{1}{2}$
- 4 9 $\frac{1}{2}$ per gallon.	- 7 6 $\frac{1}{2}$ nearly p. gal.

IX. Having the value of a wey (viz. 256 lb.) of cheese, &c. to find the value of 1 lb. or of an hundred weight; divide the value of a wey by 8, and that quotient by 4, which gives the value of 8 lb.; which divided by 8, gives the value of 1 lb.; or the value of 8 lb., multiplied by 7, gives the value of $\frac{1}{2}$ cwt.; which multiplied by 2, gives the value of an hundred weight.

23. Cheese at 2 l. 18 s. 8 d. per wey, what per lb. and per cwt.?	24. Ditto at 2 l. 8 s. per wey, what per lb. and per cwt.?
l. s. d.	l. s. d.
8 2 18 8 per wey.	8 2 8 - per wey.
4 - 7 4	4 - 6 -
8 - 1 10 price of 8 lb.	8 - 1 6 price of 8 lb.
- - 2 $\frac{1}{2}$ per lb.	- - 2 $\frac{1}{2}$ per lb.

s. d. Or 1 10 price of 8 lb. X 7 12 10 per $\frac{1}{2}$ cwt. X 2 £ 1 5 8 per cwt.	s. d. Or 1 6 price of 8 lb. X 7 10 6 per $\frac{1}{2}$ cwt. X 2 £ 1 1 -
---	--

X. A stone of wool or locks, in several manufacturing countries, is 15 lb. and 16 such stones, 8 tods, or 240 lb. make one pack;

pack; therefore having the price of a pack of wool, &c. to find the price of a tod, stone, or pound, divide the price of a pack by 8, which gives the price of a tod, which divided by 2, gives the price per stone; which divided by 3, and that quotient again by 5, gives the price of one pound.

25. Wool at 7l. 5s. per pack, what per lb.?

	l.	s.	d.
8	7	5	—
2	—	18	$1\frac{1}{2}$ per tod.
3	—	9	$\frac{3}{4}$ per stone.
5	—	3	$\frac{1}{4}$
	—	—	$7\frac{1}{4}$ per lb.

Or as 240l. = 20s., by inspection, 7l. 5s. per pack, gives $7\frac{1}{4}$ d. per lb.

26. Locks at 4l. 15s. per pack, what per lb.

	l.	s.	d.
8	4	15	—
2	—	11	$10\frac{1}{2}$ per tod.
3	—	5	$11\frac{1}{4}$ per stone.
5	—	1	$11\frac{3}{4}$
	—	—	$4\frac{3}{4}$ per lb.

Or by inspection, 4l. 15s. per pack, gives $4\frac{3}{4}$ d. per lb.

CASE III.

When the divisor is a prime number, or not composed of two or more numbers,

RULE,

Take the whole divisor, and divide as in division of integers; multiplying the remainder by that number of the next inferior denomination which makes one of its superior, adding to the product what there is in the dividend of the same denomination you are then reducing the remainder to; divide this sum as above, and so proceed in this manner till all be finished.

Chap. III. PRACTICE.

97.

1. If 53 fat Lincolnshire sheep be sold for 99 l. 16 s. 4 d. what was each sold for on an average?

	l.	s.	d.	l.	s.	d.
53)	99	16	4	(1	17 8
	46					
	20					
	936					
	406					
	35					
	12					
	424					

3. At 315 l. 3 s. 10 $\frac{1}{4}$ d. per year, what per day?

	l.	s.	d.	l.	s.	d.
365)	315	3	10 $\frac{1}{4}$	(17	3 $\frac{1}{4}$
	20					
	6303					
	2653					
	98					
	12					
	1186					
	91					
	4					
	365					

2. If 675 l. 12 s. 6 d. be equally divided amongst 138 men, what is each man's share?

	l.	s.	d.	l.	s.	d.
138)	675	12	6	(4	17 11
	123					
	20					
	2472					
	1092					
	126					
	12					
	1518					
	138					

4. A prize of 9475 l. 16 s. 8 d. being divided equally amongst 747 sailors, what is each man's share, after deducting $\frac{1}{4}$ for the captain?

	l.	s.	d.	l.	s.	d.
5)	9475	16	8			
	1895	3	4	captain.		
	747)	7580	13	4	10	2 11 2
	110					166 an.
	20					
	2213					
	719					
	12					
	8632					
	415					
	4					
	1660					

1660 remainder 166.

H

CASE

CASE IV.

To divide weights and measures.

RULE.

Weights and measures are divided exactly in the same manner as money, due regard being only had to the number of those of an inferior denomination contained in a superior one of the same species.

1. If 6 pieces of tapestry contain 227 ells Flemish, 1 quarter, 2 nails, what is the length of one piece?

El. F. qr. n.

227 1 2

 $\div 6$

Ells Fle. 37 2 3, answer.

3. What is the length of 1 piece of linen, if 21 pieces are 754 ells English, 4 quarters, 3 nails?

El. E. qr. n.

754 4 3

 $\div 7$

107 4 1

 $\div 3$

35 4 3, answer.

5. If 28 parcels of tea weigh 6 cwt. 1 qr. 13 lb. 10 oz. 12 dr. what the weight of 1 parcel?

Cwt. q. lb. oz. dr.

6 1 13 10 12

 $\div 4$

1 2 10 6 11

 $\div 7$

- - 25 7 13, answer.

2. If 20 pieces of cloth contain 438 yards, 3 quarters, what does 1 piece contain?

Yds. qr.

438 3

 $\div 4$

109 2 3

 $\div 5$

Yards 21 3 3, answer.

4. If 322 cwt. 2 qrs. 5 lb. is the weight of 25 hogheads, what is the weight of 1 hoghead?

Cwt. qr. lb.

322 2 5

 $\div 5$

64 2 1

 $\div 5$

Cwt. 12 3 17, answer.

6. If 10 silver punch-bowls of an equal weight, weigh 478 oz. 19 dwt. 14 gr. what is the weight of one?

oz. dwt.

478 19 14

 $\div 10$

Oz. 47 17 23, answer.

7. If

Chap. III. PRACTICE.

99

7. If 103 ingots of silver, of an equal weight, weigh 1225 oz. 12 dwt. 13 gr. what is the weight of one ingot?

oz. dwt. gr. oz. dr. dr.

103) 1225 12 13 (11 17 13 15

$$\begin{array}{r}
 195 \\
 \hline
 92 \\
 20 \\
 \hline
 1852 \\
 822 \\
 \hline
 101 \\
 \times 24 \\
 \hline
 2424 \\
 364 \\
 \hline
 55
 \end{array}$$

9. If 335 cwt. 1 qr. 22 lb. is the weight of 26 hogheads of sugar, what is the weight of one hoghead?

Cwt. qr. lb. cwt. qr. lb.

26) 335 1 22 (12 3 17

$$\begin{array}{r}
 75 \\
 \hline
 23 \\
 \times 4 \\
 \hline
 93 \\
 \hline
 15 \\
 \times 28 \\
 \hline
 442 \\
 182 \\
 \hline
 0
 \end{array}$$

8. If 59 parcels of tea, of an equal weight, weigh 2 qr. 1 lb. 9 oz. 14 dr. what is the weight of one parcel?

Qr. lb. oz. dr. oz. dr.

59) 2 1 9 14 (15 10

$$\begin{array}{r}
 57 \\
 \times 16 \\
 \hline
 921 \\
 331 \\
 \hline
 36 \\
 \times 16 \\
 \hline
 590 \\
 \hline
 0
 \end{array}$$

10. A common pasture contains 53 acres, 1 rood, 27 perches; another 65 acres, 2 roods, 19 perches; a third 47 acres, 3 roods; these being inclosed, are to be divided amongst 59 parishioners; how much is each man's share?

A. R. P.

$$\begin{array}{r}
 53 \quad 1 \quad 27 \\
 65 \quad 2 \quad 19 \\
 47 \quad 3 \quad -
 \end{array}$$

A. R. P.

59) 166 3 6 (2 3 12 1/3

$$\begin{array}{r}
 48 \\
 4 \\
 \hline
 195 \\
 \hline
 18 \\
 \times 40 \\
 \hline
 726 \\
 136 \\
 \hline
 1
 \end{array}$$

18 H 2

11. If

11. If 117 pieces of Holland contain 4440 ells English, and 3 nails, what doth each piece contain?

$$\begin{array}{r}
 \text{El. E. qr. n. el. E. qr. n.} \\
 117) 4440 - 3 (.37 \ 4 \ 3 \\
 \underline{930} \\
 111 \\
 \times 5 \\
 \hline
 555 \\
 87 \\
 \times 4 \\
 \hline
 351 \\
 \hline
 0 \\
 \hline
 \end{array}$$

12. The Spectator's club of fat people, though it consisted but of 15 persons, is said (N^o 9.) to weigh no less than three tons; how much at an equality was that per man?

$$\begin{array}{r}
 \text{Tons.} \\
 15) 3 - (4 \text{ cwt. anf.} \\
 \times 20 \\
 \hline
 60 \\
 \hline
 \end{array}$$

CASE V.

To reduce great hundreds (and quarters) 120 lb. to the hundred weight; to hundreds, quarters, and pounds; 112 lb. to the hundred weight.

RULE.

Consider them as small hundreds, and quarters, dividing by 2, and that quotient by 7; which last quotient added to the great weight, gives the small weight.

In 5 cwt. 2 qr. 2 lb. great weight, how much small weight? In 17 cwt. 1 qr. 21 lb. great weight, how much small weight?

$$\begin{array}{r}
 \text{Cwt. qr. lb.} \\
 2 \overline{) 5 \ 2 \ 2} \\
 7 \overline{) 2 \ 3 \ -}
 \end{array}$$

Add - 1 16

Cwt. 5 3 18, small weight.

$$\begin{array}{r}
 \text{Cwt. qr. lb.} \\
 2 \overline{) 17 \ 1 \ 21} \\
 7 \overline{) 8 \ 2 \ 14}
 \end{array}$$

1 - 26 add.

18 2 19 small weight.

CASE VI.

To reduce small weight to great weight.

2

RULE.

Chap. III. PRACTICE.

you

RULE.

Divide the hundreds and quarters, considered as great weight, by 3, and that quotient by 5; which last quotient subtracted from the small weight, leaves the great weight.

$$\begin{array}{r} \text{Cwt. qr. lb.} \\ 3 \overline{) 18 \ 2 \ 10 \text{ fm. weight.}} \\ 5 \overline{) 6 \ - \ 20} \\ \hline \text{Subtract 1} \ - \ 28 \end{array}$$

$$\begin{array}{r} \text{Cwt. qr. lb.} \\ 3 \overline{) 5 \ 3 \ 18 \text{ fm. weight.}} \\ 5 \overline{) 1 \ 3 \ 20} \\ \hline \text{Subt.} \ - \ 1 \ 16 \end{array}$$

$$17 \ 1 \ 12 \text{ gr. weight.}$$

$$5 \ 2 \ 2 \text{ gr. weight.}$$

N. B. The pounds in the first division, are taken no notice of, as being the same both in great and small weight.

A way being 256 lb. of cheese, wool, &c. in some counties of England, is composed of $8 \times 8 \times 4 = 256 \text{ lb.}$
 $= 2 \text{ cwt. } 1 \text{ qr. } 4 \text{ lb.}$

Cheese at $2\frac{3}{4} \text{ d. per lb.}$ what per way?

$$\begin{array}{r} \text{l. s. d.} \\ 2\frac{3}{4} \\ 8 \\ \hline 1 \ 10 \\ 8 \\ \hline 14 \ 8 \\ 4 \end{array}$$

Answer, 2 18 8 per way.

Or as $256 \text{ farthings} = 64 \text{ d.}$
 $= 5 \text{ s. } 4 \text{ d.} = \frac{1}{4} \text{ per lb.}$

$$\begin{array}{r} \text{s. d.} \\ 5 \ 4 \\ 4 \end{array}$$

$$1 \ 1 \ 4 = 1 \text{ d. per lb.}$$

$$2 \ 2 \ 8$$

$$16 \ - = 5 \text{ s. } 4 \text{ d. } \times 3.$$

$$2 \ 18 \ 8 = 2\frac{3}{4} \text{ d. per lb.}$$

$$\begin{array}{r} \text{lb.} \\ 120 \overline{) 256} \\ \frac{1}{2} \frac{1}{2} \frac{1}{2} \end{array}$$

£. 2 18 8 as before.

Cheese at 2 l. 8 s. per way,
 what per lb.?

$$\begin{array}{r} \text{l. s. d.} \\ 8 \overline{) 2 \ 8 \ -} \\ 8 \overline{) \ - \ 6 \ -} \\ 4 \overline{) \ - \ - \ 9} \end{array}$$

$$- \ - \ 2\frac{1}{2} \text{ per lb.}$$

In $51\frac{1}{2}$ weys of cheefe, how many hundreds?

C. qr. lb.

$$2 \text{ I } 4 = 1 \text{ wey.}$$

$$\begin{array}{r} 18 \text{ I } 4 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 109 \text{ 2 } 24 \\ \frac{1}{2}) 6 \text{ 3 } 12 \\ \hline \text{I } 2 \text{ 24} \end{array}$$

$$118 \text{ I } 4 = 51\frac{1}{2} \text{ weys.}$$

$51\frac{1}{2}$ weys of cheefe, at $2\frac{1}{2}$ d, per lb?

l. s. d.

$$\begin{array}{r} 110 \overline{) 256} \\ \underline{22} \\ 36 \\ \underline{35} \\ 14 \\ \underline{14} \\ 0 \end{array}$$

2 8 - per wey.

$$\begin{array}{r} 19 \text{ 4} \\ \hline 6 \end{array}$$

$$\begin{array}{r} 115 \text{ 4} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{1}{2}) 7 \text{ 4} \\ \hline \text{I } 16 \end{array}$$

£ 124 4, the answer.

QUESTIONS to exercise PRACTICE by DIVISION.

1. A draper bought 420 yards of broad-cloth, at the rate of $14s. 10\frac{1}{2}d.$ per ell English, how much did he pay for the whole?

s. d.

$$5 \overline{) 14 \text{ 10}\frac{1}{2} \text{ per ell English.}}$$

$$\underline{2 \text{ 11}\frac{1}{2}}$$

11 11 per yard.

10

$$\begin{array}{r} \text{£ } 5 \text{ 19 } 2 \\ \hline 7 \end{array}$$

$$\begin{array}{r} \text{£ } 41 \text{ 14 } 2 \\ \hline 6 \end{array}$$

£ 258 5 -, the answer required.

2. A draper bought of a merchant 8 packs of broad-cloth; every pack had 4 parcels in it, and each parcel contained 10 pieces; every piece being 26 yards: he gave after the rate of four pounds, seventeen shillings, and sixpence, for 6 yards; what came the whole to, and what did it cost per yard?

$$6) 4l. 17s. 6d. \quad (16s. 3d. \text{ per yard.}$$

$$\begin{array}{r} \times 2 \\ 1 \text{ 12 } 6 \end{array}$$

$$\begin{array}{r} 13 \\ 21 \text{ 2 } 6 \text{ per piece.} \end{array}$$

$$\begin{array}{r} \times 10 \\ 211 \text{ 5 - per parcel.} \end{array}$$

$$\begin{array}{r} \times 4 \\ 845 \text{ - - per pack.} \end{array}$$

$$\begin{array}{r} \times 8 \\ \text{£ } 6760 \text{ - -, the answer.} \end{array}$$

3. An

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3. An oilman bought 3 tuns of oil, which cost him 131 l. 14 s. which happened to leak out 85 gallons; this he is willing to sell again, so as to be no loser: I demand how he must sell it a gallon?

252 gallons in a tun.
3 tun.

756 gallons.
85 leaked.

1. s.
671) 151 14 (4 s. 6½ d. ⁵³/₈₇₁ the answer.
× 20

3034
350
× 12

4200
174
× 4
696

25

4. A draper bought 242 yards of broad-cloth, which cost him in all 256 l. 10 s.; for 86 yards of which he gave 1 l. 1 s. per yard; what did he give a yard for the remainder?

1. s. d.
1 1 4
× 12

1. s. d.
256 10 -
91 14 8

242
- 86

12 16 -
× 7

164 15 4 price of 156 yards.

89 12 -
2 2 8

1. s. d.
156) 164 15 4 (1 l. 1 s. 1½ d. ³⁷/₁₅₆ ans.

£ 91 14 8 price of 86 yds.

× 20
175
19
× 12
232
76
× 4
304
148

H 4

5 A

5. A gentleman, at his death, left his eldest son once and a half what he allotted his daughter; and to the young lady 1383l. less than her mother, to whom he bequeathed four times what he left towards the endowment of Hertford college, Oxon, viz. 1640 guineas: I require what he intended for his youngest son, who claimed, under the will, half as much as his mother and sister; how much less than 30000l. did the testator die worth, his debts and funeral expences being 988 l. 10s.?

20) 1640 guineas.

+ 82

£ 1722 Hertford-college.

× 4

6888 wife.

— 1383

2) 5505 daughter.

+ 2752 10s.

£ 8257 10s. eldest son.

6888

+ 5505

2) 12393

£ 6196 10s. youngest son.

l. s.

Then mother - 6888

Eldest son 8257 10

Youngest 6196 10

Daughter 5505

College - 1722

Funeral - 988 10

30000 — 29557 10s = 442 l. 10s., the answer.

6. My purse and money, quoth Dick, are worth 12s. 8d. but the money is worth seven of the purse; pray what was there in it?

s. d.

8) 12 8 purse and money.

1 7 purse.

11 1, money, the answer.

7. A

7. A dealer bought two lots of sauff, that together weighed 9 cwt. 3 qrs. 16 lb., for 97l. 17s. 6d. Their difference in point of weight was 1 cwt. 2 qrs. 16 lb.; and of price 8l. 13s. 3d. Their respective weights and values are required?

	Cwt.	qr.	lb.		l.	s.	d.
Weight	-	9	3	16	Cost	-	97 17 6
Difference	-	1	2	16			8 13 3
	2)	8	1	-		
					2)	89 4 3
Lesser lot	-	-	4	-	14		44 12 1 $\frac{1}{2}$
Greater	-	-	5	3	2		53 5 4 $\frac{1}{2}$

8. A tradesman increased his estate annually a third part, abating 100l., which he usually spent in his family; and at the end of $3\frac{1}{4}$ years, found that his net estate amounted to 3179l. 11s. 8d. Pray what had he at out-setting?

Worth at the end of $3\frac{1}{4}$ years - 3179 11 8.
 $4) 100 = 25 - -$

$3\frac{1}{4}$ years = 13 quarters $\therefore 13) 3204 11 8$
 $- 246 10 1\frac{1}{2}$

Worth at the end of 3 years - 2958 1 6 $\frac{1}{2}$
 $+ 100 - -$
 $4) 3058 1 6\frac{1}{2}$
 $764 10 4\frac{1}{2}$

Worth at the end of 2 years - 2293 11 2
 $+ 100 - -$
 $4) 2393 11 2$
 $598 7 9\frac{1}{2}$

Worth at 1 year's end - 1795 3 4 $\frac{1}{2}$
 $+ 100 - -$
 $1895 3 4\frac{1}{2}$
 $- 473 15 10$

Answer, £ 1421 7 6 $\frac{1}{2}$

R U L E.

For the aliquot part of a pound, divide the given quantity by that part; the quotient will be pounds, and the remainder so many times that part. But for the aliquot parts of a shilling, divide as before, and that quotient by 20; the last quotient will be pounds, and the remainder shillings.

But if it be a compound aliquot part, take the first aliquot part, and then the aliquot part of that part, which last quotient will answer the question.

- $\frac{1}{2}$ 737

£ 368 10 -, answer,

- $\frac{5}{7} | 873$ s. d.

£ 294 - -, answer.

- $\frac{1}{4} | 3712$ s. d.

£ 927 15 -, anſwer.

- $\frac{1}{3} \overline{) 1171}$ c. d.

£ 234 4 -, answer.

- 743, d.

£ 123 16 8, answer.

- $\frac{1}{7}$ 275 s. d.

£ 34 7 6, answer.

- $\frac{1}{18} | 765$

£ 76 10, answer.

- $\frac{1}{12} | 1768$ 2.

£ 146 15, answer.

- $$\frac{1}{30} \overline{) 757} \text{ R.}$$

£ 37 17, answer.

- ## 10. What

10. What cost 957 yards of
dowlas, at 1s. 3d.?

$$\begin{array}{r} \frac{1}{4} | 957 \quad \text{s. d.} \\ \frac{1}{4} | 239 \quad 1 \quad 3 \end{array}$$

£ 59 16 3, answer.

$$15\text{s.} + 1\text{s. } 3\text{d.} = 16\text{s. } 3\text{d.}$$

Here, and in several questions below, the remainder is looked upon to be of the same denomination with the dividend; and the second remainder is always added to the first.

12. 737 yards ditto, at 10d.?

$$\begin{array}{r} \frac{1}{4} | 737 \quad \text{s. d.} \\ \frac{3}{8} | 184 \quad - \quad 10 \end{array}$$

£ 30 14 2, answer.

$$13\text{s. } 4\text{d.} + 10\text{d.} = 14\text{s. } 2\text{d.}$$

14. 1511 yards ditto, at 5d.?

$$\begin{array}{r} \frac{5}{8} | 1511 \quad \text{s. d.} \\ \frac{1}{4} | 251 \quad 2 \quad 1 \end{array}$$

£ 31 9 7, answer.

$$7\text{s. } 6\text{d.} + 2\text{s. } 1\text{d.} = 9\text{s. } 7\text{d.}$$

Questions in practice admit of various ways of working, equally short, which serve as a proof to each other; and indeed practice is best proved by practice, though it may be proved by several other methods.

15. What cost 1511 yards of ribbon, at 6d. per yard?

$$\begin{array}{r} \frac{1}{4} | 1511 \quad \text{s. d.} \\ \frac{1}{10} | 377 \quad 1 \quad 6 \end{array}$$

£ 37 15 6, answer.

$$\text{Or } \frac{1}{4} | 1511 \quad \text{s. d.} \\ \frac{1}{10} | 755 \quad 6 \quad -$$

£ 37 15 6, answer.

$$\begin{array}{r} \frac{1}{10} | 1511 \quad \text{s. d.} \\ \frac{1}{10} | 3715 \quad 6 \end{array}$$

Remains 31 pences = 15s. 6d.

16. What

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16. What cost 1109 yards of small ribbon, at 4 d. per yard?

$$\begin{array}{r} \frac{1}{4} | 1109 \text{ s. d.} \\ \frac{1}{10} | 221 \text{ 11 4} \\ \hline \text{£ } 18 \text{ 9 8, answer.} \end{array}$$

$$\begin{array}{r} \frac{1}{4} | 1109 \text{ d.} \\ \frac{1}{10} | 369 \text{ 8} \\ \hline \text{£ } 18 \text{ 9 8, answer.} \end{array}$$

$$\begin{array}{r} \frac{1}{10} | 1109 \text{ s. d.} \\ \hline \text{£ } 18 \text{ 9 8, answer.} \end{array}$$

Remains 29 groats, or 4 times 2 s. 5 d.

17. What cost 751 yards ditto, at 3 d. per yard?

$$\begin{array}{r} \frac{1}{3} | 751 \text{ d.} \\ \frac{1}{10} | 187 \text{ 9} \\ \hline \text{£ } 9 \text{ 7 9, answer.} \end{array}$$

$$\begin{array}{r} \frac{1}{10} | 751 \text{ d.} \\ \hline \text{£ } 9 \text{ 9, answer.} \end{array}$$

Remains 31 threepences = 7 s. 9 d.

18. What cost 1511 yards of ditto, at 2 d. per yard?

$$\begin{array}{r} \frac{1}{2} | 1511 \text{ d.} \\ \frac{1}{10} | 251 \text{ 10} \\ \hline \text{£ } 12 \text{ 11 10.} \end{array}$$

$$\begin{array}{r} \frac{1}{10} | 1511 \text{ s. d.} \\ \hline 12 \text{ 11 10} \end{array}$$

Remains 31 twopences = 11 s. 10 d.

19. 1173 yards of ferreting, at $\frac{1}{2}$ d.?

$$\begin{array}{r} \frac{1}{2} | 1173 \text{ d.} \\ \frac{1}{10} | 146 \text{ 7} \frac{1}{2} \\ \hline \text{£ } 7 \text{ 6 7} \frac{1}{2} \end{array}$$

21. 1071 yards of filleting, at $\frac{1}{4}$ f.?

$$\begin{array}{r} \frac{1}{4} | 1071 \text{ d.} \\ \frac{1}{10} | 267 \text{ 2} \frac{1}{4} \\ \hline \text{£ } 3 \text{ 6 11} \frac{1}{4}, \text{ answer.} \end{array}$$

20. 713 yards ditto, at 1 d. ?

$$\begin{array}{r}
 \frac{1}{12} \overline{) 713} \quad \text{s. d.} \\
 \frac{1}{10} \overline{) 59 \ 5 \ 1} \\
 \hline
 2 \ 19 \ 5, \text{ answer.}
 \end{array}$$

22. 1075 yards ditto, at $\frac{1}{4}$?

$$\begin{array}{r}
 \frac{1}{2} \overline{) 1075} \\
 \frac{1}{12} \overline{) 537 \frac{1}{2}} \quad \text{s. d.} \\
 \frac{1}{20} \overline{) 44} = 9 \frac{1}{2} \\
 \hline
 \pounds \ 2 \ 4 \ 9 \frac{1}{2}
 \end{array}$$

23. 717 yards, at $\frac{1}{4}$ d. ?

$$\begin{array}{r}
 12 \overline{) 717} \\
 4 \overline{) 717} \ 179 \frac{1}{2} \\
 \hline
 14 \text{ s. } 11 \frac{1}{2} \text{ d., answer.}
 \end{array}$$

CASE II.

When the numerator of the fraction is more than unity, and the denominator 10,

RULE,

Multiply the given quantity by the numerator, and double the figure in the units place of the product for shillings; the figures to the left-hand will be pounds.

1. What cost 757 ells of 717 ells of long-lawn, at 18 s. per ell ?

$$\begin{array}{r}
 \overline{) 757} \\
 9 \\
 \hline
 \pounds \ 681 \ 6, \text{ answer.}
 \end{array}$$

$$\begin{array}{r}
 \overline{) 717} \\
 4 \\
 \hline
 \pounds \ 286 \ 16, \text{ answer.}
 \end{array}$$

2. 617 ells ditto, at 16 s. ?

$$\begin{array}{r}
 \overline{) 617} \\
 8 \\
 \hline
 \pounds \ 493 \ 12, \text{ answer.}
 \end{array}$$

993 ells ditto, at 6 s. ?

$$\begin{array}{r}
 \overline{) 993} \\
 3 \\
 \hline
 \pounds \ 279 \ 18, \text{ answer.}
 \end{array}$$

3. 577 ells ditto, at 14 s. ?

$$\begin{array}{r}
 \overline{) 577} \\
 7 \\
 \hline
 \pounds \ 403 \ 18
 \end{array}$$

714 ells ditto, at 4 s. ?

$$\begin{array}{r}
 \overline{) 714} \\
 2 \\
 \hline
 \pounds \ 142 \ 16, \text{ answer.}
 \end{array}$$

4. 1187 ells ditto, at 12 s. ?

$$\begin{array}{r} 1187 \\ 6 \overline{) } \\ \underline{6 } \\ 587 \\ 540 \\ \underline{47 } \\ 7 \end{array} \begin{array}{l} \text{s.} \\ \text{£ } 712 \text{ 4, answer.} \end{array}$$

933 ells ditto, at 2 s. ?

£ 93 6s., answer.

CASE III.

When the numerator is any number under 12, and the denominator 12, or under, with or without a cypher annexed,

RULE,

Multiply the given quantity by the numerator, and divide the product by the denominator; the quotient will be pounds or shillings, according to the nature of the question; and the remainder so many times the aliquot part expressed by the denominator.

1. What cost 737 yards of broad-cloth, at 18 s. 4 d. ?

$$\begin{array}{r} 737 \\ 11 \overline{) } \\ \underline{11 } \\ 128107 \end{array} \begin{array}{l} \text{s.} \quad \text{d.} \\ \text{£ } 675 \text{ 11 } 8, \text{ answer.} \end{array}$$

Rem. 7 times 1 s. 8 d. = 11 s. 8 d.

2. 371 yards ditto, at 17 s. 6 d. ?

$$\begin{array}{r} 371 \\ 7 \overline{) } \\ \underline{2597} \end{array} \begin{array}{l} \text{s.} \quad \text{d.} \\ \text{£ } 324 \text{ 12 } 6, \text{ answer.} \end{array}$$

Rem. 5 times 2 s. 6 d. = 12 s. 6 d.

3. 931 yards ditto, at 16 s. 8 d. ?

$$\begin{array}{r} 931 \\ 5 \overline{) } \\ \underline{4655} \end{array} \begin{array}{l} \text{s.} \quad \text{d.} \\ \text{£ } 775 \text{ 16 } 8, \text{ answer.} \end{array}$$

Rem. 5 times 3 s. 4 d. = 16 s. 8 d.

4. 573 yards ditto, at 16 s. ?

$$\begin{array}{r} 573 \\ 4 \overline{) } \\ \underline{2292} \end{array} \begin{array}{l} \text{s.} \\ \text{£ } 458 \text{ 8, answer.} \end{array}$$

Rem. 2 times 4 s. = 8 s.

5. 746 yards ditto, at 15s.?

$$\begin{array}{r} 746 \\ 3 \\ \hline 4 \overline{)2238} \end{array}$$

£ 559 10 s. d. —, answer.

6. 865 yards ditto, at 13 s. 4 d.?

$$\begin{array}{r} 865 \\ 2 \\ \hline 3 \overline{)2730} \end{array}$$

£ 576 13 s. d. 4, answer.

Rem. 2 times 6 s. 8 d. = 13 s. 4 d.

7. 713 yards ditto, at 12 s. 6 d.?

$$\begin{array}{r} 713 \\ 5 \\ \hline 8 \overline{)3565} \end{array}$$

£ 445 12 s. d. 6, answer.

Rem. 5 times 2 s. 6 d. = 12 s. 6 d.

8. 783 yards ditto, at 12 s.?

$$\begin{array}{r} 783 \\ 3 \\ \hline 5 \overline{)2349} \end{array}$$

£ 469 16 s. —, answer.

Rem. 4 times 4 s. = 16 s.

9. What cost 137 ells of Holland, at 11 s. 8 d.?

$$\begin{array}{r} 137 \\ 7 \\ \hline \end{array}$$

12 959

£ 79 18 s. d. 4, answer.

Rem. 11 times 1 s. 8 d. = 18 s. 4 d.

10. 537 ells ditto, at 8 s. 4 d.?

$$\begin{array}{r} 537 \\ 5 \\ \hline 12 \overline{)2685} \end{array}$$

£ 223 15 s. —, answer.

Rem. 9 times 1 s. 8 d. = 15 s.

11. 537 ells ditto, at 8 s.?

$$\begin{array}{r} 537 \\ 2 \\ \hline 5 \overline{)1074} \end{array}$$

£ 214 16 s. d. —, answer.

Rem. 4 times 4 s. = 16 s.

12. 719 ells ditto, at 7 s. 6 d.?

$$\begin{array}{r} 719 \\ 3 \\ \hline 8 \overline{)2157} \end{array}$$

£ 269 12 s. d. 6, answer.

Rem. 5 times 2 s. 6 d. = 12 s. 6 d.

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13. What cost 157 ells of Holland, at 7 s. 4 d.?

$$\begin{array}{r} 157 \\ 11 \\ \hline 30 \overline{) 1727} \\ \hline \end{array} \quad \begin{array}{l} \text{s. d.} \\ \text{£ } 57 \text{ } 11 \text{ } 4, \text{ answer.} \end{array}$$

Rem. 17 times 8 d. = 14 s. 4 d.

14. 737 lb. of bohea tea, at 5 s. 6 d.?

$$\begin{array}{r} 737 \\ 11 \\ \hline 40 \overline{) 8107} \\ \hline \end{array} \quad \begin{array}{l} \text{s. d.} \\ \text{£ } 202 \text{ } 13 \text{ } 6, \text{ answer.} \end{array}$$

Rem. 27 times 6 d. = 13 s. 6 d.

15. What cost 871 ounces of plate, at 5 s. 4 d.?

$$\begin{array}{r} 871 \\ 8 \\ \hline 30 \overline{) 6968} \\ \hline \end{array} \quad \begin{array}{l} \text{s. d.} \\ \text{£ } 232 \text{ } 5 \text{ } 4, \text{ answer.} \end{array}$$

Rem. 8 times 8 d. = 5 s. 4 d.

16. 837 yards of kersey, at 4 s. 8 d.?

$$\begin{array}{r} 837 \\ 7 \\ \hline 30 \overline{) 5859} \\ \hline \end{array} \quad \begin{array}{l} \text{s.} \\ \text{£ } 195 \text{ } 6, \text{ answer.} \end{array}$$

17. 713 yards ditto, at 4 s. 6 d.?

$$\begin{array}{r} 713 \\ 9 \\ \hline 40 \overline{) 6417} \\ \hline \end{array} \quad \begin{array}{l} \text{s. d.} \\ \text{£ } 160 \text{ } 8 \text{ } 6, \text{ answer.} \end{array}$$

Rem. 17 times 6 d. = 8 s. 6 d.

18. 379 yards ditto, at 3 s. 8 d.?

$$\begin{array}{r} 379 \\ 11 \\ \hline 60 \overline{) 4169} \\ \hline \end{array} \quad \begin{array}{l} \text{s. d.} \\ \text{£ } 69 \text{ } 9 \text{ } 8, \text{ answer.} \end{array}$$

Rem. 29 times 4 d. = 9 s. 8 d.

19. 719 yards ditto, at 3 s.?

$$\begin{array}{r} 719 \\ 3 \\ \hline 20 \overline{) 2157} \\ \hline \end{array} \quad \begin{array}{l} \text{s. d.} \\ \text{£ } 107 \text{ } 17 \text{ } -, \text{ answer.} \end{array}$$

20. 173 yards ditto, at 2 s. 9 d.?

$$\begin{array}{r} 173 \\ 11 \\ \hline 80 \overline{) 1903} \\ \hline \end{array} \quad \begin{array}{l} \text{s. d.} \\ \text{£ } 23 \text{ } 15 \text{ } 9, \text{ answer.} \end{array}$$

Rem. 63 times 3 d. = 15 s. 9 d.

I

21. What

21. What cost 1735 yards of kersey, at 3s. 6d.?

$$\begin{array}{r} 1735 \\ 7 \\ \hline 40 \overline{) 12145} \end{array} \begin{array}{l} \text{s.} \quad \text{d.} \\ \text{£} \quad 303 \quad 12 \quad 6, \text{ answer.} \end{array}$$

Rem. 25 times 6d. = 12s. 6d.

22. 931 yards ditto, at 2s. 8d.?

$$\begin{array}{r} 931 \\ 4 \\ \hline 30 \overline{) 3724} \end{array} \begin{array}{l} \text{s.} \quad \text{d.} \\ \text{£} \quad 124 \quad 2 \quad 8, \text{ answer.} \end{array}$$

23. 107 yards ditto, at 2s. 4d.?

$$\begin{array}{r} 107 \\ 7 \\ \hline 60 \overline{) 749} \end{array} \begin{array}{l} \text{s.} \quad \text{d.} \\ \text{£} \quad 12 \quad 9 \quad 8, \text{ answer.} \end{array}$$

Rem. 29 times 4d. = 9s. 8d.

24. 713 yards ditto, at 2s. 3d.?

$$\begin{array}{r} 713 \\ 9 \\ \hline 80 \overline{) 6417} \end{array} \begin{array}{l} \text{s.} \quad \text{d.} \\ \text{£} \quad 80 \quad 4 \quad 3, \text{ answer.} \end{array}$$

Rem. 17 times 3d. = 4s. 3d.

25. 795 yards ditto, at 1s. 10d.?

$$\begin{array}{r} 795 \\ 11 \\ \hline 120 \overline{) 8745} \end{array} \begin{array}{l} \text{s.} \quad \text{d.} \\ \text{£} \quad 72 \quad 17 \quad 6, \text{ answer.} \end{array}$$

Rem. 105 times 2d. = 17s. 6d.

26. What cost 173 yards of dowlas, at 1s. 9d.?

$$\begin{array}{r} 173 \\ 7 \\ \hline 80 \overline{) 1211} \end{array} \begin{array}{l} \text{s.} \quad \text{d.} \\ \text{£} \quad 15 \quad 2 \quad 9, \text{ answer.} \end{array}$$

Rem. 11 times 3d. = 2s. 9d.

27. 713 yards ditto, at 1s. 6d.?

$$\begin{array}{r} 713 \\ 3 \\ \hline 40 \overline{) 2139} \end{array} \begin{array}{l} \text{s.} \quad \text{d.} \\ \text{£} \quad 53 \quad 9 \quad 6, \text{ answer.} \end{array}$$

28. 913 yards ditto, at 1s. 4d.?

$$\begin{array}{r} 913 \\ 4 \\ \hline 60 \overline{) 3652} \end{array} \begin{array}{l} \text{s.} \quad \text{d.} \\ \text{£} \quad 60 \quad 17 \quad 4, \text{ answer.} \end{array}$$

Rem. 52 times 4d. = 17s. 4d.

29. What

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29. What cost 783 yards of dowlas, at 1 s. 3 d. ?

$$\begin{array}{r} 783 \\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ 3915 \\ \hline \end{array}$$

£ 48 18 9, answer.

Rem. 75 times 3d. = 18s. 9d.

30. 713 yards ditto, at 1 s. 2 d. ?

$$\begin{array}{r} 713 \\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 120 \\ 4991 \\ \hline \end{array}$$

£ 41 11 10, answer.

Rem. 71 times 2d. = 11s. 10d.

31. What cost 737 lb. of tobacco, at 10½ d. ?

$$\begin{array}{r} 737 \\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ 5159 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ 644 \\ \hline \end{array}$$

£ 32 4 10½, answer.

32. 673 lb. ditto, at 10 d. ?

$$\begin{array}{r} 673 \\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 120 \\ 3365 \\ \hline \end{array}$$

£ 28 - 10, answer.

Or 673

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ 3365 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ 560 \\ \hline \end{array}$$

£ 28 - 10, answer.

33. 971 lb. ditto, at 9 d. ?

$$\begin{array}{r} 971 \\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ 2913 \\ \hline \end{array}$$

£ 36 8 3, answer.

Or 971

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ 2913 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ 728 \\ \hline \end{array}$$

36 8 3

34. 1713 yards of ribbon, at 7½ d. ?

$$\begin{array}{r} 1713 \\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ 8565 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ 1070 \\ \hline \end{array}$$

£ 53 10 7½, answer.

35. What cost 1735 yards of ribbon, at 8 d. ?

$$\begin{array}{r} 1735 \\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ 3470 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ 1156 \\ \hline \end{array}$$

£ 57 16 8, answer.

Or 30 1735

$$\begin{array}{r} 8 \\ \hline \end{array}$$

£ 57 16 8, answer.

Rem. 25 times 8d. = 16s. 8d.

I 2

26. What

36. What cost 753 yards of ribbon, at $4\frac{1}{2}$ d.?

$$\begin{array}{r}
 753 \\
 3 \overline{) 2259} \quad \text{d.} \\
 8 \overline{) 282} \quad 4\frac{1}{2} \\
 \text{£ } 14 \quad 2 \quad 4\frac{1}{2} \text{ answer.}
 \end{array}$$

37. 783 yards ditto, at $5\frac{1}{2}$?

$$\begin{array}{r}
 783 \\
 2 \overline{) 5481} \quad \text{d.} \\
 8 \overline{) 685} \quad 1\frac{1}{2} \\
 20 \overline{) 342} \quad 6\frac{3}{4} \\
 \text{£ } 17 \quad 2 \quad 6\frac{3}{4} \text{ answer.}
 \end{array}$$

38. 575 yards ditto, at $3\frac{1}{4}$ d.?

$$\begin{array}{r}
 575 \\
 5 \overline{) 2875} \quad \text{d.} \\
 8 \overline{) 359} \quad 4\frac{1}{2} \\
 20 \overline{) 179} \quad 8\frac{1}{4} \\
 \text{£ } 8 \quad 19 \quad 8\frac{1}{4} \text{ answer.}
 \end{array}$$

CASE IV.

When the price is less than a pound, or a shilling, by a single aliquot part of either,

RULE,

Take that aliquot part of the quantity, which subtract from the quantity; the remainder will be the price of the whole in pounds or shillings.

1. What cost 787 yards of velvet, at 19 s. per yard?

$$\begin{array}{r}
 787 \\
 20 \overline{) 397} \\
 \text{£ } 747 \quad 13, \text{ answer.}
 \end{array}$$

2. What cost 1135 yards ditto, at 18 s. 4 d.?

$$\begin{array}{r}
 1135 \\
 12 \overline{) 9411} \quad 8 \\
 \text{£ } 1040 \quad 8 \quad 4, \text{ answer.}
 \end{array}$$

3. What

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3. What cost 785 yards of velvet, at 17 s. 6 d.?

$$\begin{array}{r} \frac{1}{10} \overline{)785} \\ 98 \quad 2 \quad 6 \end{array}$$

£ 686 17 6, answer.

4. 937 yds. ditto, at 18 s.?

$$\frac{1}{10} \overline{)937} \\ 93 \quad 14$$

£ 843 6, answer.

5. 731 yards ditto, at 16 s. 8 d.?

$$\frac{1}{4} \overline{)731} \\ 121 \quad 16 \quad 8$$

£ 609 3 4, answer.

6. 573 yds. ditto, at 16 s.?

$$\frac{1}{3} \overline{)573} \\ 114 \quad 12$$

£ 458 8, answer.

7. 1537 yds. ditto, at 15 s.?

$$\frac{1}{4} \overline{)1537} \\ 384 \quad 5$$

£ 1152 15, answer.

8. 536 yards ditto, at 13 s. 4 d.?

$$\frac{1}{3} \overline{)536} \\ 178 \quad 13 \quad 4$$

£ 357 6 8, answer.

9. 1511 yards ditto, at 19 s. 10 d.?

$$\frac{1}{10} \overline{)1511} \\ 12 \quad 11 \quad 10$$

£ 1488 8 2, answer.

10. 1109 yards ditto, at 19 s. 9 d.?

$$\frac{1}{10} \overline{)1109} \\ 13 \quad 17 \quad 3$$

£ 1095 2 9, answer.

11. 1109 yards ditto, at 19 s. 8 d.?

$$\frac{1}{10} \overline{)1109} \\ 18 \quad 9 \quad 8$$

£ 1090 10 4, answer.

12. 1511 yards ditto, at 19 s. 6 d.?

$$\frac{1}{10} \overline{)1511} \\ 37 \quad 15 \quad 6$$

£ 1473 4 6, answer.

13. 737 yards ditto, at 19 s. 4 d.?

$$\frac{1}{10} \overline{)737} \\ 24 \quad 11 \quad 4$$

£ 712 8 8, answer.

1. What cost 731 yards of broad-cloth, at 18 s. 4 d.?

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} | 731$$

365	10
182	15
121	16 8

£ 670 1 8, anf.

Or,

$$\frac{1}{10} + \frac{1}{20} | 731$$

657	18
12	3 8

£ 670 1 8, answer.

2. 957 yards ditto, at 17 s. 8 d.?

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} | 957$$

478	10
319	-
47	17

£ 845 7, answer.

Or,

$$\frac{1}{10} + \frac{1}{12} | 957$$

765	12
79	15

£ 845 7, answer.

3. 107 yards ditto, at 16 s. 4 d.?

$$\frac{1}{10} + \frac{1}{25} | 107$$

85	12
1	15 8

£ 87 7 8, answer.

4. 377 yards ditto, at 15 s. 4 d.?

$$\frac{1}{10} + \frac{1}{5} | 377$$

226	4
62	16 8

£ 289 - 8, answer.

5. 317 yards ditto, at 14 s. 3 d.?

$$\frac{1}{10} + \frac{1}{20} | 317$$

221	18
3	19 3

£ 225 17 3, answer.

6. 101 yards ditto, at 12 s. 2 d.?

$$\frac{1}{10} + \frac{1}{50} | 101$$

60	12
-	16 10

£ 61 8 10, ansf.

7. What cost 713 gallons of rum, at 11 s. 4 d. per gallon?

$$\frac{1}{10} + \frac{1}{5} | 713$$

285	4
118	16 8

£ 404 - 8, answer.

8. 571 gallons ditto, at 10 s. 8 d.?

$$\frac{1}{2} + \frac{1}{10} | 571$$

285	10
19	- 8

£ 304 10 8, answer.

9. What cost 109 gallons of rum, at 9 s. 8 d. per gallon?

$$\frac{1}{10} + \frac{1}{10} \overline{)109}$$

43	12
9	1 8

£ 52 13 8, answer.

10. 137 gallons ditto, at 9 s. 2 d.?

$$\frac{1}{4} + \frac{1}{8} \overline{)137}$$

45	13	4
17	2	6

£ 62 15 10, answer.

11. 719 gallons ditto, at 8 s. 3 d.?

$$\frac{1}{10} + \frac{1}{20} \overline{)719}$$

287	12
8	19 9

£ 296 11 9, answer.

12. 473 yards of Holland, at 8 s. 4 d.?

$$\frac{1}{4} + \frac{1}{10} \overline{)473}$$

118	5
78	16 8

£ 197 1 8, answer.

13. 157 yards ditto, at 7 s. 4 d.?

$$\frac{1}{2} + \frac{1}{10} \overline{)157}$$

52	6	8
5	4	8

£ 57 11 4, answer.

14. 703 ells ditto, at 6 s. 11 d. per ell?

$$\frac{1}{2} + \frac{1}{20} \overline{)703}$$

234	6	8
8	15	9

£ 243 2 5, answer.

15. 959 ells ditto, at 5 s. 10 d.?

$$\frac{1}{5} + \frac{1}{10} \overline{)959}$$

159	16	8
119	17	6

£ 279 14 2, answer.

16. 371 ells ditto, at 4 s. 2 d.?

$$\frac{1}{2} + \frac{1}{20} \overline{)371}$$

74	4
3	1 10

£ 77 5 10, answer.

17. 873 yards of kersey, at 3 s. 10 d. per yard?

$$\frac{1}{5} + \frac{1}{20} \overline{)873}$$

145	10
21	16 6

£ 167 6 6, answer.

18. 379 yards ditto, at 3 s. 8 d.?

$$\frac{1}{5} + \frac{1}{20} \overline{)379}$$

63	3	4
6	6	4

£ 69 9 8, answer.

19. What

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19. What cost 891 yards of kerley, at 3s. 7d. per yard?

$$\frac{1}{8} + \frac{1}{10} \overline{)891}$$

148	10
11	2 9

£ 159 12 9, answer.

20. 1735 yards ditto, at 3s. 6d.?

$$\frac{1}{8} + \frac{1}{20} \overline{)1735}$$

216	17	6
86	15	-

£ 303 12 6, answer.

21. 907 yards ditto, at 3s. 2d.?

$$\frac{1}{8} + \frac{1}{25} \overline{)907}$$

113	7	6
30	4	8

£ 143 12 2, answer.

22. 719 yards ditto, at 3s.?

$$\frac{1}{10} + \frac{1}{20} \overline{)719}$$

71	18
35	19

£ 107 17, answer.

23. 873 yards ditto, at 2s. 10d.?

$$\frac{1}{5} + \frac{1}{20} \overline{)873}$$

109	2	6
14	11	-

£ 123 13 6, answer.

24. 173 yards ditto, at 2s. 9d.?

$$\frac{1}{8} + \frac{1}{10} \overline{)173}$$

21	12	6
2	3	3

£ 23 15 9, answer.

25. 931 yards ditto, at 2s. 8d.?

$$\frac{1}{8} + \frac{1}{25} \overline{)931}$$

116	7	6
7	15	2

£ 124 2 8, answer.

26. 107 yards ditto, at 2s. 4d.?

$$\frac{1}{10} + \frac{1}{25} \overline{)107}$$

10	14
1	15 8

£ 12 9 8, answer.

27. 735 yards of Irish cloth, at 2s. 3d.?

$$\frac{1}{10} + \frac{1}{20} \overline{)735}$$

73	10
9	3 9

£ 82 13 9, answer.

28. 317 yards ditto, at 2s. 2d.?

$$\frac{1}{10} + \frac{1}{25} \overline{)317}$$

31	14
2	12 10

£ 34 6 10, answer.

29. What

9. What cost 109 gallons of rum, at 9 s. 8 d. per gallon?

$$\frac{1}{10} + \frac{1}{12} | 109$$

$$\begin{array}{r} 43 \ 12 \\ 9 \ 1 \ 8 \end{array}$$

£ 52 13 8, answer.

10. 137 gallons ditto, at 9 s. 2 d.?

$$\frac{1}{3} + \frac{1}{6} | 137$$

$$\begin{array}{r} 45 \ 13 \ 4 \\ 17 \ 2 \ 6 \end{array}$$

£ 62 15 10, answer.

11. 719 gallons ditto, at 8 s. 3 d.?

$$\frac{1}{10} + \frac{1}{20} | 719$$

$$\begin{array}{r} 287 \ 12 \\ 8 \ 19 \ 9 \end{array}$$

£ 296 11 9, answer.

12. 473 yards of Holland, at 8 s. 4 d.?

$$\frac{1}{4} + \frac{1}{5} | 473$$

$$\begin{array}{r} 118 \ 5 \\ 78 \ 16 \ 8 \end{array}$$

£ 197 1 8, answer.

13. 157 yards ditto, at 7 s. 4 d.?

$$\frac{1}{2} + \frac{1}{5} | 157$$

$$\begin{array}{r} 52 \ 6 \ 8 \\ 5 \ 4 \ 8 \end{array}$$

£ 57 11 4, answer.

14. 703 ells ditto, at 6 s. 11 d. per ell?

$$\frac{1}{2} + \frac{1}{20} | 703$$

$$\begin{array}{r} 234 \ 6 \ 8 \\ 8 \ 15 \ 9 \end{array}$$

£ 243 2 5, answer.

15. 959 ells ditto, at 5 s. 10 d.?

$$\frac{1}{5} + \frac{1}{5} | 959$$

$$\begin{array}{r} 159 \ 16 \ 8 \\ 119 \ 17 \ 6 \end{array}$$

£ 279 14 2, answer.

16. 371 ells ditto, at 4 s. 2 d.?

$$\frac{1}{5} + \frac{1}{25} | 371$$

$$\begin{array}{r} 74 \ 4 \\ 3 \ 1 \ 10 \end{array}$$

£ 77 5 10, answer.

17. 873 yards of kersey, at 3 s. 10 d. per yard?

$$\frac{1}{5} + \frac{1}{20} | 873$$

$$\begin{array}{r} 145 \ 10 \\ 21 \ 16 \ 6 \end{array}$$

£ 167 6 6, answer.

18. 379 yards ditto, at 3 s. 8 d.?

$$\frac{1}{5} + \frac{1}{25} | 379$$

$$\begin{array}{r} 63 \ 3 \ 4 \\ 6 \ 6 \ 4 \end{array}$$

£ 69 9 8, answer.

19. What

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19. What cost 891 yards of Kersey, at 3s. 7d. per yard?

$$\frac{1}{2} + \frac{1}{10} \overline{)891}$$

148	10
11	2 9

£ 159 12 9, answer.

20. 1735 yards ditto, at 3s. 6d.?

$$\frac{1}{2} + \frac{1}{10} \overline{)1735}$$

216	17	6
86	15	-

£ 303 12 6, answer.

21. 907 yards ditto, at 3s. 2d.?

$$\frac{1}{2} + \frac{1}{10} \overline{)907}$$

113	7	6
30	4	8

£ 143 12 2, answer.

22. 719 yards ditto, at 3s.?

$$\frac{1}{10} + \frac{1}{100} \overline{)719}$$

71	18
35	19

£ 107 17, answer.

23. 873 yards ditto, at 2s. 10d.?

$$\frac{1}{2} + \frac{1}{10} \overline{)873}$$

109	2	6
14	11	-

£ 123 13 6, answer.

24. 173 yards ditto, at 2s. 9d.?

$$\frac{1}{2} + \frac{1}{10} \overline{)173}$$

21	12	6
2	3	3

£ 23 15 9, answer.

25. 931 yards ditto, at 2s. 8d.?

$$\frac{1}{2} + \frac{1}{10} \overline{)931}$$

116	7	6
7	15	2

£ 124 2 8, answer.

26. 107 yards ditto, at 2s. 4d.?

$$\frac{1}{10} + \frac{1}{100} \overline{)107}$$

10	14
1	15 8

£ 12 9 8, answer.

27. 735 yards of Irish cloth, at 2s. 3d.?

$$\frac{1}{10} + \frac{1}{100} \overline{)735}$$

73	10
9	3 9

£ 82 13 9, answer.

28. 317 yards ditto, at 2s. 2d.?

$$\frac{1}{10} + \frac{1}{100} \overline{)317}$$

31	14
2	12 10

£ 34 6 10, answer.

29. What

29. What cost 137 yards of Irish cloth, at 1 s. 11 d.?

$$\frac{1}{12} + \frac{1}{10} \overline{)137}$$

11	8	4
1	14	3

£ 13. 2 7, answ.

30. 795 yards ditto, at 1 s. 10 d.

$$\frac{1}{12} + \frac{1}{10} \overline{)795}$$

66	5
6	12 6

£ 72 17 6, answ.

31. 713 yards ditto, at 1 s. 6 d.?

$$\frac{1}{10} + \frac{1}{10} \overline{)713}$$

35	13
17	16 6

£ 53 9 6, answ.

32. 913 yards ditto, at 1 s. 4 d.?

$$\frac{1}{10} + \frac{1}{10} \overline{)913}$$

45	13
15	4 4

£ 60 17 4, answ.

33. 873 yards of dowlas, at 1 s. 3 d.?

$$\frac{1}{10} + \frac{1}{10} \overline{)873}$$

43	13
10	18 3

£ 54 11 3, answer,

34. 713 yards ditto, at 1 s. 2 d.?

$$\frac{1}{10} + \frac{1}{10} \overline{)713}$$

35	13
5	18 10

£ 41 11 10, answ.

35. 757 yards ditto, at 11½ d.?

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} \overline{)757}$$

378	6
252	4
94	7½

£ 36 - 5½, an.

36. 371 yards ditto, at 9½ d.?

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} \overline{)371}$$

185	6
61	10
46	4½
293	8½

£ 14 13 8½, an.

37. 1713 yards of ribbon, at 8½ d. per yard?

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} \overline{)1713}$$

856	6
214	1½
142	9
1213	4½

£ 60 13 4½

38. What

38. What cost 587 yards of ribbon, at 7 d. per yard?

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

587	
195	8
146	9
342	5

£ 17 2 5, answer.

39. 713 yards ditto, at 6½ d.?

$$\frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{3}{4}$$

713	
178	3
118	10
89	1½
386	2½

£ 19 6 2½, anf.

40. 731 yards ditto, at 5½ d.?

$$\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

731	
243	8
91	4½
335	½

£ 16 15 ½, answer.

41. 373 yards ditto, at 5d.?

$$\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$$

373	
93	3
62	2
155	5

£ 7 15 5, answer.

42. 715 yards ditto, at 3½ d.?

$$\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

715	
119	2
89	4½
208	6½

£ 10 8 6½, answer.

43. 757 yards ditto, at 2½ d.?

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

757	
94	7½
63	1
157	8½

£ 7 17 8½, answer.

CASE VI.

When the price of the integer is a farthing, or farthings joined with pence, or with shillings and pence,

RULE,

Work for the shillings and pence, as before directed; then observe what part of any of the foregoing lines the farthing or farthings are, which take, and then add all together.

1. What

1. What cost 715 yards of tape, at $1\frac{1}{4}$ d. per yard?

$$\begin{array}{r}
 \frac{1}{12} \overline{)715} \\
 \frac{1}{4} \quad 59 \quad 7 \\
 \underline{14} \quad 10\frac{3}{4} \\
 \frac{1}{20} \quad 74 \quad 5\frac{3}{4} \\
 \underline{\quad} \\
 \text{£ } 3 \quad 14 \quad 5\frac{3}{4}, \text{ answer.}
 \end{array}$$

2. 495 yards ditto, at $1\frac{3}{4}$ d.?

$$\begin{array}{r}
 \frac{1}{8} \overline{)495} \\
 \frac{1}{6} \quad 61 \quad 10\frac{5}{8} \\
 \underline{10} \quad 3\frac{3}{8} \\
 \frac{1}{10} \quad 72 \quad 2\frac{3}{8} \\
 \underline{\quad} \\
 \text{£ } 3 \quad 12 \quad 2\frac{3}{8}, \text{ answer.}
 \end{array}$$

3. 351 yards of small ribbon, at $2\frac{1}{4}$ d.?

$$\begin{array}{r}
 \frac{1}{2} \overline{)351} \\
 \frac{1}{2} \quad 58 \quad 6 \\
 \underline{7} \quad 3\frac{1}{2} \\
 \frac{1}{20} \quad 65 \quad 9\frac{1}{2} \\
 \underline{\quad} \\
 \text{£ } 3 \quad 5 \quad 9\frac{1}{2}, \text{ answer.}
 \end{array}$$

4. 741 yards ditto, at $2\frac{3}{4}$ d.?

$$\begin{array}{r}
 \frac{1}{4} \overline{)741} \\
 \text{Subt. } \frac{1}{12} \quad 185 \quad 3 \\
 \underline{15} \quad 5\frac{1}{2} \\
 \frac{1}{20} \quad 169 \quad 9\frac{1}{2} \\
 \underline{\quad} \\
 \text{£ } 8 \quad 9 \quad 9\frac{1}{2}, \text{ answer.}
 \end{array}$$

5. 243 yards ditto, at $3\frac{1}{4}$ d.?

$$\begin{array}{r}
 \frac{1}{4} \overline{)243} \\
 \frac{1}{12} \quad 60 \quad 9 \\
 \underline{5} \quad \frac{3}{4} \\
 \frac{1}{20} \quad 65 \quad 9\frac{3}{4} \\
 \underline{\quad} \\
 \text{£ } 3 \quad 5 \quad 9\frac{3}{4}, \text{ answer.}
 \end{array}$$

6. 747 yards ditto, at $3\frac{1}{4}$ d.?

$$\begin{array}{r}
 \frac{1}{4} \overline{)747} \\
 \frac{1}{4} \quad 186 \quad 9 \\
 \underline{46} \quad 8\frac{1}{4} \\
 \frac{1}{20} \quad 233 \quad 5\frac{1}{4} \\
 \underline{\quad} \\
 \text{£ } 11 \quad 13 \quad 5\frac{1}{4}, \text{ answer.}
 \end{array}$$

7. 714 yards ditto, at $4\frac{1}{4}$ d.?

$$\begin{array}{r}
 \frac{1}{4} \overline{)714} \\
 \frac{1}{12} \quad 178 \quad 6 \\
 \underline{59} \quad 6 \\
 \underline{14} \quad 10\frac{1}{2} \\
 \frac{1}{20} \quad 252 \quad 10\frac{1}{2} \\
 \underline{\quad} \\
 \text{£ } 12 \quad 12 \quad 10\frac{1}{2}, \text{ answer.}
 \end{array}$$

8. 291 yards ditto, at $4\frac{3}{4}$ d.?

$$\begin{array}{r}
 \frac{1}{4} \overline{)291} \\
 \frac{1}{6} \quad 72 \quad 9 \\
 \underline{36} \quad 4\frac{1}{2} \\
 \underline{6} \quad \frac{1}{4} \\
 \frac{1}{20} \quad 115 \quad 2\frac{1}{4} \\
 \underline{\quad} \\
 \text{£ } 5 \quad 15 \quad 2\frac{1}{4}, \text{ answer.}
 \end{array}$$

9. What

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9. What cost 747 yards of ribbon, at $5\frac{1}{4}$ d. per yard?

$$\begin{array}{r} \frac{1}{2} | 747 \\ \hline \frac{1}{4} | 249 \\ \hline \frac{1}{4} | 62 \quad 3 \\ \hline 15 \quad 6\frac{1}{2} \\ \hline \frac{1}{20} | 326 \quad 9\frac{1}{2} \end{array}$$

£ 16 16 $9\frac{1}{2}$, answer.

10. 210 yds. ditto, at $5\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{4} + \frac{1}{8} | 210 \\ \hline \frac{1}{4} | 52 \quad 6 \\ \hline 35 \quad - \\ \hline 13 \quad 1\frac{1}{2} \\ \hline \frac{1}{20} | 100 \quad 7\frac{1}{2} \end{array}$$

£ 5 - $7\frac{1}{2}$, answ.

11. 737 yds. ditto, at $6\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{2} | 737 \\ \hline \frac{1}{4} | 245 \quad 8 \\ \hline \frac{1}{8} | 122 \quad 10 \\ \hline 15 \quad 4\frac{1}{4} \\ \hline \frac{1}{20} | 383 \quad 10\frac{1}{4} \end{array}$$

£ 19 3 $10\frac{1}{4}$, answer.

12. 1173 yards ditto, at $6\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{2} | 1173 \\ \hline \frac{1}{4} | 586 \quad 6 \\ \hline 73 \quad 3\frac{1}{2} \\ \hline \frac{1}{20} | 659 \quad 9\frac{1}{2} \end{array}$$

£ 32 19 $9\frac{1}{2}$, answer.

2

13. 1131 yards ditto, at $7\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{2} | 1131 \\ \hline \frac{1}{6} | 565 \quad 6 \\ \hline \frac{1}{4} | 94 \quad 3 \\ \hline 23 \quad 6\frac{1}{2} \\ \hline \frac{1}{20} | 683 \quad 3\frac{1}{2} \end{array}$$

£ 34 3 $3\frac{1}{2}$, answ.

14. 711 yds. ditto, at $7\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{2} + \frac{1}{4} | 711 \\ \hline \frac{1}{4} | 237 \\ \hline \frac{1}{4} | 177 \quad 9 \\ \hline 44 \quad 5\frac{1}{2} \\ \hline \frac{1}{20} | 459 \quad 2\frac{1}{4} \end{array}$$

£ 22 19 $2\frac{1}{4}$, answ.

15. 495 yds. ditto, at $8\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{2} | 495 \\ \hline \frac{1}{3} | 247 \quad 6 \\ \hline \frac{1}{8} | 82 \quad 6 \\ \hline 10 \quad 3\frac{1}{4} \\ \hline \frac{1}{20} | 340 \quad 3\frac{1}{4} \end{array}$$

£ 17 - $3\frac{1}{4}$, answ.

16. 1157 yards ditto, at $8\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{2} | 1157 \\ \hline \frac{1}{3} + \frac{1}{8} | 578 \quad 6 \\ \hline 192 \quad 10 \\ \hline 72 \quad 3\frac{1}{2} \\ \hline \frac{1}{20} | 843 \quad 7\frac{1}{2} \end{array}$$

£ 42 3 $7\frac{1}{2}$, answ.

17. What

17. What cost 527 yards of ribbon, at $9\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{2} | 527 \\ \hline \frac{1}{2} | 263 \quad 6 \\ \frac{1}{12} | 131 \quad 9 \\ \hline 10 \quad 11\frac{3}{4} \\ \hline \frac{1}{20} | 406 \quad 2\frac{3}{4} \end{array}$$

£ 20 6 $2\frac{3}{4}$, answer.

18. 715 yds. ditto, at $9\frac{3}{4}$ d.?

$$\begin{array}{r} \frac{1}{2} | 715 \\ \hline \frac{1}{2} | 357 \quad 6 \\ \frac{1}{4} | 178 \quad 9 \\ \hline 44 \quad 8\frac{1}{2} \\ \hline \frac{1}{20} | 580 \quad 11\frac{3}{4} \end{array}$$

£ 29 - $11\frac{3}{4}$, answer.

19. 785 yds. ditto, at $10\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{2} | 785 \\ \hline \frac{1}{2} | 392 \quad 6 \\ \frac{1}{12} | 196 \quad 3 \\ \frac{1}{4} | 65 \quad 5 \\ \hline 16 \quad 4\frac{1}{4} \\ \hline \frac{1}{20} | 670 \quad 6\frac{1}{4} \end{array}$$

£ 33 10 $6\frac{1}{4}$, answer.

20. 9113 yards ditto, at $10\frac{1}{2}$ d.?

$$\begin{array}{r} \frac{1}{2} + \frac{1}{2} | 9113 \\ \hline \frac{1}{2} | 4556 \quad 6 \\ 3037 \quad 8 \\ \hline 569 \quad 6\frac{3}{4} \\ \hline \frac{1}{20} | 8163 \quad 8\frac{3}{4} \end{array}$$

£ 408 3 $8\frac{3}{4}$, answer.

21. 415 yds. ditto, at $11\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{2} + \frac{1}{2} | 415 \\ \hline 207 \quad 6 \\ \frac{1}{4} | 138 \quad 4 \\ \hline 34 \quad 7 \\ \hline 8 \quad 7\frac{3}{4} \\ \hline \frac{1}{20} | 389 \quad -\frac{3}{4} \end{array}$$

£ 19 9 $-\frac{3}{4}$, answ.

22. 797 yds. ditto, at $11\frac{3}{4}$ d.?

$$\begin{array}{r} \frac{1}{2} + \frac{1}{2} + \frac{1}{4} | 797 \\ \hline 398 \quad 6 \\ 265 \quad 8 \\ \hline \frac{1}{8} | 99 \quad 7\frac{1}{2} \\ \hline 16 \quad 7\frac{1}{4} \\ \hline \frac{1}{20} | 780 \quad 4\frac{3}{4} \end{array}$$

£ 39 - $4\frac{3}{4}$, anf.

23. 371 lb. of tobacco, at 1s. $-\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{20} | 371 \\ \hline 9 \quad 5 \quad 6 \\ \frac{1}{2} | 4 \quad 12 \quad 9 \\ \hline \frac{1}{12} | 4 \quad 12 \quad 9 \\ \hline 7 \quad 8\frac{3}{4} \end{array}$$

£ 18 18 $8\frac{3}{4}$, answer.

24. 171 lb. ditto, at 1s. $-\frac{3}{4}$ d.?

$$\begin{array}{r} \frac{1}{20} | 171 \\ \hline 4 \quad 5 \quad 6 \\ \frac{1}{2} | 4 \quad 5 \quad 6 \\ \hline 10 \quad 8\frac{1}{2} \end{array}$$

£ 9 1 $8\frac{1}{2}$, answer.

Chap. III. PRACTICE.

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25. What cost 907 lb. of tobacco, at 1 s. $1\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{20} | 907 \\ \frac{1}{12} \frac{1}{4} \begin{array}{r} 45 \ 7 \\ 3 \ 15 \ 7 \\ 18 \ 10\frac{3}{4} \end{array} \\ \hline \pounds \ 50 \ 1 \ 5\frac{3}{4}, \text{ answer.} \end{array}$$

29. 9081 lb. ditto, at 1 s. $3\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{20} | 9081 \\ \frac{1}{4} \frac{1}{12} \begin{array}{r} 454 \ 1 \\ 113 \ 10 \ 3 \\ 9 \ 9 \ 2\frac{1}{4} \end{array} \\ \hline \pounds \ 577 \ - \ 5\frac{1}{4}, \text{ ansf.} \end{array}$$

26. 175 lb. ditto, at 1 s. $1\frac{3}{4}$ d.?

$$\begin{array}{r} \frac{1}{20} | 175 \\ \frac{1}{8} \frac{1}{6} \begin{array}{r} 8 \ 15 \\ 1 \ 1 \ 10\frac{1}{2} \\ 3 \ 7\frac{1}{4} \end{array} \\ \hline \pounds \ 10 \ - \ 6\frac{1}{4}, \text{ answer.} \end{array}$$

30. 173 lb. ditto, at 1 s. $3\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{20} | 173 \\ \frac{1}{4} \frac{1}{4} \begin{array}{r} 8 \ 13 \\ 2 \ 3 \ 3 \\ 10 \ 9\frac{3}{4} \end{array} \\ \hline \pounds \ 11 \ 7 \ - \frac{1}{4}, \text{ answer.} \end{array}$$

27. 137 lb. ditto, at 1 s. $2\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{20} | 137 \\ \frac{1}{6} \frac{1}{8} \begin{array}{r} 6 \ 17 \\ 1 \ 2 \ 10 \\ 2 \ 10 \end{array} \\ \hline \pounds \ 8 \ 2 \ 8\frac{1}{4}, \text{ answer.} \end{array}$$

31. 957 lb. ditto, at 1 s. $4\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{20} | 957 \\ \frac{1}{4} \frac{1}{4} \frac{1}{4} \begin{array}{r} 47 \ 17 \\ 11 \ 19 \ 3 \\ 3 \ 19 \ 9 \\ 19 \ 11\frac{1}{4} \end{array} \\ \hline \pounds \ 64 \ 15 \ 11\frac{1}{4}, \text{ answer.} \end{array}$$

28. 713 lb. ditto, at 1 s. $2\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{20} + \frac{1}{20} | 713 \\ \frac{1}{8} \begin{array}{r} 23 \ 15 \ 4 \\ 17 \ 16 \ 6 \\ 2 \ 4 \ 6\frac{3}{4} \end{array} \\ \hline \pounds \ 43 \ 16 \ 4\frac{1}{4}, \text{ ansf.} \end{array}$$

32. 875 ells of Irish cloth, at 1 s. $4\frac{1}{4}$ d. per ell?

$$\begin{array}{r} \frac{1}{20} | 875 \\ \frac{1}{4} \frac{1}{8} \begin{array}{r} 43 \ 15 \\ 10 \ 18 \ 9 \\ 5 \ 9 \ 4\frac{1}{2} \\ 18 \ 2\frac{3}{4} \end{array} \\ \hline \pounds \ 61 \ 1 \ 4\frac{1}{4}, \text{ answer.} \end{array}$$

33. What

33. What cost 879 ells of Irish cloth, at 1 s. 5 $\frac{1}{4}$ d. per ell?

$\frac{1}{20}$	879		
$\frac{1}{3}$	43	19	
$\frac{1}{4}$	14	13	
$\frac{1}{4}$	3	13	3
		18	3 $\frac{1}{4}$

£ 63 3 6 $\frac{1}{4}$, answer.

34. 871 ells ditto, at 1 s. 5 $\frac{1}{4}$ d.?

$\frac{1}{20}$	871		
$\frac{1}{3} + \frac{1}{4}$	43	11	
	14	10	4
$\frac{1}{6}$	5	8	10 $\frac{1}{2}$
		18	1 $\frac{3}{4}$

£ 64 8 4 $\frac{1}{4}$, answ.

35. 171 ells ditto, at 1 s. 6 $\frac{1}{4}$ d.?

$\frac{1}{20}$	171		
$\frac{1}{3}$	8	11	
$\frac{1}{4}$	2	17	
$\frac{1}{4}$	7	8	6
		3	6 $\frac{1}{2}$

£ 13 - - $\frac{3}{4}$, answ.

36. 137 ells ditto, at 1 s. 6 $\frac{1}{4}$ d.?

$\frac{1}{20}$	137		
$\frac{1}{3}$	6	17	
$\frac{1}{4}$	3	8	6
		8	6 $\frac{1}{2}$

£ 10 14 - $\frac{3}{4}$, answ.

37. 875 ells ditto, at 1 s. 7 $\frac{1}{4}$ d.?

$\frac{1}{20}$	875		
$\frac{1}{3}$	43	15	
$\frac{1}{6}$	21	17	6
$\frac{1}{4}$	3	12	11
		18	2 $\frac{1}{4}$

£ 70 3 7 $\frac{1}{4}$, answer.

38. 173 ells ditto, at 1 s. 7 $\frac{1}{4}$ d.?

$\frac{1}{20}$	173		
$\frac{1}{3}$	8	13	
$\frac{1}{4}$	4	6	6
$\frac{1}{6}$	1	1	7 $\frac{1}{2}$
		3	7 $\frac{1}{4}$

£ 14 4 8 $\frac{1}{4}$, answ.

39. 375 ells ditto, at 1 s. 8 $\frac{1}{4}$ d.?

$\frac{1}{20}$	375		
$\frac{1}{3}$	18	15	
$\frac{1}{4}$	9	7	6
$\frac{1}{4}$	3	2	6
		7	9 $\frac{1}{4}$

£ 31 12 9 $\frac{1}{4}$, answer.

40. 721 ells ditto, at 1 s. 8 $\frac{1}{4}$ d.?

$\frac{1}{20}$	721		
$\frac{1}{3} + \frac{1}{4}$	36	1	
	18	-	6
	6	-	2
	2	5	- $\frac{1}{4}$

£ 62 6 8 $\frac{1}{4}$, answer.

41. What

41. What cost 307 ells of Irish cloth, at 1s. 9½d. per ell?

$$\begin{array}{r}
 \frac{1}{10} | 307 \\
 \hline
 \frac{1}{2} \quad 15 \quad 7 \\
 \frac{1}{2} \quad 7 \quad 13 \quad 6 \\
 \frac{1}{4} \quad 3 \quad 16 \quad 9 \\
 \quad 19 \quad 2\frac{1}{4} \\
 \hline
 \text{£ } 27 \quad 16 \quad 5\frac{1}{4}, \text{ answer.}
 \end{array}$$

42. 317 ells ditto, at 1s. 9½d.

$$\begin{array}{r}
 \frac{1}{10} | 317 \\
 \hline
 \frac{1}{2} \quad 15 \quad 17 \\
 \frac{1}{2} \quad 7 \quad 18 \quad 6 \\
 \frac{1}{4} \quad 3 \quad 19 \quad 3 \\
 \quad 6 \quad 7\frac{1}{4} \\
 \hline
 \text{£ } 28 \quad 1 \quad 4\frac{1}{4}, \text{ answer.}
 \end{array}$$

43. 107 ells ditto, at 1s. 10½d.

$$\begin{array}{r}
 \frac{1}{12} | 107 \\
 \hline
 \frac{1}{10} \quad 8 \quad 18 \quad 4 \\
 \frac{1}{8} \quad 17 \quad 10 \\
 \quad 2 \quad 2\frac{1}{2} \\
 \hline
 \text{£ } 9 \quad 18 \quad 4\frac{1}{2}, \text{ answer.}
 \end{array}$$

44. 199 ells ditto, at 1s. 10½d.

$$\begin{array}{r}
 \frac{1}{12} | 199 \\
 \hline
 \frac{1}{10} \quad 16 \quad 11 \quad 8 \\
 \frac{1}{8} \quad 2 \quad 1 \quad 5\frac{1}{2} \\
 \quad 4 \quad 1\frac{1}{2} \\
 \hline
 \text{£ } 18 \quad 17 \quad 3\frac{1}{2}, \text{ answer.}
 \end{array}$$

45. 147 ells ditto, at 1s. 11½d.

$$\begin{array}{r}
 \frac{1}{12} + \frac{1}{10} | 147 \\
 \hline
 \frac{1}{12} \quad 12 \quad 5 \\
 \quad 1 \quad 16 \quad 9 \\
 \quad 3 \quad -\frac{1}{2} \\
 \hline
 \text{£ } 14 \quad 4 \quad 9\frac{1}{2}, \text{ ans.}
 \end{array}$$

46. 175 ells ditto, at 1s. 11½d.

$$\begin{array}{r}
 \frac{1}{12} + \frac{1}{10} | 175 \\
 \hline
 \frac{1}{4} \quad 14 \quad 11 \quad 8 \\
 \quad 2 \quad 3 \quad 9 \\
 \quad 10 \quad 11\frac{1}{2} \\
 \hline
 \text{£ } 17 \quad 6 \quad 4\frac{1}{2}, \text{ ans.}
 \end{array}$$

CASE VII.

When the integer is pounds, shillings, pence, and farthings,

RULE,

Multiply the given quantity by the pounds; and proceed with the shillings, pence, and farthings, as in the foregoing cases.

K

1. What

1. What cost 137 yards of
brocade, at 1l. 17s. $6\frac{1}{4}$ d.
per yard?

$$\begin{array}{r} \frac{7}{10} + \frac{1}{5} + \frac{1}{10} = 137 \\ 14s. - d. = \begin{array}{r} 95 \ 18 \\ 3 \ 4 = 22 \ 16 \ 8 \\ 2(\frac{1}{5}) = 1 \ 2 \ 10 \\ \hline 2 \ 10\frac{1}{4} \end{array} \\ \text{Answer } \pounds \ 257. - \ 4\frac{1}{4} \end{array}$$

2. 2710 cwt. of sugar, at
2l. 3s. $7\frac{1}{2}$ d.?

$$\begin{array}{r} \frac{1}{5} + \frac{1}{10} = 271 \\ \times \\ \hline 5420 \\ 3s. 4d. = 451 \ 13 \ 4 \\ 3(\frac{1}{5}) = 33 \ 17 \ 6 \\ \hline 5 \ 12 \ 11 \\ \text{Answer } \pounds \ 5911 \ 3 \ 9 \end{array}$$

3. 741 cwt. ditto, at 2l.
13s. $7\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{10} = 741 \\ \times 2 \\ \hline 1482 \\ \frac{1}{5} = 494 \\ 13s. 4d. = 9 \ 5 \ 3 \\ 3(\frac{1}{5}) = 15 \ 5\frac{1}{4} \\ \hline \text{Answer } \pounds \ 1986 - \ 8\frac{1}{4} \end{array}$$

4. 947 cwt. of hops, at
4l. 15s. $10\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{7}{10} + \frac{1}{5} = 947 \\ \times 4 \\ \hline 3788 \\ 14s. - d. = 662 \ 18 - \\ 1 \ 8(\frac{1}{5}) = 78 \ 18 \ 4 \\ \hline 7 \ 17 \ 10 \\ 19 \ 8\frac{1}{4} \\ \text{Answer } \pounds \ 4538 \ 13 \ 10\frac{1}{4} \end{array}$$

5. 457 laft of colefeed, at
14l. 17s. $9\frac{1}{2}$ d. per laft?

$$\begin{array}{r} \frac{7}{10} + \frac{1}{5} = 457 \\ \times 14 \\ \hline 6398 \\ 14s. - d. = 319 \ 18 \\ 3 \ 4(\frac{1}{5}) = 76 \ 3 \ 4 \\ \hline 9 \ 10 \ 5 \\ 19 \ \frac{1}{2} \\ \text{Answer } \pounds \ 6804 \ 10 \ 9\frac{1}{2} \end{array}$$

6. 375 cwt. hops, at 3l.
7s. $11\frac{1}{4}$ d.?

$$\begin{array}{r} \frac{1}{10} = 375 \\ \times 3 \\ \hline 1125 \\ 7s. 6d. = 140 \ 12 \ 6 \\ 4 \ \frac{1}{4} = 6 \ 5 - \\ \hline 1 \ 11 \ 3 \\ 7 \ 9\frac{1}{4} \\ \text{Answer } \pounds \ 1273 \ 16 \ 6\frac{1}{4} \end{array}$$

CASE VIII.

When the given quantities are of several denominations,
RULE,

RULE,

Find the value of the integers, as in the foregoing cases; and for the lesser denomination in the given quantity, if they are the aliquot part of an integer, divide the given price thereby; but if they are not aliquot parts, divide them into such, or of each other, as you can most conveniently; then add all together, their sum will be the answer.

ALIUOT PARTS in CLOTH MEASURE.

One yard the integer.

Qrs. n.		
2	-	$\frac{1}{2}$
1	-	$\frac{1}{4}$
-	=	$\frac{1}{8}$

One quarter the integer.

Nail.		
1	-	$\frac{1}{4}$
2	-	$\frac{1}{2}$

One ell English integer.

Qrs. n.		
2	2	$\frac{1}{2}$
1	1	$\frac{1}{4}$
1	-	$\frac{1}{8}$
-	2	$\frac{1}{10}$
-	1	$\frac{1}{20}$

One French ell integer.

Qrs. n.		
3	-	$\frac{1}{2}$
2	-	$\frac{1}{3}$
1	2	$\frac{1}{4}$
1	-	$\frac{1}{6}$
-	3	$\frac{1}{8}$
-	2	$\frac{1}{12}$

One Flemish ell integer.

Qrs. n.		
1	2	$\frac{1}{2}$
1	-	$\frac{1}{3}$
-	3	$\frac{1}{4}$
-	2	$\frac{1}{6}$
-	1	$\frac{1}{12}$

1. 713 yds. 3 qrs. 2 n. of kersey, at 7 s. 9 $\frac{1}{2}$ d. per yard?

$\frac{1}{2}$	+	$\frac{3}{10}$	713	3	3
6	8	=	237	13	4
1	-	=	35	13	-
-	$1\frac{1}{2}$	=	4	9	$1\frac{1}{2}$
Qrs.	2	=	3	$10\frac{1}{4}$	
	1	=	1	$11\frac{1}{2}$	
Nails	3	=	1	$5\frac{1}{2}$	

Answer £ 278 2 9

2. 17 ells E. - qrs. 2 n. of gold brocade, at 3l. 10s. 9 d.?

$\frac{1}{2}$	17	-	2
	3		
		s. d.	
	51	-	-
$\frac{1}{10}$	8	10	= 10 -
	8	6	= - 6
$\frac{1}{2}$	4	3	= - 3
2 nails	7	$\frac{1}{4}$	= $\frac{1}{10}$ price.

£ 60 9 9 $\frac{1}{4}$, answer.

K 2

3. What

3. What cost 19 French ells, 4 qrs. 3 n. of Brussels lace, at 3l. 19s. 11d.?

	3	19	11
			3
	11	19	9
			6
	71	18	6
3 n. = $\frac{1}{4}$	3	19	11
		9	11 $\frac{1}{4}$

£ 76 8 4 $\frac{1}{4}$ answer.

4. What cost 71 French ells 1 qr. ditto, at 2l. 17s. 8 $\frac{1}{2}$ d.?

F. E. qr.

$\frac{1}{2} + \frac{1}{4}$	71	1	
$+\frac{1}{8}$		2	
	142		s. d.
	35	10	- = 10 -
	23	13	4 = 6 8
$\frac{1}{4}$	3	11	- = 1 -
	-	2	11 $\frac{1}{2}$ = - $\frac{1}{2}$
1 qr.	-	9	7 $\frac{1}{4}$ = $\frac{1}{8}$ price.

£ 205 6 10 $\frac{1}{4}$.

5. What cost 709 French ells, 5 qrs. 3 n. of ditto, at 14s. 7 $\frac{1}{2}$ d.?

F. E. qr. n.

$\frac{7}{10} + \frac{1}{40}$	709	5	3	
				s. d.
	406	6	-	= 14 -
$\frac{1}{6}$	17	14	6	= - 6
$\frac{1}{4}$	2	19	1	= - 1
	-	14	9 $\frac{1}{4}$	= - $\frac{1}{4}$
3 - =	-	7	3 $\frac{1}{2}$	= $\frac{1}{2}$ } p.
2 - =	-	4	10 $\frac{1}{2}$	= $\frac{1}{2}$ }
- 3 = $\frac{1}{4}$	-	1	9 $\frac{1}{4}$	

£ 518 8 3 $\frac{1}{4}$

6. What cost 719 ells Flemish, 2 qrs. 3 n. of fine Holland, at 1l. 10s. 9 $\frac{1}{4}$ d.?

E. Flemish.

$\frac{1}{2} + \frac{1}{10}$	719		
	359	10	- = 10s.
$\frac{1}{4}$	23	19	4 = - 8d.
$\frac{1}{4}$	2	19	11 = - 1
qr. n.	-	14	11 $\frac{1}{2}$ = -
1 2 =	-	15	4 $\frac{1}{2}$ = $\frac{1}{2}$ }
1 - $\frac{1}{4}$	-	10	3 = $\frac{1}{4}$ }
- 1 =	-	2	6 $\frac{1}{4}$ =

£ 1107 12 5, answer.

7. What cost 4 pieces of ribbon, each 17 yards, 1 qr. 3 nails, at 1s. 1 $\frac{1}{2}$ d. per yard?

Y. qr. n.

	17	1	3
			4
$\frac{1}{10}$	69	3	- , at 13 $\frac{1}{2}$ d.
qrs. $\frac{1}{4}$	3	9	-
2 - =	-	8	7 $\frac{1}{2}$
1 - =	-	-	6 $\frac{3}{4}$ } $\frac{1}{2}$ }
	-	-	3 $\frac{1}{4}$ } $\frac{1}{4}$ }

£ 3 18 5 $\frac{1}{2}$, answer.

8. What cost 13 ells, 2 qrs. 2 n. of Holland, at 3s. 7 $\frac{1}{2}$ d. per ell English?

$\frac{1}{2}$	-	3	7 $\frac{1}{2}$
			13
	2	7	1 $\frac{1}{2}$ qr. n.
		1	9 $\frac{1}{4}$ = 2 2

£ 2 8 11 $\frac{1}{4}$, answer.

ALIQUOT

ALIQOT PARTS *in* TROY WEIGHT.

One ounce the integer.

One pennyweight integer.

dwt. gr.		
10	-	= $\frac{1}{2}$
6	16	= $\frac{1}{3}$
5	-	= $\frac{1}{4}$
4	-	= $\frac{1}{5}$
3	8	= $\frac{1}{6}$
2	12	= $\frac{1}{8}$
2	-	= $\frac{1}{10}$
1	16	= $\frac{1}{12}$
1	-	= $\frac{1}{20}$

gr.		
12	=	$\frac{1}{2}$
8	=	$\frac{1}{3}$
6	=	$\frac{1}{4}$
4	=	$\frac{1}{6}$
3	=	$\frac{1}{8}$
2	=	$\frac{1}{12}$
N. B. 4 l. per		
os. is 2d.		
per grain.		

9. A silver gilt punch bowl, weight 49 oz. 2 dwt. 12 gr. what comes it to at 8 s. 11 $\frac{1}{4}$ d. per ounce?

oz. dwt. gr.		
$\frac{1}{10} + \frac{1}{10} + \frac{1}{10}$	49	2 12
	19	12 - at 8 s. - d.
	1	12 8 at - 8
dwt. gr. $\frac{1}{4}$	-	12 3 at - 3
2 12	-	3 $\frac{1}{4}$
	-	1 $1\frac{1}{4} = \frac{1}{8}$ price.

£ 22 1 1, answer.

10. A pair of chased silver salts, weight 7 oz. 5 dwt. at 8 s. 9 $\frac{1}{4}$ d. per ounce?

$\frac{1}{4}$	8	9 $\frac{1}{4}$
		7
3	1	8 $\frac{1}{4}$
	2	2 $\frac{1}{4}$
£ 3 3 10 $\frac{1}{2}$, answer.		

11. I demand the value of a service of plate, weight 971 oz. 15 dwt. 16 gr. at 3 l. 19 s. 11 $\frac{1}{4}$ d. per ounce?

oz. dwt. gr.		
	971	15 16
		3
	29	13
	87	3 18
	80	18 4
$\frac{1}{4}$	12	2 9
dwt. $\frac{1}{4}$	3	- 8 $\frac{1}{4}$
10 -	1	19 11 $\frac{1}{4} = \frac{1}{2}$ oz.
4 -		15 11 $\frac{1}{4}$
1 16		6 7 $\frac{1}{4}$
£ 386 4 $\frac{1}{2}$, answer		

K 3

ALIQUOT

3. What cost 19 French ells, 4 qrs. 3 n. of Bruffels lace, at 3l. 19 s. 11 d.?

	3	19	11
			3
	11	19	9
			6
	71	18	6
3 n. = $\frac{1}{4}$	3	19	11
		9	11 $\frac{1}{2}$

£ 76 8 4 $\frac{1}{2}$, answer.

4. What cost 71 French ells 1 qr. ditto, at 2l. 17 s. 8 $\frac{1}{2}$ d.?

F. E. qr.

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$	71	1	
$+$	2		
	142		s. d.
	35	10	- = 10
	23	13	4 = 6 8
$\frac{1}{4}$	3	11	- = 1 -
	-	2	11 $\frac{1}{2}$ = - $\frac{1}{2}$
1 qr.	-	9	7 $\frac{1}{2}$ = $\frac{1}{8}$ price.

£ 205 6 10 $\frac{1}{2}$.

5. What cost 709 French ells, 5 qrs. 3 n. of ditto, at 14 s. 7 $\frac{1}{2}$ d.?

F. E. qr. n.

$\frac{7}{10} + \frac{1}{4} + \frac{1}{8}$	709	5	3	
				s. d.
	496	6	-	= 14 -
$\frac{1}{10}$	17	14	6	= - 6
$\frac{1}{4}$	2	19	1	= - 1
	-	14	9 $\frac{1}{4}$	= - $\frac{1}{4}$
3 - =	-	7	3 $\frac{1}{2}$	= $\frac{1}{2}$
2 - =	-	4	10 $\frac{1}{2}$	= $\frac{1}{2}$
- 3 = $\frac{1}{4}$	-	1	9 $\frac{1}{2}$	

£ 518 8 3 $\frac{1}{2}$

6. What cost 719 ells Flemish, 2 qrs. 3 n. of fine Holland, at 1l. 10 s. 9 $\frac{1}{2}$ d.?

E. Flemish.

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$	719			
	359	10	-	= 10 s.
$\frac{1}{4}$	23	19	4	= - 8 d.
$\frac{1}{8}$	2	19	11	= - 1
qr. n.	-	14	11 $\frac{1}{2}$	= -
1 2 =	-	15	4 $\frac{1}{2}$	= $\frac{1}{2}$
1 - $\frac{1}{4}$	-	10	3	= $\frac{1}{2}$
- 1 =	-	2	6 $\frac{1}{2}$	=

£ 1107 12 5, answer.

7. What cost 4 pieces of ribbon, each 17 yards, 1 qr. 3 nails, at 1 s. 1 $\frac{1}{2}$ d. per yard?

Y. qr. n.

	17	1	3
			4
$\frac{1}{10}$	69	3	-
			at 13 $\frac{1}{2}$ d.
qrs. $\frac{1}{4}$	3	9	-
2 - =	-	8	7 $\frac{1}{2}$
1 - =	-	-	6 $\frac{1}{2}$
	-	-	3 $\frac{1}{2}$

£ 3 18 5 $\frac{1}{2}$, answer.

8. What cost 13 ells, 2 qrs. 2 n. of Holland, at 3 s. 7 $\frac{1}{2}$ d. per ell English?

$\frac{1}{2}$	-	3	7 $\frac{1}{2}$
			13
	2	7	1 $\frac{1}{2}$
		1	9 $\frac{1}{4}$
			qr. n.
			= 2 2

£ 2 8 11 $\frac{1}{2}$, answer.

ALIQUOT

ALICOT PARTS *in* TROY WEIGHT.

One ounce the integer.

One pennyweight integer.

dwt. gr.	
10	— = $\frac{1}{2}$
6 16	— = $\frac{1}{3}$
5	— = $\frac{1}{4}$
4	— = $\frac{1}{5}$
3 8	— = $\frac{1}{6}$
2 12	— = $\frac{1}{8}$
2	— = $\frac{1}{10}$
1 16	— = $\frac{1}{12}$
1	— = $\frac{1}{20}$

gr.	
12	— = $\frac{1}{2}$
8	— = $\frac{1}{3}$
6	— = $\frac{1}{4}$
4	— = $\frac{1}{6}$
3	— = $\frac{1}{8}$
2	— = $\frac{1}{12}$
N. B. 4 l. per oz. is 2 d. per grain.	

9. A silver gilt punch bowl, weight 49 oz. 2 dwt. 12 gr. what comes it to at 8 s. 11 $\frac{1}{2}$ d. per ounce?

oz. dwt. gr.	
$\frac{1}{10} + \frac{1}{10} + \frac{1}{10}$	49 2 12
	19 12 — at 8 s. — d.
	1 12 8 at — 8
dwt. gr. $\frac{1}{4}$	— 12 3 at — 3
2 12	— 3 — $\frac{1}{4}$
	— 1 1 $\frac{1}{4}$ = $\frac{1}{2}$ price.

£ 22 1 1, answer.

10. A pair of chased silver salts, weight 7 oz. 5 dwt. at 8 s. 9 $\frac{1}{2}$ d. per ounce?

$\frac{1}{4}$	8 9 $\frac{1}{2}$
	7
3 1	8 $\frac{1}{4}$
2	2 $\frac{1}{4}$
£ 3 3 10 $\frac{1}{2}$, answer.	

11. I demand the value of a service of plate, weight 971 oz. 15 dwt. 16 gr. at 3 l. 19 s. 11 $\frac{1}{2}$ d. per ounce?

oz. dwt. gr.	
	971 15 16
	3
	29 13
	873 18
	80 18 4
$\frac{1}{4}$	12 2 9
dwt.	3 — 8 $\frac{1}{4}$
10 —	1 19 11 $\frac{1}{2}$ = $\frac{1}{2}$ oz.
4 —	15 11 $\frac{1}{2}$
1 16	6 7 $\frac{1}{4}$
£ 386 4 $\frac{1}{2}$, answer	

K 3

ALICOT

ALIQOT PARTS *in* AVERDUPOISE WEIGHT.

One tun the integer.

Cwt.	qr.	lb.	
10	-	-	$\frac{1}{2}$
5	-	-	$\frac{1}{4}$
4	-	-	$\frac{1}{8}$
2	3	12	$\frac{1}{16}$
2	2	-	$\frac{1}{32}$
2	-	-	$\frac{1}{64}$
1	-	-	$\frac{1}{128}$

One hundred integer.

Qr.	lb.	
2	= 56	$\frac{1}{2}$
1	= 28	$\frac{1}{4}$
	16	$\frac{1}{8}$
	14	$\frac{1}{16}$

 $\frac{1}{2}$ Cwt. = 56 lb. integer.

Qr.	lb.	
1	= 28	$\frac{1}{2}$
	14	$\frac{1}{4}$
	8	$\frac{1}{8}$
	7	$\frac{1}{16}$

 $\frac{1}{4}$ Cwt. or 28 lb. integer.

14	=	$\frac{1}{2}$
7	=	$\frac{1}{4}$
4	=	$\frac{1}{8}$
2	8	$\frac{1}{16}$

One pound the integer.

oz.	
8	= $\frac{1}{2}$
4	= $\frac{1}{4}$
2	= $\frac{1}{8}$

One ounce the integer.

dr.	
8	= $\frac{1}{2}$
4	= $\frac{1}{4}$
2	= $\frac{1}{8}$

12. What cost 73 cwt. 1 qr. of sugar, at 3 l. 15 s. 7 d. per cwt.?

	Cwt.	
$\frac{6}{10} + \frac{1}{10}$	73 1	
$\frac{1}{10}$	3	
	219	
	43 16	
	12 3 4	
	- 18 3	
1 qr. =	- 18 10 $\frac{1}{2}$	= $\frac{1}{4}$ p.
	<u>£ 276 16 5 $\frac{1}{2}$</u>	anf.

13. What cost 73 l cwt. 3 qrs. of hops, at 3 l. 18 s. 7 $\frac{1}{2}$ d. per cwt.?

	Cwt.	qr.	
$\frac{9}{10} + \frac{1}{10}$	73 l 3		
	3		
	2193		
	657 18		
	18 5 6		
	4 11 4 $\frac{1}{2}$		
	1 19 3 $\frac{1}{2}$	= $\frac{1}{4}$ p.	
	- 19 7 $\frac{1}{2}$	= $\frac{1}{4}$ p.	
	<u>£ 2876 13 10</u>		answ.

14. What

14. What cost 7 cwt. 2 qrs. 10 lb. of treacle, at 1 l. 17 s. 8 d. per cwt.?

	l.	s.	d.
$\frac{1}{2}$	1	17	8
			7
	13	3	8
$\frac{1}{7}$	-	18	10
$\frac{1}{4}$	-	2	8 $\frac{1}{2}$
	-	-	8

£ 14 5 10 $\frac{1}{2}$, answer.

15. 17 cwt. 1 qr. 12 lb. at 1 l. 19 s. 8 d. per cwt.?

$\frac{1}{10} + \frac{1}{12}$	17
qr. lb.	15 6
1 -	1 8 4
	- 9 11 = $\frac{1}{4}$ price.
7 -	- 2 5 $\frac{1}{4}$ = $\frac{1}{4}$ of laft.
4 -	- 1 5 = $\frac{1}{4}$ of dit.
1 -	- - 4 $\frac{1}{2}$ = $\frac{1}{4}$ of laft.

£ 34 8 6, answer.

17. What cost 17 hogheads of treacle, at 1 l. 12 s. 7 d. per cwt. each hoghead weighing 5 cwt. 2 qrs. 8 lb.?

Cwt. qr. lb.

5	2	8
		3

16	2	24
		6

100	1	4
5	2	8

94	12	24
----	----	----

56	8	-
----	---	---

2	7	-
---	---	---

-	7	10
---	---	----

2	-	-
---	---	---

16	-	16 3 $\frac{1}{2}$ = $\frac{1}{2}$
----	---	------------------------------------

8	-	4 7 $\frac{1}{2}$ = $\frac{1}{2}$
---	---	-----------------------------------

-	2	3 $\frac{1}{2}$
---	---	-----------------

£ 154 6 1, answer.

16. What cost the freight of 7 ton, 13 cwt. 3 qrs. 19 lb., at 14 l. 17 s. 9 d. per ton?

$\frac{1}{2} + \frac{1}{7}$	14 17 9
	7
	104 4 3
$\frac{1}{10}$	7 8 10 $\frac{1}{2}$
	2 2 6 $\frac{1}{2}$
	14 10 $\frac{1}{2}$
	11 = $\frac{1}{2}$ of $\frac{1}{2}$ cwt.

£ 114 11 5 $\frac{1}{2}$, answer.

$\frac{1}{2}$	14 10 $\frac{1}{2}$
$\frac{1}{8}$	7 5 $\frac{1}{2}$
	- 11 d.

£ 4

18. What

ALÍQUOT PARTS *in* AVERDUPOISE WEIGHT.

One tun the integer.

Cwt.	qr.	lb.	
10	-	-	$\frac{1}{2}$
5	-	-	$\frac{1}{4}$
4	-	-	$\frac{1}{5}$
2	3	12	$\frac{1}{7}$
2	2	-	$\frac{1}{8}$
2	-	-	$\frac{1}{10}$
1	-	-	$\frac{1}{20}$

One hundred integer,

Qr.	lb.	
2	= 56	$\frac{1}{2}$
1	= 28	$\frac{1}{4}$
	16	$\frac{1}{7}$
	14	$\frac{1}{8}$

 $\frac{1}{2}$ Cwt. = 56 lb. integer.

Qr.	lb.	
1	= 28	$\frac{1}{2}$
	14	$\frac{1}{4}$
	8	$\frac{1}{7}$
	7	$\frac{1}{8}$

 $\frac{1}{4}$ Cwt. or 28 lb. integer.

14	=	$\frac{1}{2}$
7	=	$\frac{1}{4}$
4	=	$\frac{1}{7}$
2	8	=

One pound the integer.

oz.	
8	= $\frac{1}{2}$
4	= $\frac{1}{4}$
2	= $\frac{1}{8}$

One ounce the integer.

dr.	
8	= $\frac{1}{2}$
4	= $\frac{1}{4}$
2	= $\frac{1}{8}$

12. What cost 73 cwt. 1 qr. of sugar, at 3 l. 15 s. 7 d. per cwt.?

	Cwt.	
$\frac{6}{10} + \frac{1}{10}$	73 1	
$\frac{1}{10}$	3	
	219	
	43 16	
	12 3 4	
	- 18 3	
1 qr. =	- 18 10 $\frac{1}{4}$	= $\frac{1}{4}$ p.
	£ 276 16 5 $\frac{1}{4}$	anf.

13. What cost 73 l cwt. 3 qrs. of hops, at 3 l. 18 s. 7 $\frac{1}{2}$ d. per cwt.?

	Cwt.	qr.	
$\frac{10}{10} + \frac{1}{10}$	73 l	3	
	3		
	2193		
	657 18		
$\frac{1}{4}$	18 5 6		
	4 11 4 $\frac{1}{2}$		
	1 19 3 $\frac{1}{2}$	= $\frac{1}{2}$	
	- 19 7 $\frac{1}{4}$	= $\frac{1}{4}$	} p.
	£ 2876 13 10		answ.

14. What

14. What cost $\frac{1}{2}$ cwt. 2 qrs. 10 lb. of treacle, at 1 l. 17 s. 8 d. per cwt.?

	l.	s.	d.
$\frac{1}{2}$	1	17	8
			7
	13	3	8
$\frac{1}{7}$	-	18	10
$\frac{1}{4}$	-	2	8 $\frac{1}{4}$
	-	-	8

£ 14 5 10 $\frac{1}{4}$, answer.

15. 17 cwt. 1 qr. 12 lb. at 1 l. 19 s. 8 d. per cwt.?

$\frac{1}{10} + \frac{1}{12}$	17		
qr. lb.	15	6	
1	1	8	4
	-	-	9 11 = $\frac{1}{4}$ price.
	7	-	2 5 $\frac{1}{2}$ = $\frac{1}{4}$ of last.
	4	-	1 5 = $\frac{1}{2}$ of dit.
	1	-	- 4 $\frac{1}{2}$ = $\frac{1}{4}$ of last.

£ 34 8 6, answer.

17. What cost 17 hogheads of treacle, at 1 l. 12 s. 7 d. per cwt. each hoghead weighing 5 cwt. 2 qrs. 8 lb.?

Cwt. qr. lb.

5	2	8
		3

16	2	24
		6

100	1	4
5	2	8

94	12	24
56	8	-

qr.	lb.	
2	-	-
16	-	16 3 $\frac{1}{2}$ = $\frac{1}{2}$
8	-	4 7 $\frac{1}{2}$ = $\frac{1}{7}$
	-	2 3 $\frac{1}{2}$

£ 154 6 1, answer.

16. What cost the freight of 7 ton, 13 cwt. 3 qrs. 19 lb., at 14 l. 17 s. 9 d. per ton?

$\frac{1}{2} + \frac{1}{7}$	14	17	9
			7
	104	4	3
$\frac{1}{10}$	7	8	10 $\frac{1}{2}$
	2	2	6 $\frac{1}{2}$
	14	10	10 $\frac{1}{2}$
			11 = $\frac{1}{2}$ of $\frac{1}{2}$ cwt.

£ 114 11 5 $\frac{1}{2}$, answer.

$\frac{1}{2}$	14	10 $\frac{1}{2}$
$\frac{1}{8}$	7	5 $\frac{1}{2}$
	-	11 d.

£ 4

18. What

18. What cost the freight of 37 tons, 19 cwt. 3 qrs. at 19 l. 19 s. 2 d. per ton?

T. cwt. qrs.					
	37	19	3		
	19				
	<hr/>			l.	s. d.
$\frac{1}{10}$	703	-	-	at 19	- - per ton.
	35	3	-	at -	19 -
cwt.	-	6	2	at -	- 2
10 -	9	19	7	$= \frac{1}{2}$	} price.
5 -	4	19	$9\frac{1}{2}$	$= \frac{1}{2}$	
4 = $\frac{1}{2}$	3	19	10	$= \frac{1}{2}$	
		9	$11\frac{1}{2}$		
		4	$11\frac{3}{4}$		
<hr/>					
£	758	3	4,	answer.	

ALIQUOT PARTS *in* LAND MEASURE.

One acre the integer.

R. P.

2	-	=	$\frac{1}{2}$
1	-	=	$\frac{1}{4}$
-	32	=	$\frac{1}{32}$
-	20	=	$\frac{1}{20}$
-	16	=	$\frac{1}{16}$

One rood the integer.

Poles.

20	=	$\frac{1}{20}$
10	=	$\frac{1}{10}$
8	=	$\frac{1}{8}$
5	=	$\frac{1}{5}$
4	=	$\frac{1}{4}$
2	=	$\frac{1}{2}$

19. What is the rent of 713 acres, 3 roods, 39 perches of flax-land, at 3 l. 17 s. 6 d. per acre?

A. R. P.					
	713	3	39		
	$\times 3$				
	2139				
	356	10	-		
	178	5	-		
	89	2	6		
	1	18	9	} price.	
	-	19	4 $\frac{1}{2}$		
	-	9	8 $\frac{1}{2}$		
	-	4	10		
R. P.					
2	-				
1	-				
20		$\frac{1}{2}$			
10					
5					
4					
		1	11 $\frac{1}{2}$		

£ 2766 14 6, answer.

20. What

20. What is the rent of 21. 17 acres, - roods, 10
17 acres, 3 roods, and 35 perches, at 2l. 13s. 6d. per
perches of flax-land, at 4l. acre?
per acre?

A. R. P.	
17	3 35
4	
<hr/>	
R. P.	68
2 -	2 - -
1 -	1 - -
- 20	- 10 -
- 10	- 5 -
- 5	- 2 6
<hr/>	
£ 71 17 6, answer.	

A. R. P.	
$\frac{1}{2} + \frac{1}{20}$	17 - 10
	2
<hr/>	
	34
	10 4
$\frac{1}{2}$	- 17 -
	- 8 6
Per. 10	- 3 4 = $\frac{1}{2}$ of $\frac{1}{2}$
<hr/>	
£ 45 12 10, answer.	

N. B. This belongs to Example 21.

$\frac{1}{4}$	2 13 6
<hr/>	
$\frac{1}{2}$	13 4 $\frac{1}{2}$
<hr/>	
	3 4

Captain JOHN ELFORD,

1763,

September the 12th.

Bought of THOMAS CHEESEMONGER.			
Cwt. qr. lb.	l. s. d.	l. s. d.	
5 2 24 at 1 17	4 per cwt.	- 10 13	4
4 2 10 at 1 12	6	- 7 9	1½
- - - at 1 10	each	- 18	-
2 3 25 at 2 16	10 per cwt.	- 8 8	11½
- - - at - 3	4 per stone	- 8 3	4
- - - at 8 3	8 each	- 98	4

£ 150 18 8½

Mr. JONATHAN MARRIOT,

1763, August 25.

Bought of GEORGE GROCER and Company.			
Cwt. qrs. lb.	l. s. d.	l. s. d.	
17 2 17 at 1 13	10 per cwt.	29 17	2½
12 1 19 at 1 14	5	- 21 7	5
4 - 12 at 4 19	4	- 20 7	11½
1 - 15 at 2 16	4	- 3 3	10½
1 3 19 at 3 12	4	- 6 18	10½
2 1 19 at 1 19	1	- 4 14	6½
2 2 12 at 1 18	4	- 4 19	11½

£ 91 9 9½

Cwt.

Chap. III. PRACTICE.

199

Cwt. qr. lb. l. s. d.

5 2 24 at 1 17 4

$$\begin{array}{r} 5 \\ 9 \ 6 \ 8 \\ - 18 \ 8 \\ \hline \frac{1}{7} - 5 \ 4 \\ - 2 \ 8 \\ \hline \pounds 10 \ 13 \ 4 \end{array}$$

4 2 10 at 1 12 6

$$\begin{array}{r} 4 \\ 6 \ 10 \ - \\ \hline \frac{1}{7} - 16 \ 3 \\ \frac{1}{4} - 2 \ 3\frac{3}{4} \\ - - 6\frac{1}{4} \\ \hline \pounds 7 \ 9 \ 1\frac{1}{2} \end{array}$$

2 3 25 at 2 16 10

$$\begin{array}{r} 2 \\ 5 \ 13 \ 8 \\ \frac{1}{2} 8 \ 5 \\ \hline \frac{1}{4} - 14 \ 2\frac{1}{2} \\ \frac{1}{8} - 8 \ 1\frac{1}{4} \\ - 3 \ 6\frac{1}{2} \\ - 1 \ - \\ \hline \pounds 8 \ 8 \ 11\frac{1}{4} \end{array}$$

17 2 17 at 1 13 10

$$\begin{array}{r} 4 \\ 6 \ 15 \ 4 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 27 \ 1 \ 4 \\ 1 \ 13 \ 10 \\ \hline \frac{1}{7} \frac{1}{7} - 10 \ 11 \\ \frac{1}{4} - 2 \ 5 \\ - 2 \ 1\frac{1}{4} \\ - - 7\frac{1}{2} \\ \hline \pounds 29 \ 17 \ 2\frac{1}{2} \end{array}$$

Cwt. qr. lb. $\frac{1}{7}$ l. s. d.

12 1 19 at 1 14 5

$$\begin{array}{r} 12 \\ 20 \ 13 \ - \\ - 8 \ 7\frac{1}{4} \\ \hline \frac{1}{7} - 4 \ 11 \\ \frac{1}{8} - - 7\frac{3}{4} \\ - - 3\frac{1}{2} \\ \hline \pounds 21 \ 7 \ 5 \end{array}$$

4 - 12 at 4 19 4

$$\begin{array}{r} 4 \\ 19 \ 17 \ 4 \\ \hline \frac{1}{7}) 2 \ 9 \ 8 \ \frac{1}{2} - 7 \ 1 \\ \hline 3 \ 6\frac{1}{2} \\ \pounds 20 \ 7 \ 11\frac{1}{2} \end{array}$$

1 - 15 at 2 16 4

$\frac{1}{7}) 11. 8s. 2d.$

$$\begin{array}{r} 4 \ \frac{1}{4} \\ - 3 \ 6\frac{1}{4} \\ \hline \pounds 3 \ 3 \ 10\frac{1}{2} \end{array}$$

1 3 19 at 7 3 12 4

$$\begin{array}{r} 1 \ 16 \ 2 \\ - 18 \ 1 \\ \hline \frac{1}{2} - 10 \ 4 \\ \frac{1}{2} - 1 \ 3\frac{1}{2} \\ - - 7\frac{3}{4} \\ \hline \pounds 6 \ 18 \ 10\frac{1}{2} \end{array}$$

2 1 19 at $\frac{1}{4}$ 1 19 1

$$\begin{array}{r} 2 \\ 3 \ 18 \ 2 \\ \hline \frac{1}{8}, \frac{1}{7}) 19s. 6\frac{1}{2}d. \end{array}$$

$$\begin{array}{r} - 9 \ 9\frac{1}{2} \\ \frac{1}{2} - 2 \ 9\frac{1}{2} \\ - 2 \ 5\frac{1}{4} \\ - 1 \ 4\frac{1}{2} \\ \hline \pounds 4 \ 14 \ 6\frac{1}{2} \end{array}$$

2 2 12 at $\frac{1}{2}$ 1 18 4

$$\begin{array}{r} 2 \\ 3 \ 16 \ 8 \\ \hline \frac{1}{7} - 19 \ 2 \\ - 2 \ 8\frac{1}{2} \\ - 1 \ 4\frac{1}{2} \\ \hline \pounds 4 \ 19 \ 11 \end{array}$$

The

Chap. III. PRACTICE.

199

Cwt. qr. lb. l. s. d.

5 2 24 at 1 17 4

5
9 6 8
- 18 8
 $\frac{1}{7}$ - 5 4
- 2 8

£ 10 13 4

4 2 10 at 1 12 6

4
6 10 -
 $\frac{1}{7}$ - 16 3
 $\frac{1}{4}$ - 2 $3\frac{3}{4}$
- - $6\frac{1}{4}$

£ 7 9 $1\frac{1}{2}$

2 3 25 at 2 16 10

2
5 13 8
- 8 5
 $\frac{1}{4}$ - 14 $2\frac{1}{2}$
 $\frac{1}{7}$ - 8 $1\frac{1}{2}$
- 3 $6\frac{1}{2}$
- 1 -

£ 8 8 $11\frac{1}{4}$

17 2 17 at 1 13 10

4
6 15 4
4

27 1 4

1 13 10
 $\frac{1}{7}$ - 16 11
 $\frac{1}{4}$ - 2 5
- 2 $1\frac{1}{4}$
- - $7\frac{1}{4}$

£ 29 17 $2\frac{1}{2}$

Cwt. qr. lb. $\frac{1}{7}$ l. s. d.

12 1 19 at 1 14 5

12
20 13 -
- 8 $7\frac{1}{4}$
 $\frac{1}{7}$ - 4 11
 $\frac{1}{4}$ - - $7\frac{1}{4}$
- - $3\frac{1}{2}$

£ 21 7 5

4 - 12 at 4 19 4

4
19 17 4
- 7 1
3 $6\frac{1}{2}$

£ 20 7 $11\frac{1}{2}$

1 - 15 at 2 16 4

1 1 11. 8s. 2d. 4 $\frac{1}{4}$

- 3 $6\frac{1}{4}$

£ 3 3 $10\frac{1}{2}$

1 3 19 at $\frac{1}{7}$ 3 12 4

1 16 2
- 18 1
 $\frac{1}{7}$ - 10 4
 $\frac{1}{4}$ - 1 $3\frac{1}{2}$
- - $7\frac{1}{2}$

£ 6 18 $10\frac{1}{4}$

2 1 19 at $\frac{1}{4}$ 1 19 1

2
3 18 2

$\frac{1}{7}$, $\frac{1}{4}$ 19s. $6\frac{1}{2}$ d. - 9 $9\frac{1}{4}$

$\frac{1}{2}$ - 2 $9\frac{1}{2}$
- 2 $5\frac{1}{4}$
- 1 $4\frac{1}{2}$

£ 4 14 $6\frac{1}{2}$

2 2 12 at $\frac{1}{7}$ 1 18 4

2
3 16 8
 $\frac{1}{7}$ - 19 2
- 2 $8\frac{1}{4}$
- 1 $4\frac{1}{2}$

£ 4 19 11

The

TO BENJ. BUILDER, for Work and Materials in his House at Henley Park, Surry.

[illegible]

8 107
f 205

1. What

1. What will the carriage of 17 cwt. 3 qrs. 11 lb. come to, at the rate of 7 s. the hundred?

7 s. per cwt.									
17									
<hr/>									
£	5	19	-	-	-	-	-	Carriage of 17	cwt. qt. lb.
	-	3	6	-	-	-	-	-	- 2 -
	-	1	9	-	-	-	-	-	- 1 -
	-	-	5 ¹	-	-	-	-	-	- 7 -
	-	-	3	-	-	-	-	-	- 4 -
<hr/>									
£	6	4	11 ¹	the answer.					
<hr/>									
				Cwt.	17	3	11		
<hr/>									

2. A draper bought 56 pieces of kersey, each piece containing 34 ells English, at the rate of 5 s. 4 d. per ell Flemish. What did the whole come to?

56 pieces	3) 5 s. 4 d. per ell Flemish.
34	1 9 1
<hr/>	<hr/>
224	7 1 1
168	per yard.
<hr/>	<hr/>
4) 1904 ells English in all.	
476	
<hr/>	
2380 yards in all.	
<hr/>	

	Pence.	s.	d.	l.	s.	d.
$\frac{17}{10} \frac{1}{4}$) 2380	=	198	4	=	9 18 4.
<hr/>						
	595					
	238					
	9	18	4			
	3	6	1 1			
<hr/>						
	846	4	5 1	the answer.		

3. If one ounce of silver (plate) bullion cost 5 s. 4 1 d. what will be the value of 14 ingots, each weighing 28 oz. 15 pwts. 12 grains?

Oz. dwt. gr.

28 15 12

2

57 11

7

$\frac{1}{5}$ 402 17 - at 5 s. $4\frac{1}{2}$ d. per oz. $\frac{1}{5}$ 5 $4\frac{1}{2}$

$\frac{1}{4}$	80	8	$\frac{1}{4}$	2	$8\frac{1}{4}$
$\frac{1}{8}$	20	2	$\frac{1}{8}$	1	4
$\frac{1}{16}$	6	14	$\frac{1}{16}$		$6\frac{1}{2}$
$\frac{1}{32}$		16			
$\frac{1}{64}$		4			
		9			
		$6\frac{1}{2}$			

108 5 $3\frac{1}{2}$, the answer required. 4 $6\frac{1}{2}$ p. of 17 dwt.

CASE IX.

DUODECIMALS.

Duodecimals are so called, because they decrease by twelves from the place of feet, towards the right-hand; the inches I call primes, the next seconds, thirds, &c. according to their distance from feet.

This rule is sometimes called cross-multiplication.

RULES for multiplying DUODECIMALS.

Having under the multiplicand written the corresponding denomination of the multiplier; multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier; write each result under its respective term, and carry an unit for ever 12, from each lower denomination to its next superior.

2. In the same manner multiply each term in the multiplicand by the prime (or inches) in the multiplier, and write down the result of each term, one place removed to the right-hand of those in the multiplicand.

3. In the like manner multiply with the seconds (or parts of an inch) setting down the result, one place still further to the right-hand; and the sum of all these give the product required.

Let it be required to multiply 9 feet $7\frac{3}{4}$ inches by 7 feet $10\frac{1}{4}$ inches.

Thus

$$\begin{array}{l} \text{F. inch.} \quad \text{f.} \quad \text{' } \quad \text{"} \\ \text{Thus } 9 \quad 7\frac{1}{2} = 9 \quad 7 \quad 9 \\ \text{And } 7 \quad 10\frac{1}{2} = 7 \quad 10 \quad 3 \end{array}$$

$$\begin{array}{r} 67 \quad 6 \quad 3 \quad - \quad - \\ 8 \quad - \quad 5 \quad 6 \quad - \\ 2 \quad 4 \quad 11 \quad 3 \\ \hline 75 \quad 9 \quad 1 \quad 5 \quad 3, \text{ the product.} \end{array} \quad \left. \begin{array}{l} \text{multiplic.} \times \left\{ \begin{array}{l} 7 \text{ feet.} \\ 10 \text{ primes.} \\ 3 \text{ seconds.} \end{array} \right. \right.$$

It will often happen, that the feet in the given multiplicand are so many, that to multiply them by the lesser denominations, and take $\frac{1}{12}$ th of the product as before directed, will require some work to be done on spare paper, which may be avoided by observing the following

R U L E.

Multiply the feet first; then instead of multiplying by the primes or inches, take an aliquot part of the multiplicand, according to their corresponding inches; thus, for 1 prime or inch, take $\frac{1}{12}$ of the multiplicand, for 2 inches, take $\frac{1}{6}$, for 3 take $\frac{1}{4}$, for 4 take $\frac{1}{3}$, for 5 take $\frac{1}{4} + \frac{1}{6}$ or $\frac{2}{3} + \frac{1}{6}$, for 6 take $\frac{1}{2}$, for 7 take $\frac{1}{2} + \frac{1}{6}$, or $\frac{2}{3} + \frac{1}{6}$, for 8 take $\frac{2}{3} + \frac{1}{6}$, or $\frac{5}{6}$, for 9 take $\frac{3}{4} + \frac{1}{4}$, for 10 take $\frac{2}{3} + \frac{1}{6}$, and for 11 inches take $\frac{1}{2} + \frac{1}{4} + \frac{1}{6}$; and in like manner for seconds or parts, only observing that the last quotes are only $\frac{1}{12}$ part of the foregoing, and must accordingly be put one place further toward the right-hand:

Let it be required to multiply 368 feet $7\frac{1}{2}$ inches, by 9 feet $4\frac{1}{2}$ inches?

$$\begin{array}{r} \text{Feet.} \quad \text{' } \quad \text{"} \\ 368 \quad 7 \quad 6 \\ 9 \quad 4 \quad 9 \\ \hline 3317 \quad 7 \quad 6 \\ 122 \quad 10 \quad 6 \quad - \quad - \quad = \frac{1}{12}, \text{ or 4 primes.} \\ 15 \quad 4 \quad 3 \quad 9 \quad - \quad = \frac{1}{6} \text{ of } \frac{1}{12}, \text{ or 6 seconds.} \\ 7 \quad 8 \quad 1 \quad 10 \quad 6 \quad = \frac{1}{2} \text{ of the last, or 3 seconds.} \\ \hline 3463 \quad 6 \quad 5 \quad 7 \quad 6 \end{array}$$

Feet.	/	''				
279	5	3				
7	9	6				
1956	-	9	-	-		
139	8	7	6	-	=	$\frac{1}{2}$ or 6 primes.
69	10	3	9	-	=	$\frac{1}{2}$ of the last, or 3 primes.
11	7	8	7	6	=	$\frac{1}{6}$ of the last, or 6 seconds.
2177	3	4	10	6		

But if the multiplier also be a large number, multiply the feet into each other; then for the primes and seconds in the multiplier, proceed as in the last examples; and for the primes and seconds in the multiplicand, take aliquot parts of the feet in the multiplier; the sum of all will be the answer required.

Feet.	/	''				
187	10	3				
73	7	9				
561						
1309						
93	11	1	6	-	=	$\frac{1}{2}$ for 6 primes.
15	7	10	3	-	=	$\frac{1}{6}$ of the last for 1 prime.
7	9	11	1	6	=	$\frac{1}{2}$ of the last for 6 seconds.
3	10	11	6	9	=	$\frac{1}{2}$ of the last for 3 seconds.
36	6	-	-	-	=	$\frac{1}{2}$ of 73 feet for 6 primes
24	4	-	-	-	=	$\frac{1}{3}$ of 73 feet for 4 primes
1	6	3	-	-	=	$\frac{1}{4}$ of 73 primes for 3''
13834	8	1	5	3		

CHAPTER IV.

VULGAR FRACTIONS.

SECT. I.

NOTATION.

A FRACTION, or broken number, is that which represents a part of any thing proposed, and is expressed by two numbers, placed one above the other, with a line drawn betwixt them.

L

Thus

Thus $\left\{ \begin{array}{l} \frac{3}{4} \text{ Numerator.} \\ \frac{3}{4} \text{ Denominator.} \end{array} \right.$

The denominator, or number placed underneath the line, denotes how many equal parts the integer or whole thing is supposed to be divided into, being only the divisor in division; and the numerator or number placed above the line, shews how many of these parts are contained in the fraction.

A vulgar fraction is either proper, improper, simple, or compound.

A proper fraction is such, whose numerator is less than its denominator, as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, or $\frac{1}{11}$, &c.

An improper fraction is such, whose numerator is equal to, or greater than its denominator, as $\frac{5}{5}$, $\frac{18}{7}$, $\frac{271}{11}$, $\frac{387}{5}$, &c.

Here note, that if the numerator and denominator are equal, the fraction is equal to an integer.

A simple fraction hath only one numerator and denominator, whether it be proper or improper, as $\frac{1}{2}$, $\frac{4}{3}$, $\frac{5}{7}$, $\frac{2}{3}$, &c.

A compound fraction, or fraction of a fraction, hath several numerators and denominators connected together by the particle of, as $\frac{2}{7}$ of $\frac{3}{4}$ of $\frac{1}{2}$, by which is meant first, that the integer or whole thing is divided into five equal parts, three of which parts make $\frac{3}{4}$, which fraction is divided into eight equal parts, and seven of those parts taken, viz. $\frac{7}{8}$ of $\frac{3}{4}$; then this fraction is divided into seven equal parts, and two of those parts taken, viz. $\frac{2}{7}$ of $\frac{7}{8}$ of $\frac{3}{4}$.

Suppose, for instance, a pound sterling to be so divided.

5	20s.
-	—
	4s. $\times 3 = 12s. = \frac{3}{4} \text{ } \pounds.$
-	—
8	12s.
-	—
	1s. 6d. $\times 7 = 10s. 6d. = \frac{7}{8} \text{ of } \frac{3}{4} \text{ of } \pounds 1.$
-	—
7	10s. 6d.
-	—
	1s. 6d. $\times 2 = 3s. = \frac{2}{7} \text{ of } \frac{7}{8} \text{ of } \frac{3}{4} \text{ of a pound sterling.}$

A mixed number is a whole number with a fraction annexed, as $5\frac{3}{7}$, which is read five and three-sevenths; $21\frac{1}{2}$ is twenty-one and one half, &c.

S E C T.

SECTION II.

REDUCTION of VULGAR FRACTIONS.

IN order to facilitate the doctrine of vulgar fractions, I shall premise the following

A X I O M.

If both the numerator and denominator of a fraction be multiplied or divided by one and the same number, the fraction will retain the same value.

Viz. $\frac{2}{7} \times \frac{3}{3} = \frac{6}{21}$, and $\frac{12}{16} \div \frac{4}{4} = \frac{3}{4}$; that is, if the numerator 7 and the denominator 9 be each multiplied by the same number, viz. by 3, the produced fraction, viz. $\frac{21}{27}$, and the proposed one $\frac{2}{9}$ are equal, as the numerator and denominator of the first are in the same proportion as the numerator and denominator of the second.

Also if the numerator 12, and the denominator 16, be each divided by the same number, viz. by 4, the fractions $\frac{3}{4}$ and $\frac{12}{16}$ for the same reason are equal.

C A S E I.

To reduce a compound fraction into a single one.

R U L E.

Multiply all the numerators into one another for a numerator, and all the denominators into one another for the denominator.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{11}$ into a single fraction.

$$\frac{3 \times 2 \times 5 \times 4}{4 \times 3 \times 6 \times 11} = \frac{120}{792}, \text{ the single fraction required.}$$

If a numerator of one term in a compound fraction be equal to a denominator in another term, cancel or reject both, and divide those numerators and denominators which are divisible by each other, or by the same number; which quotients multiplied into the remaining numerators and denominators, reduce the compound fraction to a single one in its lowest terms.

Let the last example, viz. $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{4}{11}$ be reduced into a single fraction, and its lowest terms.

$$\frac{3}{4} \text{ of } \frac{2}{3} \text{ of } \frac{5}{6} \text{ of } \frac{4}{11} = \frac{5}{33} = \frac{120}{792}.$$

2. Let $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$ be reduced into a single fraction in its lowest terms.

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{4}{5} \text{ of } \frac{3}{4} = \frac{1}{12}, \text{ as was required.}$$

C A S E II.

To reduce mixed numbers and integers into improper fractions.

I shall divide this case into three parts.

I. If the integer have no assigned denominator.

R U L E.

An unit subscribed must be the denominator.

$$\text{Thus } 7 = \frac{7}{1}, \quad 12 = \frac{12}{1}, \quad 56 = \frac{56}{1}, \quad 248 = \frac{248}{1}, \text{ \&c.}$$

II. If the integer have an assigned denominator.

R U L E.

Multiply the integer by the assigned denominator, the product is the numerator to the assigned denominator.

Reduce 17 into a fraction whose denominator shall be 12.

$$\text{Thus } 17 \times 12 = 204 \text{ numerator, } \therefore \frac{204}{12} = 17.$$

III. If the integer have a fraction annexed.

R U L E.

Multiply the integer by the denominator, and to the product add the numerator; the sum is the numerator to the denominator of the annexed fraction.

Let

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Let $7\frac{7}{8}$, $21\frac{1}{2}$ and $119\frac{1}{4}$ be reduced into improper fractions.

First, $7 \times 8 + 7 = 63$, $21 \times 27 + 19 = 586$, and $119 \times 38 + 35 = 4557$.

Therefore $7\frac{7}{8} = \frac{63}{8}$, $21\frac{1}{2} = \frac{586}{27}$, and $119\frac{1}{4} = \frac{4557}{38}$.

CASE III.

To reduce an improper fraction into its equivalent, whole, or mixed number.

RULE.

Divide the numerator by the denominator, the quotient gives the integer, and under the remainder (if any) subscribe the denominator.

Reduce $\frac{63}{8}$, $\frac{586}{27}$, $\frac{4557}{38}$, into their equal, whole, or mixed numbers.

$$8) \frac{63}{7} \left(7\frac{7}{8} = \frac{63}{8}, 27) \frac{586}{19} \left(21\frac{1}{2} = \frac{586}{27}, \text{ and} \right.$$

$$38) \frac{4557}{35} \left(119\frac{1}{4} = \frac{4557}{38} \right.$$

2. Let $\frac{204}{12}$, $\frac{364}{7}$, and $\frac{108}{3}$, be reduced into their equivalent, whole, or mixed numbers.

$$12) 204 \left(17 = \frac{204}{12}, 7) 364 \left(52 = \frac{364}{7}, \text{ and } 3) 108 \left(36 = \frac{108}{3} \right.$$

CASE IV.

To abbreviate or reduce fractions into their lowest or least denomination.

If the numerator and denominator are even numbers, take half the one, and half the other, as often as may be; and when either of them fall out to be an odd number, then divide them by any number that you can discover will divide both numerator and denominator without any remainder.

Or, by finding the greatest common measure by the following

L 3

RULE.

R U L E.

Divide the greater number by the lesser, and that divisor by the remainder (if there be any) and so on continually until there be no remainder left. Then will the last divisor be the greatest common measure, which if it happen to be 1, then are they prime numbers, and are already in their lowest terms; but if otherwise, divide the numbers by the last divisor, and their quotients will be their least terms required.

1. Let $\frac{192}{336}$ be reduced into its lowest terms.

$$2) \frac{192}{336} \left(\frac{96}{168} \right) \frac{48}{84} \left(\frac{24}{42} \right) \frac{12}{21} \left(\frac{4}{7} \right) \dots \frac{4}{7} = \frac{192}{336}.$$

By finding the common measure.

$$192 \overline{) 336} \left(\frac{1}{144} \right) \frac{192}{48} \left(\frac{1}{144} \right) \left(\frac{1}{3} \text{ Common measure } 48 \right) \frac{192}{336} \left(\frac{4}{7} \right).$$

2. What is $\frac{1036}{1184}$ in its lowest terms?

$$2) \frac{1036}{1184} \left(\frac{518}{592} \right) \frac{259}{296} \left(\frac{7}{8} \right) = \frac{1036}{1184}.$$

By finding the common measure.

$$1036 \overline{) 1184} \left(\frac{1}{184} \right) \frac{1036}{1036} \left(\frac{1}{148} \right) \frac{1036}{1184} \left(\frac{7}{8} \right), \text{ as before.}$$

C A S E V.

To alter or change different fractions into one denomination, retaining the same value.

R U L E.

Multiply all the denominators into each other for a new and common denominator, and each numerator into all the denominators but its own for a new numerator.

1. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, into fractions, having one common denominator.

First,

First, $4 \times 9 \times 5 = 180$, common denominator.

Also $\left. \begin{array}{l} 3 \times 9 \times 5 = 135 \\ 7 \times 4 \times 5 = 140 \\ 2 \times 4 \times 9 = 72 \end{array} \right\}$ numerators.

Therefore $\frac{1}{4} = \frac{135}{180}$, $\frac{2}{9} = \frac{140}{180}$, and $\frac{3}{5} = \frac{72}{180}$.

2. Reduce $\frac{1}{2}$, $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$, and $3\frac{2}{7}$, into fractions, having one common denominator.

First $\frac{1}{2}$ of $\frac{2}{4}$ of $\frac{1}{5} = \frac{1}{10}$, and $3\frac{2}{7} = \frac{26}{7}$.

The fractions reduced to single ones will be $\frac{1}{2}$, $\frac{1}{10}$, and $\frac{26}{7}$.

First, $3 \times 10 \times 7 = 210$, common denominator.

Also $\left. \begin{array}{l} 2 \times 10 \times 7 = 140 \\ 1 \times 7 \times 3 = 21 \\ 26 \times 3 \times 10 = 780 \end{array} \right\}$ numerators.

$\therefore \frac{1}{2} = \frac{140}{210}$, $3\frac{2}{7} = \frac{26}{7} = \frac{780}{210}$. And $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5} = \frac{1}{10} = \frac{21}{210}$.

If there be two denominators already alike, you need multiply but by one of them, as in the following example.

3. Reduce $\frac{1}{2}$, $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$, 5, and $25\frac{3}{5}$ into fractions, having one common denominator.

First $\frac{1}{2}$ of $\frac{1}{8}$ of $\frac{1}{5} = \frac{1}{80}$, $5 = \frac{5}{1}$, and $25\frac{3}{5} = \frac{128}{5}$.

In single ones, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{128}{5}$.

$8 \times 1 \times 5 = 40$, common denominator.

$\left. \begin{array}{l} 7 \times 1 \times 5 = 35 \\ 1 \times 5 = 5 \\ 5 \times 8 \times 5 = 200 \\ 128 \times 8 = 1024 \end{array} \right\}$ numerators.

Viz. $\frac{1}{2} = \frac{35}{40}$, $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5} = \frac{1}{60} = \frac{5}{40}$, $5 = \frac{5}{1} = \frac{200}{40}$.

And $25\frac{3}{5} = \frac{128}{5} = \frac{1024}{40}$.

2. When there are only two fractions to be reduced, if one of the denominators is a multiple of the other, divide; and by the quote multiply the numerator and denominator of that fraction which hath the least denominator, and the fraction thus found will be equivalent to the given ones.

Reduce $\frac{3}{7}$ and $\frac{4}{8}$ to a common denominator.

First, $28 \div 7 = 4$. Then $\frac{3 \times 4}{7 \times 4} = \frac{12}{28}$.

$\therefore \frac{3}{7}$ and $\frac{4}{8}$ are the fractions required.

3. Or if both of the denominators have a common multiple, divide each of the denominators thereby, and multiply the contrary numerators and denominators by each contrary quotient.

Let $\frac{3}{8}$ and $\frac{4}{6}$ be fractions proposed to be reduced.

As 2 will measure 6 and 8, their respective quotes being 3 and 4.

Then $3 \times 8 = 4 \times 6 = 24$, the common denominator.

Also $5 \times 4 = 20$, and $3 \times 3 = 9$, the numerators.

$\therefore \frac{5}{8} = \frac{15}{24}$, and $\frac{3}{6} = \frac{9}{24}$, the fraction required.

Reduce $\frac{7}{8}$ and $\frac{11}{15}$ to a common denominator.

Divide by 5. . . 4 and 3 are the quotes.

Then 20×3 , or $15 \times 4 = 60$, the common denominator.

Also $7 \times 3 = 21$, and $11 \times 4 = 44$, numerators.

Therefore $\frac{7}{8} = \frac{21}{24}$, and $\frac{11}{15} = \frac{44}{40}$, are the fractions required.

C A S E VI.

To reduce a fraction to an equivalent one of any other assigned denominator; viz. to find a numerator, which, with the assigned denominator, will make a fraction equivalent to the proposed one, when possible.

R U L E.

Multiply the assigned denominator by the numerator of the proposed fraction, and divide the product by the denominator; the quote (if there be no remainder) is the numerator sought.

Reduce $\frac{3}{4}$ to an equivalent fraction, having for its denominator 28.

Thus

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Thus $28 \times 3 = 84$; then $84 \div 4 = 21$, the numerator; that is, $\frac{3}{4} = \frac{21}{4}$.

Whenever the denominator assigned is divisible (without a remainder) by the denominator of the given fraction, the thing is possible, otherwise not.

CASE VII.

To find whether one fraction be greater or lesser in value than another.

RULE.

Multiply the numerators into each other's denominator, and if the products are equal, the fractions are so; otherwise the numerator of the greatest fraction multiplied by the denominator of the other, will be the greatest product.

Which is the fraction of the greatest value, viz. $\frac{7}{8}$, or $\frac{5}{6}$?

Thus $7 \times 6 = 42$; but $5 \times 8 = 40$, consequently $\frac{7}{8}$ is the fraction of the greater value.

Let $\frac{3}{4}$ and $\frac{2}{3}$ be fractions proposed.

Then $3 \times 28 = 84$; and $4 \times 21 = 84$. Here the products, and also the value of the fractions, are equal.

CASE VIII.

To reduce coins, weights, measures, &c. into fractions.

RULE.

Reduce the coin, weight, &c. into the lowest name mentioned for a numerator; and put the number of those parts contained in an unit of the integer, to which the proposed fraction is to be reduced for the denominator; then reduce the fraction into its lowest terms.

Reduce 7 s. 3 d. into a fraction, a pound being the integer.

12

87 pence, the fraction will be $\frac{87}{120} \text{ l.}$

$3) \frac{87}{120} (= \frac{29}{40} \text{ l. in its lowest terms} = 7 \text{ s. } 3 \text{ d.}$

Reduce 4 s. 7½ d. into a fraction, a pound being the integer.

12

55
4

$\frac{55}{4} = \frac{221}{8} \text{ l.} = 4 \text{ s. } 7\frac{1}{2} \text{ d., as was required.}$

223 farthings.

Reduce

Reduce $4\frac{1}{2}$ d. into the fraction of a shilling.

$$\frac{4}{18} \text{ farthings.}$$

$$6) \frac{18}{48} (= \frac{1}{2} = 4\frac{1}{2} \text{ d. as was required.})$$

Reduce 3 cwt. 2 qrs. 21 lb. into a fraction, 1 cwt. being the integer.

3 cwt. 2 qrs. 21 lb.

$$\begin{array}{r} 4 \\ \hline 14 \\ 28 \\ \hline 112 \\ 28 \\ \hline 413 \end{array} \quad 7) \frac{413}{112} = \frac{59}{16} = 3 \text{ cwt. 2 qr. 21 lb. as was req.}$$

Reduce 27 oz. 17 pwt. 18 gr. into a fraction, one ounce Troy being the integer.

$$\begin{array}{r} \text{oz. pwt. gr.} \\ 27 \ 17 \ 18 \\ 20 \\ \hline 557 \\ 24 \\ \hline 2228 \\ 1114 \\ \hline 6) \frac{13386}{480} = \frac{2232}{80} = \frac{558}{20} = 27 \text{ oz. 17 pwt. 18 gr.} \end{array}$$

C A S E IX.

To reduce a fraction of an unit of a higher denomination to an equivalent fraction of an unit of a lower species of the same kind with the higher.

R U L E.

Multiply the numerator of the given fraction, by the number of units in the next inferior species that make an unit of the denomination of your fraction, and that product multiply

multiply by the number of units in the next inferior denomination that make an unit of the last denomination, and thus proceed till you come to the lowest you design; then make the last product a numerator to the denominator of the fraction given.

1. Reduce $\frac{1}{14}$ l. to an equivalent fraction in the denomination of 1 d.

First, $3 \times 20 = 60$, and $60 \times 12 = 720$, numerator.

$$\begin{array}{ccc} \text{D.} & \text{D.} & \text{£.} \\ \frac{720}{14} = \frac{360}{7} = \frac{3}{14}, & \text{as was required.} & \end{array}$$

2. Reduce $\frac{1}{4}$ of a shilling to the fraction of a farthing.

First, $4 \times 12 = 48$, and $48 \times 4 = 192$, numerator.

$$\begin{array}{c} \text{qr.} \\ \therefore \frac{192}{5} = \frac{1}{4} \text{ of a shilling, as was required.} \end{array}$$

3. Reduce $\frac{1}{3}$ cwt. to the fraction of 1 lb.

Thus $4 \times 2 = 8$, and $8 \times 28 = 224$, numerator.

$$\begin{array}{c} \text{lb.} \\ \therefore \frac{224}{3} = \frac{1}{3} \text{ cwt. as was required.} \end{array}$$

CASE X.

To reduce a fraction of an unit of a lower denomination to an equivalent fraction in the denomination of an higher.

RULE.

Multiply the denominator by the number of units in the given fraction that is equal to an unit of the next superior denomination, and the product by such a number of units of its denomination, as is equal to an unit of the next above it; and thus go on till you come to the highest species required, and the last product is a denominator to the numerator of the fraction given.

1. Reduce $\frac{1}{4}$ of a farthing into the fraction of 1 l.

$$8 \times 4 \times 12 \times 20 = 7680, \text{ denominator.}$$

$$\text{So that } \frac{1}{4} \text{ of a farthings} = \frac{5}{7680} = \frac{1}{1536} \text{ l.}$$

Or

Or by compound fractions, $\frac{1}{4}$ of a farthing = $\frac{1}{8}$ of $\frac{1}{4}$ of

$$\frac{1}{12} \text{ of } \frac{1}{70} = \frac{1}{1536}$$

2. Reduce $\frac{1}{4}$ oz. into the fraction of 1 cwt.

$$7 \times 16 \times 28 \times 4 = 12544, \text{ denominator.}$$

$$\therefore \frac{4}{12544} = \frac{1}{3136} = \frac{4}{7}$$

That is, $\frac{1}{7}$ of $\frac{1}{16}$ of $\frac{1}{28}$ of $\frac{1}{4}$ = $\frac{1}{3136}$ = $\frac{4}{7}$, as before.

C A S E XI.

To find the value of a fraction in coin, weight, measure, time, &c.

R U L E.

Multiply the numerator of the given fraction by the number of units of the next inferior species that makes one of the denomination of your fraction, and divide the product by the denominator; the quotient is so many integers of that lower species; and if there is a remainder, proceed as before, still reducing and dividing, till you come to the lowest species; and the several quotients, with the remainder (if any, which is always the numerator of a fraction of the lowest species) are the answer.

1. What is the value of $\frac{1}{7}$ of a pound?

$$\begin{array}{r} 5 \\ \times 20 \\ \hline 100 \end{array} \begin{array}{l} \text{s. d. qr.} \\ 14 \quad 3 \quad 1\frac{1}{7}, \text{ answer.} \end{array}$$

$$\begin{array}{r} 2 \\ \times 12 \\ \hline 24 \\ 3 \\ \times 4 \\ \hline 12 \\ (5) \end{array}$$

2. What is the value of $\frac{1}{710}$ cwt.?

$$\begin{array}{r} 17 \\ 4 \\ \hline 68 \\ \times 28 \\ \hline 544 \\ 136 \\ \hline 1904 \end{array} \begin{array}{l} \text{lb. oz. dr.} \\ 10 \quad 11 \quad 2\frac{1}{2}, \text{ anf.} \end{array}$$

$$\begin{array}{r} 124 \\ \times 16 \\ \hline 1984 \\ 204 \\ 26 \\ \hline 416 \\ (60) \end{array} \begin{array}{l} 2) \frac{60}{178} (\frac{30}{89} \end{array}$$

3. What

3. What is the value of $\frac{4}{3}$ of a shilling?

$$\begin{array}{r} 4 \\ \times 12 \\ \hline 48 \end{array} \text{ d. f. } 2\frac{2}{3} \text{ answer.}$$

$$\begin{array}{r} 3 \\ \times 4 \\ \hline 12 \\ (2) \end{array}$$

4. What is the value of $\frac{3}{4}$ of a degree?

$$\begin{array}{r} 3 \\ \times 60 \\ \hline 180 \end{array} (22' 30'', \text{ answer.})$$

$$\begin{array}{r} 4 \\ \times 60 \\ \hline 240 \end{array}$$

5. What is the value of $\frac{6}{7}$ hundred weight?

$$\begin{array}{r} 6 \\ 4 \\ \hline 7) 24 \end{array} (3 \text{ qr. lb. } 12, \text{ answer.})$$

$$\begin{array}{r} 3 \\ \times 28 \\ \hline 84 \end{array}$$

6. What is the value of $\frac{2}{3}$ of $\frac{1}{4}$ of a year?

$$\frac{2}{3} \text{ of } \frac{1}{4} = \frac{1}{6}.$$

Seconds in a year = $31556937 \times 10 = 315569370$.

$$\begin{array}{r} 60) 315569370 \\ 180 \\ \hline 21) 315569370 \end{array} (15027112 \frac{1}{2} = \frac{6}{7} \text{ seconds.})$$

$$\begin{array}{r} 105 \\ 56 \\ 149 \\ 23 \\ 27 \\ 60 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 60) 250451 \text{ minutes } 52'' \frac{6}{7}. \\ 24) 4174 \text{ hours } 11' 52'' \frac{6}{7}. \\ 173 \text{ days } 22 \text{ h. } 11' 52'' \frac{6}{7}, \text{ answer.} \end{array}$$

If the fraction to be valued be an improper one, divide the numerator by the denominator, and the quotient is an integer of the same species with the fraction; then reduce the remainder as before.

7. What is the value of $\frac{77}{8}$ of an ounce troy?

$$\frac{77}{8}$$

$\frac{77}{8}$ oz. dwt. gr.
 $\frac{77}{8}$ 77 (9 12 12, the answer.

$$\begin{array}{r} 5 \\ \times 20 \\ \hline 100 \\ 4 \\ \times 24 \\ \hline 96 \\ \hline \end{array}$$



S E C T. III:

ADDITION of FRACTIONS.

IN order to prepare fractions for addition or subtraction, all compound fractions must be reduced to single ones; and if they are of different denominations, they must be brought into the same denomination, and reduced, so as all the fractions shall have one common denominator.

R U L E.

Add together all the numerators, for a new numerator, under which subscribe the common denominator.

1. Add $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ together.

First, $\frac{1}{3} = \frac{4}{12}$, $\frac{1}{4} = \frac{3}{12}$ and $\frac{1}{5} = \frac{2}{12}$, per reduction.

Then $10 + 15 + 12 = 37$, the new numerator.

$\therefore \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{37}{12} = 3\frac{1}{12}$, the sum required.

2. Add $3\frac{1}{2} + \frac{1}{4} + \frac{1}{5}$ of $\frac{1}{7} + 7$ into one sum.

First,

$$\left. \begin{array}{l} \frac{5}{8} = \frac{25}{40} \\ \frac{7}{8} = \frac{35}{40} \\ \frac{1}{2} \text{ of } 7 = \frac{28}{40} \end{array} \right\} \text{ per reduction.}$$

Then $25 + 35 + 28 = 88$, and $\frac{88}{40} = 2\frac{11}{10}$.

$\therefore 3 + 7 + 2\frac{11}{10} = 12\frac{11}{10}$, the sum required.

2

3. Add

3. Add $\frac{2}{7}$ of 15l. + $3\frac{1}{2}$ l. + $\frac{1}{7}$ of $\frac{2}{7}$ of $\frac{2}{7}$ of a pound + $\frac{2}{7}$ of $\frac{2}{7}$ of a shilling into one sum.

$$\left. \begin{array}{l} \text{First, } \frac{2}{7} \text{ of } 15\text{l.} = \frac{30}{7} = 4\frac{2}{7} \\ \frac{1}{7} \text{ of } \frac{2}{7} \text{ of } \frac{2}{7} = \frac{2}{7^3} = \frac{2}{343} \\ \frac{2}{7} \text{ of } \frac{2}{7} \text{ s.} = \frac{2}{7^2} \text{ s.} = \frac{2}{49} \text{ s.} = \frac{2}{49} \times \frac{1}{20} = \frac{2}{980} \end{array} \right\} \begin{array}{l} 3\frac{1}{2} \\ \\ \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{Reduced into pounds and} \\ \text{fractions of a pound} \\ \text{sterling.} \end{array}$$

And $\frac{2}{7} = \frac{2}{70}$, $\frac{1}{7} = \frac{1}{70}$, $\frac{1}{7} = \frac{1}{70}$, and $\frac{1}{70}$, with one common denominator.

Then $20 + 30 + 10 + 1 = 61$, numerator.

$\therefore 4 + 3 + \frac{61}{70} = 7\frac{61}{70} = 7\text{l. } 17\text{s. } 5\frac{1}{2}\text{d.}$, the answer.



SECT. IV.

SUBTRACTION of FRACTIONS.

THE fractions being prepared, as before directed in addition, then,

RULE,

Subtract one numerator from the other, and their difference will be a new numerator, under which subscribe the common denominator.

1. Subtract $\frac{2}{3}$ of $\frac{2}{7}$, from $\frac{2}{7}$.

$$\text{First } \frac{2}{7} \text{ of } \frac{2}{7} = \frac{2}{21}, \text{ and } \frac{2}{3} = \frac{14}{21}.$$

$$\therefore \frac{14}{21} - \frac{2}{21} = \frac{12}{21} = \frac{4}{7}, \text{ the answer required.}$$

2. What number is that, from which if you deduct the $\frac{1}{16}$ of $\frac{47}{19}$, and to the remainder add $\frac{1}{16}$ of $\frac{47}{19}$, the sum will be 3?

$$\begin{aligned} \text{First, } \frac{1}{16} \text{ of } \frac{47}{19} &= \frac{47}{304}. \text{ Then } 3 = \frac{912}{304}; \text{ and } \frac{912}{304} - \frac{47}{304} \\ &= \frac{865}{304} = \frac{21625}{7600}. \end{aligned} \quad \text{Also}$$

Also $\frac{1}{25}$ of $\frac{7}{8} = \frac{266}{7600} \therefore \frac{21625}{7600} + \frac{266}{7600} = 2\frac{5691}{7600}$, the ans.

3. What number is that, to which if you add $\frac{1}{11}$ of 12 + $\frac{1}{15}$ of 27, and from the total subtract $\frac{1}{7}$ of $7\frac{1}{2} - \frac{2}{3}$ of $1\frac{1}{4}$, the remainder shall be 8?

First, $\frac{1}{7}$ of $7\frac{1}{2} = \frac{60}{24}$, and $\frac{2}{3}$ of $1\frac{1}{4} = \frac{29}{24} \therefore \frac{60}{24} - \frac{29}{24} = 1\frac{7}{24} = 1\frac{1463}{576}$.

Then $8 + 1\frac{1463}{576} = 9\frac{1463}{576}$. Also $\frac{1}{11}$ of $\frac{12}{1} = \frac{228}{209}$; and

$\frac{1}{15}$ of $\frac{27}{1} = \frac{297}{209}$.

$\therefore \frac{228}{209} + \frac{297}{209} = \frac{525}{209} = 2\frac{353}{209}$. Lastly, $9\frac{1463}{576} - 2\frac{353}{209} = 6\frac{3911}{576}$, the number sought.



S E C T. V.

MULTIPLICATION of FRACTIONS.

TO prepare fractions for either multiplication or division, reduce compound fractions to single ones; bring mixed numbers into improper fractions, and express whole numbers fraction-wise; also reduce fractions into their lowest terms. Then,

R U L E,

Multiply the numerators into one another for a new numerator, and the denominators one into another for a new denominator.

1. Multiply $\frac{3}{7}$ into $\frac{5}{6}$. First, $3 \times 5 = 15$, and $7 \times 6 = 42$.

Answer, $\frac{3}{7} \times \frac{5}{6} = \frac{15}{42} = \frac{5}{14}$.

2. Multiply $\frac{2}{11}$ into $\frac{2}{3}$ of $\frac{7}{7}$.

First, $\frac{2}{3}$ of $\frac{7}{7} = \frac{2}{1}$. Then $7 \times 2 = 14$, and $11 \times 7 = 77$.

Answer, $\frac{2}{11} \times \frac{2}{3}$ of $\frac{7}{7} = \frac{14}{77} = \frac{2}{11}$.

3. Mul-

3. Multiply $7\frac{2}{3}$ into $5\frac{1}{4}$. First $7\frac{2}{3} = \frac{67}{9}$, and $5\frac{1}{4} = \frac{23}{4}$.

Then $67 \times 23 = 1541$, and $9 \times 4 = 36$.

Answer $7\frac{2}{3} \times 5\frac{1}{4} = \frac{1541}{36} = 42\frac{25}{36}$.

4. Multiply $2\frac{1}{2}$ by $\frac{1}{7}$, and this product by 2, and this again by $\frac{1}{2}$ of $\frac{1}{8}$.

First $2\frac{1}{2} = \frac{5}{2}$, $2 = \frac{2}{1}$, and $\frac{1}{2}$ of $\frac{1}{8} = \frac{1}{16}$.

Then $\frac{5}{2} \times \frac{2}{1} \times \frac{1}{16} \times \frac{1}{8} = \frac{25}{144}$, the answer.

Hence it may be observed, that if the multiplier be a proper fraction, the product will always be less than the multiplicand.



S E C T. VI.

DIVISION of FRACTIONS.

THE fractions being prepared as directed for multiplication, division may be thus performed.

R U L E.

Multiply the numerator of the dividend into the denominator of the dividing fraction for a numerator, and the other numerator and denominator together for a new denominator.

1. Divide $\frac{3}{4}$ by $\frac{2}{5}$ $\frac{2}{5})\frac{3}{4}(\frac{15}{8} = 1\frac{7}{8}$, answer.

2. Divide $\frac{5}{7}$ l. by $\frac{2}{3}$ of a shilling. First $\frac{2}{3}$ shil. $\frac{2}{60}$ l.

Then $\frac{2}{60} = \frac{1}{30})\frac{5}{7}(\frac{150}{7} = 21\frac{1}{7}$ l. = 21 l. 8 s. $6\frac{1}{2}$ d. + $\frac{1}{2}$ farth.

3. Divide $\frac{3}{5}$ by 7. Thus $\frac{7}{1})\frac{3}{5}(\frac{3}{35}$, the answer required.

4. Divide $4\frac{1}{2}$ by $5\frac{1}{2}$. First $4\frac{1}{2} = \frac{14}{3}$, and $5\frac{1}{2} = \frac{38}{7}$.

M

Then

Then $\frac{38}{7} \frac{14}{3} \left(\frac{28}{114} = \frac{49}{57} \right)$, the answer required.

5. Divide $\frac{2}{3}$ of $\frac{3}{7}$ by $\frac{5}{6}$ of $\frac{3}{4}$. First $\frac{2}{3}$ of $\frac{3}{7} = \frac{2}{7}$, and $\frac{5}{6}$ of $\frac{3}{4} = \frac{5}{8}$.

Then $\frac{5}{8} \frac{2}{7} \left(\frac{16}{35} \right)$, the quotient sought.

If the divisor and dividend have both the same denominator, the quotient may be found, by dividing one numerator by another.

6. Divide $3\frac{1}{4}$ by $\frac{3}{4} \dots \frac{3}{4} \frac{15}{4} \left(5, \text{ the answer.} \right)$

7. Divide $\frac{5}{17}$ by $\frac{13}{17} \dots \frac{13}{17} \frac{5}{17} \left(\frac{5}{13}, \text{ the answer.} \right)$

2. If the divisor and dividend have each the same numerator; divide one of the denominators by the other, which will give the quotient required.

8. Divide $\frac{4}{17}$ by $\frac{4}{5} \dots \frac{4}{5} \frac{4}{17} \left(\frac{5}{17}, \text{ the answer.} \right)$

9. Divide $\frac{7}{9}$ by $\frac{7}{25} \dots \frac{7}{25} \frac{7}{9} \left(2\frac{7}{9}, \text{ the answer.} \right)$

3. If the numerator and denominator of the dividend can be divided without a remainder, by the numerator and denominator of the divisor, their quotients will answer the question.

10. Divide $\frac{9}{28}$ by $\frac{3}{7} \dots \frac{3}{7} \frac{9}{28} \left(\frac{3}{4}, \text{ the answer.} \right)$

4. If a number can be found, that will divide both the numerators, or both the denominators (viz. those of the divisor and dividend) without a remainder; use those quotients instead of the given numerators and denominators, which will give the result in its lowest terms.

11. Divide $\frac{12}{35}$ by $\frac{9}{14} \dots 3 \frac{3}{14} \frac{4}{25} \left(\frac{8}{15}, \text{ the answer.} \right)$

QUESTIONS to exercise VULGAR FRACTIONS.

1. A lad having got 4000 nuts, in his return was met by mad Tom, who took from him $\frac{5}{8}$ of $\frac{2}{3}$ of his whole stock. Raving Ned lights on him afterwards, and forces $\frac{7}{10}$ of $\frac{1}{2}$ of the remainder from him. Unlucky positive Jack found him, and required $\frac{3}{4}$ of $\frac{1}{4}$ of what he had left. Smiling Dolly was by promise to have $\frac{1}{4}$ of a quarter of what nuts he brought home. How many then had the boy left?

$$\frac{5}{8} \text{ of } \frac{2}{3} \text{ of } 4000 = \frac{4000}{1666\frac{2}{3}} \text{ Mad Tom took.}$$

$$\frac{2}{5} \text{ of } \frac{5}{6} \text{ of } \frac{7000}{3} = \frac{2333\frac{1}{3}}{583\frac{1}{3}} \text{ left. Raving Ned took.}$$

$$\frac{7}{10} \text{ of } \frac{17}{20} \text{ of } 1750 = \frac{1750}{1041\frac{1}{4}} \text{ left. positive Jack took.}$$

$$\frac{3}{4} \text{ of } \frac{1}{4} \text{ of } \frac{2835}{4} = \frac{708\frac{3}{4}}{132\frac{3}{4}} \text{ left. smiling Dolly had.}$$

$$\underline{575\frac{3}{4}}, \text{ the answer.}$$

2. There is a number, which if divided by $\frac{16}{3}$ of $\frac{3}{16}$ will quote $\frac{361}{37}$; pray what is the square of that number?

$$\frac{16}{3} \text{ of } \frac{3}{16} = \frac{1}{1}, \text{ which neither multiplies or divides.}$$

$$\therefore \frac{361}{37} \times \frac{361}{37} = \frac{130321}{1369} = 95\frac{286}{1369}, \text{ the answer.}$$

3. There is a number, which if multiplied by $\frac{3}{4}$ of $\frac{7}{9}$ of $2\frac{1}{2}$, will produce no more than 1; what is the cube of that number?

$$\frac{3}{4} \text{ of } \frac{7}{9} \text{ of } \frac{11}{4} = \frac{77}{48} \text{) } \frac{1}{1} \text{ } \frac{48}{77}$$

$$\text{Then } \frac{48}{77} \times \frac{48}{77} \times \frac{48}{77} = \frac{110592}{450583}, \text{ the answer.}$$

4. Four figures of 9 may be so placed and disposed of, as to denote and read for 100, neither more nor less; pray how is that to be done?

$$\text{Answer } 99\frac{9}{9} = 100.$$

M 2

5. Kitty

5. Kitty told her brother George, that though her fortune on her marriage took 19312 l. out of the family, it was but $\frac{1}{3}$ of two year's rent, heaven be praised, of his yearly income; pray what was that?

$$\begin{array}{r} \frac{2}{5}) \frac{19312}{1} (\frac{96560}{3} \\ \frac{2}{1}) \frac{96560}{3} (\frac{96560}{6} = 16093 \text{ l. } 6 \text{ s. } 8 \text{ d. per annum.} \end{array}$$

6. A merry young fellow in a small time got the better of $\frac{1}{3}$ of his fortune; by advice of his friends he then gave 2200 l. for an exempt's place in the guards; his profusion continued till he had no more than 880 guineas left, which he found by computation was just $\frac{1}{5}$ part of the money after the commission was bought; pray what was his fortune at first?

$$810 \text{ guineas} = 924 \text{ l.}$$

$$\text{Then } \frac{3}{20}) \frac{24}{1} (\frac{18480}{3} = 6160 \text{ l.}$$

$$2200 + 6160 = 8360 = \frac{4}{5} \text{ of his whole fortune.}$$

$$\frac{4}{5}) \frac{8360}{1} (\frac{41800}{4} = 10450 \text{ l. the answer.}$$

7. A certain captain sends out $\frac{1}{3}$ of his soldiers + 10, and there remained $\frac{1}{2}$ + 15; how many soldiers had he?

$$\frac{1}{3} \text{ or } \frac{2}{6} + 10 = \text{what he sent out.}$$

$$\text{And } \frac{1}{2} \text{ or } \frac{3}{6} + 15 = \text{what remained.}$$

$$\text{Their sum } \frac{5}{6} + 25 = \text{number of soldiers.}$$

$$\text{Hence } 25 = \frac{1}{6} \text{ of the soldiers.}$$

$$\therefore 25 \times 6 = 150, \text{ the answer required.}$$

8. A certain gentleman hires a servant, and promises him 24 pounds yearly wages, together with a cloak. At eight months end the servant obtains leave to go away, and instead of his wages receives a cloak + 13 pounds; how much did the cloak cost?

Abby's Analyst.
As

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As 8 months = $\frac{2}{3}$ year; therefore, at 8 months end, his due is $\frac{2}{3}$ of 24 l. (= 16 l.) + $\frac{1}{3}$ of the cloak.

Then 16 l. — 13 l. = 3 l. = value of $\frac{1}{3}$ of the cloak.

$\therefore 3 \text{ l.} \times 3 = 9 \text{ l.}$ the answer required.

9. If a man gain 33 crowns a week, how much must he spend a week to have 500 crowns, together with the expence of four weeks, remaining at the year's end?

Albby's Analyst.

First $30 \times 52 = 1560$ crowns gained in a year.

Also $1560 - 500 = 1060$, the dividend.

And $52 \div 4 = 56$, the divisor.

$\therefore 56 \mid 1060 (18\frac{1}{2} \text{ crowns spent} = 4 \text{ l. } 14 \text{ s. } 7\frac{1}{2} \text{ d. per week.}$

And $30 - 18\frac{1}{2} = 11\frac{1}{2} \text{ crowns} = 2 \text{ l. } 15 \text{ s. } 4\frac{1}{2} \text{ d. saved.}$

10. A country spark address'd a charming she,

In whom all lovely features did agree!

But he not skill'd i'th' art (you may presage,)

Was too solicitous to know her age.

The lady smil'd at his prepos't'rous rul'

Of courtship; but to satisfy the fool,

Made him this answer with a gen'rous air

(A lofty charm peculiar to the fair.)

My age is that, if multiply'd by three,

And two-sevenths of that product tripl'd be,

The square root of two-ninths of that is four;

And now farewell, I'll never see you more.

Your fond impertinence has caus'd this rage:

'Tis clownish fure to ask a woman's age.

So you're desir'd to assist him, or perchance,

The spark must still remain in ignorance. *Ladies Diary.*

First $4 \times 4 = 16$. Then $\frac{2}{9} \mid \frac{16}{1} \left(\frac{144}{2} = 72 \right)$.

Also $3 \mid 72 (24$, and $\frac{2}{7} \mid \frac{24}{1} \left(\frac{168}{2} = 84 \right)$.

$\therefore 3 \mid 84 (28$, the answer required.

11. A person having about him a certain number of crowns, said, If $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$ of what he had, were added together, they would make just 45; how many crowns had he about him?

$$\frac{1}{4} = \frac{3}{12}, \frac{1}{3} = \frac{4}{12}, \text{ and } \frac{1}{6} = \frac{2}{12}.$$

M 3

Then

$$\text{Then } \frac{3}{12} + \frac{4}{12} + \frac{2}{12} = \frac{9}{12} = \frac{3}{4} = 45.$$

$$\therefore \frac{3}{4}) \frac{45}{1} \left(\frac{180}{3} = 60, \text{ the answer.} \right.$$

12. A schoolmaster being asked how many scholars he had, answered: If I had as many, and $\frac{1}{2}$ as many, and $\frac{1}{4}$ as many, I should have 99; how many had he?

$$\text{First } 1 = \frac{4}{4}, \frac{1}{2} = \frac{2}{4}.$$

$$\text{Then } \frac{4}{4} + \frac{4}{4} + \frac{2}{4} + \frac{1}{4} = \frac{11}{4} = 99, \text{ per quest.}$$

$$\therefore \frac{11}{4}) \frac{99}{1} \left(\frac{396}{11} = 36 \text{ scholars. Q. E. F.} \right.$$

13. When I wrote this, if to my age you add,
 $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ (thereof) with $\frac{1}{5}$ more,
 The number 25 will then be had;
 Ingenious Tyro's, pray my age explore.

$$\text{First } 1 = \frac{30}{30}, \frac{1}{2} = \frac{15}{30}, \frac{1}{3} = \frac{10}{30}, \text{ and } \frac{1}{5} = \frac{6}{30}.$$

$$\text{Also } \frac{30}{30} + \frac{15}{30} + \frac{10}{30} + \frac{6}{30} = \frac{61}{30}.$$

$$\text{And } 25 - \frac{3}{5} = 24\frac{2}{5} = \frac{122}{5} = \frac{732}{30}.$$

$$\therefore \frac{61}{30}) \frac{732}{30} \left(12 \text{ years, the answer required.} \right.$$

14. What number is that, which added to its $\frac{1}{2}$ + its $\frac{1}{4}$ + 3, makes 108?

$$\text{First } \frac{1}{1} = \frac{4}{4}, \frac{1}{2} = \frac{2}{4}. \text{ Also } 108 - 3 = 105.$$

$$\text{Then will } \frac{4}{4} + \frac{2}{4} + \frac{1}{4} = \frac{7}{4} = 105.$$

$$\therefore \frac{7}{4}) \frac{105}{1} \left(\frac{420}{7} = 60, \text{ the answer.} \right.$$

15. Admit there is 212 l. 14 s. 7 d. to be divided amongst a captain, four men, and a boy; the captain to have a share and half; the men each a share, and the boy $\frac{1}{2}$ of a share; what ought each person to have?

$$1\frac{1}{2} = \frac{3}{2} = \frac{9}{6} \text{ capt. } \frac{4}{1} = \frac{24}{6} \text{ men, and } \frac{2}{6} \text{ boy.}$$

$$\text{Then } \frac{9}{6} + \frac{24}{6} + \frac{2}{6} = \frac{35}{6} = 2121. 14s. 7d. = 51055d.$$

$$\therefore \frac{35}{6} \left(\frac{51055}{1} \right) \left(\frac{306330}{35} \right) = 8752\frac{2}{3} = 361. 8s. 4\frac{2}{3}d.$$

l. s. d.

$$\begin{array}{r} \text{And } 2) 36 \quad 9 \quad 4\frac{2}{3} \quad \} = 54 \quad 14 \quad -\frac{2}{3} \text{ captain.} \\ \quad + 18 \quad 4 \quad 8\frac{2}{3} \quad \} \\ \quad 36 \quad 9 \quad 4\frac{2}{3} \times 4 = 145 \quad 17 \quad 5\frac{2}{3} \text{ men.} \\ 3) 36 \quad 9 \quad 4\frac{2}{3} \quad (= 12 \quad 3 \quad 1\frac{2}{3} \text{ boy.} \end{array} \quad \left. \vphantom{\begin{array}{l} 36 \\ 18 \\ 36 \\ 36 \end{array}} \right\} \text{Q. E. F.}$$

$$\underline{\underline{\pounds 212 \quad 14 \quad 7}}}$$

16. There is a cistern with three unequal cocks, containing 60 gallons of water; and if the greatest cock be opened, it will be empty in one hour; and if the second cock be opened, it will be empty in two hours; if the third be opened, it will be empty in three hours: now I demand in what time it will be empty, if all run together?

The first empties $\frac{1}{1}$ } cistern in an $\left\{ \frac{6}{\frac{1}{1}} \right\}$ their sum = $\frac{1}{\frac{1}{6}}$ all
 The second - - $\frac{1}{2}$ } hour = $\left\{ \frac{3}{\frac{1}{2}} \right\}$ running.
 And the third - - $\frac{1}{3}$ }

$$\therefore \frac{11}{6} \left(\frac{60}{1} \right) \left(\frac{360}{11} \right) = 32\frac{8}{11} \text{ minutes, the answer.}$$

17. A gentleman has an orchard of fruit trees, one-half of the trees bearing apples, one-fourth pears, one-sixth plums, and one-fifty of them bearing cherries; how many fruit trees in all grow in the said orchard?

$$\text{First } \frac{1}{2} = \frac{6}{12} \text{ apples, } \frac{1}{4} = \frac{3}{12} \text{ pears, } \frac{1}{6} = \frac{2}{12} \text{ plums.}$$

$$\text{Then } \frac{6}{12} + \frac{3}{12} + \frac{2}{12} = \frac{11}{12}. \text{ Also } \frac{12}{12} - \frac{11}{12} = \frac{1}{12} \text{ cherries.}$$

= 50.

$$\begin{array}{l} \therefore 50 \times 6 = 300 \text{ apples.} \\ \text{Also } 50 \times 3 = 150 \text{ pears.} \\ \text{Again } 50 \times 2 = 100 \text{ plums.} \\ \text{And } - - - - - 50 \text{ cherries.} \end{array}$$

$$\text{In all - } \underline{\underline{600.}} \quad \text{Q. E. F.}$$

M 4

18. Five

18. Five persons discoursing about their ages, said the second to the first, my age is the double of your's; and said the third to the first, my age is as much, and $\frac{1}{4}$ as much as your's; then said the fourth to the second and third, my age is as much as both yours added together; but said the fifth, my age is three times as much as the age of the first, and the sum of all our ages make just 168 years; what was the age of each?

$$\begin{array}{r} 1 \\ 2 \\ 1\frac{1}{4} \\ 3\frac{1}{4} \\ 3 \\ \hline \end{array}$$

$10\frac{1}{2} = \frac{21}{2}$ divisor for the first person's age.

$$\begin{array}{l} \therefore \frac{21}{2} \bigg) \frac{168}{1} \left(\frac{336}{21} = 16 \text{ first} \right. \\ \text{Also} - 16 \times 2 = 32 \text{ second} \\ \text{Again} - 16 \times 1\frac{1}{4} = 20 \text{ third} \\ \text{Likewise } 32 + 20 = 52 \text{ fourth} \\ \text{And} - - 16 \times 3 = 48 \text{ fifth} \\ \hline \text{Sum } 168 \end{array} \left. \vphantom{\begin{array}{l} \frac{21}{2} \bigg) \frac{168}{1} \\ \frac{336}{21} \\ 16 \times 2 \\ 16 \times 1\frac{1}{4} \\ 32 + 20 \\ 16 \times 3 \end{array}} \right\} \text{person's age.}$$

CHAPTER V.

DECIMAL FRACTIONS.

SECT. I.

NOTATION.

THE word decimal is derived from *decem* (ten) and denotes the nature of its numbers; because the integer, or whole thing, whether it be coin, weight, measure, time, &c. is supposed to be divided into ten equal parts, and every one of those parts into ten other equal parts, &c. *ad infinitum*.

The integer being thus divided by imagination into 10, 100, 1000, 10000, &c. is the denominator to the decimal fraction :

Thus $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$, &c.

These denominators are seldom or never set down, but only the numerators ; and when the numerators do not consist of so many places as the denominator hath cyphers, the said places in the numerator must be supplied by cyphers prefixed on the left-hand. So $\frac{3}{10}$ is wrote .3, $\frac{17}{100}$ is .05, $\frac{17}{1000}$ is .017, and $\frac{17}{10000}$ is .0051, &c.

Also mixed numbers are expressed thus, viz. 8.7 is 8 and 7 tenths, 59.017 is 59 and 17 thousandths, or parts of a thousand, &c.

Cyphers at the end, namely at the right-hand of a decimal, do neither augment or diminish its value ; for 5, .50, .500, .5000, and .50000, are decimals having the same value, being each equal to $\frac{1}{2}$, as may be found by abbreviation of vulgar fractions.

Cyphers prefixed to decimals, decrease their value in a tenfold proportion, by removing them further from the integer.

Thus { $.5 = 5$ tenth parts.
 $.05 = 5$ parts of an hundred.
 $.005 = 5$ parts of a thousand.
 $.0005 = 5$ parts of ten thousand.
 $.00005 = 5$ parts of an hundred thousand, &c.

In whole numbers, the first place above (that is, on the left-hand of) the place of units, signifies tens of units ; but in fractions, the first place beneath (that is, on the right-hand of) the place of units, denotes tenth-parts of 1, or unity, and is called the first place of decimal parts, or place of primes ; likewise the second place above the place of units, signifies hundreds of units ; but the second place beneath the place of units, expresses hundredth parts of unity, and is called the second place of decimals, or place of seconds ; so that as the value of the places in integers ascend in a tenfold proportion from the place of units towards the left-hand, so the value of the places of decimals descend in a tenfold proportion beneath the place of units towards the right-hand.

A TABLE for NOTATION of INTEGERS and DECIMALS,

8	7	2	3	6	5	.	8	2	3	5	3	7	8	5
Units place.							Parts of an hundred millions.							
Tens.							Parts of ten millions.							
Hundreds.							Parts of a million.							
Thousands.							Parts of an hundred thousand.							
Tens of thousands.							Thousandth parts.							
Hundreds of thousands,							Hundredth parts.							
&c.							Tenth.							

It may be observed by the foregoing table, that the places of integers, or whole numbers, are separated from the decimal parts by a point, that the numbers on the left-hand of the point expresses 872365 integers, or units; and that the number on the right-hand of the point shews .82353785 parts of 1 (or an integer) supposed to be divided into 100000000 equal parts.

Hence, if the separating point, in any mixed or fractional number, be moved one place towards the left-hand, then every figure, and consequently the whole expression, is but a tenth part of what it was before; that is, it is divided by 10; if it be moved two places, it is divided by 100; if three places, by 1000, &c. But if the separating point be moved towards the right-hand, then the whole expression is multiplied by 10, 100, 1000, &c. according as it is moved one, two, or three places.

There are several ways of reading or expressing a decimal, as supposing the decimal parts in the table were to be read in words, viz. .82353785.

First, They may be reduced to, and express as vulgar fractions, viz. $\frac{82353785}{100000000}$.

Secondly, By calling them primes, seconds, &c. according to their distance from the separating point, viz. 8 primes, 2 seconds, 3 thirds, 5 fourths, 3 fifths, 7 sixths, 8 sevenths, and 5 eighths.

Thirdly, Thus 82 millions, 353 thousand, 785 eights.

Fourthly, Or thus, 8, 2, 3, 5, 3, 7, 8, 5 of a decimal.

S E C T.

SECT. II.

REDUCTION of DECIMALS,

CASE I.

To reduce a vulgar fraction into a decimal.

RULE.

Annex cyphers to the numerator, till it be equal to, or greater than the denominator; then divide by the denominator, and the quotient will be the decimal sought.

If, after you have made use of all the cyphers annexed to the numerator, there be a remainder, annex cyphers thereto, and continue your division, till it divide off or arrive to what degree of exactness you think proper.

Always observe to set a point betwixt the numerator and the cyphers annexed thereto, and that the quotient have as many places as you annex cyphers to the numerator and remainders; and if it be deficient, let the want be supplied by prefixing as many cyphers to the quotient as it falls short.

EXAMPLE.

Reduce $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, and $\frac{1}{32}$, into decimals. Thus,

2) 1.0) .5 . . . 4 (1.00 (.25 . . . 4) 3.00 (.75.

Also 8) 1.000 (.125, and 16) 1.0000 (.0625.

2. Reduce $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{1}{12}$, into decimals.

5) 1.0 (.2 = $\frac{1}{5}$. 25) 2.00 (.08 = $\frac{2}{25}$. 125) 3.000 (.024 = $\frac{1}{41.67}$

Those decimals that are reduced from such a vulgar fraction, whose numerator with cyphers annexed is an aliquot part of, or can be measured by its denominator, are finite or terminate decimals; as the decimals resulting from the foregoing examples.

No fraction will produce a finite decimal, but such whose denominator is 2 or 5, and their multiples.

But such as are produced from a vulgar fraction, whose numerator with cyphers annexed is no aliquot part of, or cannot be measured by its denominator, will be indeterminate, or endless.

In circulating decimals, if one figure only repeats, it is called a single repetend; as for example,

3. Suppose the decimal of $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{5}{5}$, $\frac{6}{5}$, $\frac{7}{5}$, &c. was required.

$$9)1.0000$$

$$9)2.0000$$

$$9)3.0000$$

$$.1111, \&c. = \frac{1}{9}, .2222, \&c. = \frac{2}{9}, .3333 = \frac{3}{9} = \frac{1}{3}.$$

To avoid the trouble of writing down unnecessary figures, a single repetend is denoted by the repeating digit dashed; that is, the decimal $.1111, \&c. = .\dot{1} = \frac{1}{9}$, $.2222, \&c. = .\dot{2} = \frac{2}{9}$. Also $.3333, \&c. = .\dot{3} = \frac{3}{9} = \frac{1}{3}$, $.4 = \frac{4}{9}$, $.5 = \frac{5}{9}$, $.6 = \frac{6}{9} = \frac{2}{3}$, $.7 = \frac{7}{9}$, $.8 = \frac{8}{9}$, and $.9 = \frac{9}{9} = 1$.

4. Suppose it was required to reduce $\frac{1}{12}$, $\frac{1}{36}$, and $\frac{1}{54}$, into decimals.

$$12)1.000$$

$$36)5.000$$

$$960)31.0000000$$

$$.08\dot{3} = \frac{1}{12}$$

$$.13\dot{8} = \frac{1}{36}$$

$$.032291\dot{6} = \frac{1}{54}$$

The decimals resulting from these last examples are called mixed single repetends.

5. Let $\frac{2}{11}$, $\frac{1}{7}$, and $\frac{17}{86}$, be reduced into decimals.

$$11)2.000000$$

$$7)3.00000$$

$$286)17.0000000$$

$$.181818, \&c. = .\dot{1}8$$

$$.\dot{4}285\dot{7}$$

$$.059440\dot{8}.$$

Those decimals in which two or more figures circulate, are called compound repetends; and the manner of distinguishing them, is by dashing the first and last figure of the repetend, by which means we make one place of the repetend sufficient, as in the last example.

In a compound repetend, any one of the circulating figures may be made the first of the repetend; for instance, in the repetend $8.632\dot{8}325325, \&c.$ it may be made $8.63\dot{4}53$, or $8.632\dot{8}3\dot{4}$. And by this means any two or more repetends may be made to begin and end in the same place; and then they are said to be conterminous.

5. Let $\frac{13}{3731}$ be reduced to a decimal.

3731) 13.000 (.0034843205, &c.

18070

31460

16120

11960

7670

20800

2145

The decimal resulting from the last example, is called an approximate decimal, having some places true, and the rest uncertain; these approximating decimals are sometimes wrote with the signs + or —, to denote whether the last figure is greater or less than just: thus, .0034843205 +, or .0034843206 —; the first signifies that the decimal is greater than .0034843205, by some uncertain figures; and the second, viz. .0034843206 —, denotes that the true decimal exceeds .0034843205, and is less than .0034843206.

CASE II.

To reduce coins, weights, measures, &c. into decimals.

RULE I.

Reduce the different species into one, viz. the lowest denomination they consist of, for a dividend; then reduce the integer into the same denomination for a divisor; the result will be the decimal required.

RULE II.

Write the given denominations or parts orderly under each other, the inferior or least parts being uppermost; let these be the dividends.

Against each part on the left-hand, write the number thereof contained in one of its superior; let these be divisors.

Then beginning with the upper one, write the quotient of each division, as decimal parts on the right-hand of the dividend next below it; and let this mixed number be divided by its divisor, &c. till all be finished, and the last quotient will be the decimal sought.

RULE

RULE III.

The decimal may be readily found by the rule of practice, namely, by considering the next inferior denomination as aliquot parts of the integer; and those still lower as aliquot parts of the superior ones, or of each other; the sum of all those aliquot parts will be the decimal required.

Ex. I. Let $3\frac{1}{4}$ d. be reduced to a decimal, a pound sterling being the integer.

By RULE I.

$$\begin{array}{r}
 3\frac{1}{4} \text{ d.} \\
 \underline{4} \\
 960 \overline{) 15.0} (.015625 \text{ l.} = 3\frac{1}{4} \text{ d.} \\
 \underline{540} \\
 600 \\
 \underline{240} \\
 480
 \end{array}$$

By RULE II.

$$\begin{array}{r}
 4 \overline{) 3.00} \\
 12 \overline{) 3.75} \\
 20 \overline{) 0.3125} \\
 \hline
 0.15625
 \end{array}$$

The decimal as before =

 $3\frac{1}{4}$ d.

By RULE III.

$$\begin{array}{l}
 3 \text{ d.} = \frac{1}{10} \text{ of } 1 \text{ l.} = .0125 \\
 \frac{1}{4} \text{ d.} = \frac{1}{4} \text{ of } 3 \text{ d.} = .003125
 \end{array}$$

$$\text{Sum } \underline{\text{£}.015625}$$

The decimal sought.

2. What decimal of a pound is 5 s. $7\frac{1}{4}$ d.5 s. $7\frac{1}{4}$ d.

$$\begin{array}{r}
 12 \\
 \underline{67} \\
 4 \\
 96.0 \overline{) 27.1} (.28229, \&c. \\
 \underline{790} \\
 220 \\
 \underline{280} \\
 880 \\
 \hline
 (16)
 \end{array}$$

By RULE II.

$$\begin{array}{r}
 4 \overline{) 3.00} \\
 12 \overline{) 7.75} \\
 20 \overline{) 5.64583} \\
 \hline
 0.2822916 = 5 \text{ s. } 7\frac{1}{4} \text{ d.}
 \end{array}$$

By

By RULE III.

$$\begin{aligned} 5 \text{ s.} &= \frac{1}{2} \text{ of } 1 \text{ l.} = .25 \\ 6 \text{ d.} &= \frac{1}{16} \text{ of } 5 \text{ s.} = .025 \\ 1\frac{1}{2} &= \frac{1}{4} \text{ of } 6 \text{ d.} = .00625 \\ \frac{1}{4} &= \frac{1}{8} \text{ of } 1\frac{1}{2} \text{ d.} = .001041\bar{6} \end{aligned}$$

$$\text{Viz. } 5 \text{ s. } 7\frac{1}{4} \text{ d.} = \underline{\underline{\text{£} .282291\bar{6}}}.$$

3. What decimal of a pound is equal to 19s. 11d.?

19s. 11d. <u>12</u>	By RULE II.						
24.0) 23.9(.99583 = 19s. 11d. 230 140 200 80	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">12</td> <td style="border-left: 1px solid black; padding-left: 5px;">11.000</td> </tr> <tr> <td style="padding-right: 5px;">20</td> <td style="border-left: 1px solid black; padding-left: 5px;">19.916</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><u>0.99583 = 19s. 11d.</u></td> </tr> </table>	12	11.000	20	19.916		<u>0.99583 = 19s. 11d.</u>
12	11.000						
20	19.916						
	<u>0.99583 = 19s. 11d.</u>						

By RULE III.

$$\begin{aligned} 19 \text{ s.} - \text{d.} &= .95 \\ - 6 &= \frac{1}{2} \text{ of } 1 \text{ s.} = .025 \\ - 3 &= \frac{1}{4} \text{ of } 6 \text{ d.} = .0125 \\ - 2 &= \frac{1}{8} \text{ of } 6 \text{ d.} = .0083\bar{3} \end{aligned}$$

$$19 \text{ s. } 11 \text{ d.} = \underline{\underline{\text{£} .9958\bar{3}}}.$$

4. What decimal part of an hundred weight is 2 qrs. 12 lb. 12 oz.?

qr. lb. oz. 2 12 12 <u>28</u> 68 <u>16</u>	By RULE II.												
1792) 1100.0(.6138393 2480 6880 15040 7040 16640 512	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">16 viz. 4</td> <td style="border-left: 1px solid black; padding-left: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">4)</td> <td style="border-left: 1px solid black; padding-left: 5px;">12 . . 3.0</td> </tr> <tr> <td style="padding-right: 5px;">28 - 4</td> <td style="border-left: 1px solid black; padding-left: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">7)</td> <td style="border-left: 1px solid black; padding-left: 5px;">12.75 . . 3.1875</td> </tr> <tr> <td style="padding-right: 5px;">4</td> <td style="border-left: 1px solid black; padding-left: 5px;"><u>2.4553571 +</u></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><u>0.61383 -</u></td> </tr> </table> </td> </tr> </table> </td> </tr> </table>	16 viz. 4	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">4)</td> <td style="border-left: 1px solid black; padding-left: 5px;">12 . . 3.0</td> </tr> <tr> <td style="padding-right: 5px;">28 - 4</td> <td style="border-left: 1px solid black; padding-left: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">7)</td> <td style="border-left: 1px solid black; padding-left: 5px;">12.75 . . 3.1875</td> </tr> <tr> <td style="padding-right: 5px;">4</td> <td style="border-left: 1px solid black; padding-left: 5px;"><u>2.4553571 +</u></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><u>0.61383 -</u></td> </tr> </table> </td> </tr> </table>	4)	12 . . 3.0	28 - 4	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">7)</td> <td style="border-left: 1px solid black; padding-left: 5px;">12.75 . . 3.1875</td> </tr> <tr> <td style="padding-right: 5px;">4</td> <td style="border-left: 1px solid black; padding-left: 5px;"><u>2.4553571 +</u></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><u>0.61383 -</u></td> </tr> </table>	7)	12.75 . . 3.1875	4	<u>2.4553571 +</u>		<u>0.61383 -</u>
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4)	12 . . 3.0												
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7)	12.75 . . 3.1875												
4	<u>2.4553571 +</u>												
	<u>0.61383 -</u>												

By

By Rule III.

	qr. lb. oz.
$\frac{1}{2}$ = .52 - -
$\frac{1}{7}$ of 2 qr. = .0714286	= - 8 -
$\frac{1}{2}$ of 8 lb. = .0357143	= - 4 -
$\frac{1}{4}$ of 4 lb. = .00446428	= - - 8
$\frac{1}{8}$ of 8 oz. = .00223214	= - - 4
	<hr/>
	.6138393 = 2 12 12, as before.

5. What decimal part of a pound troy are 10 oz. 18 dwt. 16 grs.?

oz. dwt. gr.	By Rule II.
10 18 16	
24	24 { 4 16
<hr/>	6 40
218	20 18.6
24	12 10.93
<hr/>	0.97 lb. troy.
872	
436	
<hr/>	
576.0) 524.8 (.97 = 10 18 16	
640	
<hr/>	
64	
<hr/>	

6. What decimal part of a degree of a circle are 48° 37' 54"?

48° 37' 54"	By Rule II.
60	
<hr/>	60 54.0
2917	60 37.9
60	60 48.6316
<hr/>	0.810527 = 48° 37' 54"
216.000) 175.074 (.810527	
227	
1140	
600	
<hr/>	
1680	
<hr/>	

7. What

7. What decimal part of a foot = $10\frac{1}{4}$ inches?

$10\frac{1}{4}$ inches.

By RULE II.

$$\begin{array}{r} 4 \\ \hline 48 \overline{) 41.0} (.8541\phi \\ \underline{260} \\ 200 \\ 80 \\ 320 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \overline{) 1.00} \\ 12 \overline{) 10.25} \\ \hline 0.8541\phi = 10\frac{1}{4} \text{ inches.} \end{array}$$

8. What decimal part of a gallon of ale = 133 cubic inches?

$$\begin{array}{r} 282 \overline{) 133.0} (.4716312- \\ \underline{2020} \\ 460 \\ \underline{1780} \\ 5880 \\ \underline{340} \\ 580, \&c. \end{array}$$

9. What decimal part of a year = 217 days, 7 hours, 18 minutes?

$$\begin{array}{r} \text{d.} \quad \text{h.} \quad \text{m.} \\ 217 \quad 7 \quad 18 \\ \underline{24} \\ 868 \\ \underline{434} \\ 5215 \\ \underline{60} \\ 312918 \\ \underline{60} \end{array}$$

$$\begin{array}{r} 31556937 \overline{) 18775080.0} (.594958883367 \\ \dots\dots\dots 299661150 \\ 156487170 \\ 302504220 \\ 185817870 \\ \hline \&c. \end{array}$$

CASE III.

To reduce any decimal into the equivalent known parts of coin, weight, measure.

N

RULE.

R U L E.

Multiply the given number by the number of units contained in the next inferior denomination, cutting off as many figures from the product as the given decimal consists of; then multiply the remaining parts (if any) by the next lower denomination, cutting off as before; and thus proceed till you have converted your decimals, or come to the lowest part; and the several figures to the left-hand of the separating points, will be the several parts of the quantity required.

What known parts of coin are equal to .015625 l.?

$$\begin{array}{r}
 .015625 \\
 20 \\
 \hline
 .312500 \\
 12 \\
 \hline
 3.750000 \quad \text{Answer, } 3\frac{1}{4} \text{ d.} \\
 4 \\
 \hline
 3.000000
 \end{array}$$

What known parts of coin are equal .2822916 l.?

$$\begin{array}{r}
 .2822916 \\
 20 \\
 \hline
 s. \text{ ---} \\
 5.6458333 \\
 12 \\
 \hline
 d. \text{ ---} \\
 7.7500000 \\
 4 \\
 \hline
 3.0000000 \quad \text{Answer, } 5 \text{ s. } 7\frac{1}{4} \text{ d.}
 \end{array}$$

What known coin equals .99583 l.?

$$\begin{array}{r}
 .99583 \\
 20 \\
 \hline
 s. \text{ ---} \\
 19.91666 \\
 12 \\
 \hline
 d. \text{ ---} \\
 11.00000 \quad \text{Answer, } 19 \text{ s. } 11 \text{ d.}
 \end{array}$$

What

What known weight is .6138393 of a cwt.?

$$\begin{array}{r}
 .6138393 \\
 \text{qr. } \underline{\hspace{1cm}} \\
 2.4553572 \\
 \underline{\hspace{1cm}} \\
 36428576 \\
 9107144 \quad \text{Answer, 2 qr. 12 lb. 12 oz.} \\
 \hline
 12.7499916 \\
 \underline{\hspace{1cm}} \\
 11.9998656 = 12 \text{ oz. very near.}
 \end{array}$$

CASE IV.

To reduce a decimal into its least equivalent vulgar fraction.

1st. If the decimal be finite,

RULE,

Under the given decimal write an unit, with as many cyphers as the decimal consists of places; then divide both the numerator and denominator by the greatest common measure, which gives the least equivalent vulgar fraction required.

1. Required the least vulgar fractions equivalent to .5, .25, .75, .125, and .0625?

$$\begin{aligned}
 \text{Answer } .5 &= \frac{5}{10} = \frac{1}{2}, \quad .25 = \frac{25}{100} = \frac{1}{4}, \quad .75 = \frac{75}{100} = \frac{3}{4}, \\
 .125 &= \frac{125}{1000} = \frac{1}{8}, \quad \text{and } .0625 = \frac{625}{10000} = \frac{1}{16}.
 \end{aligned}$$

2. What is the least vulgar fraction equal to .625 and .5625?

$$\text{Answer, } .625 = \frac{625}{1000} = \frac{5}{8}, \quad \text{and } .5625 = \frac{5625}{10000} = \frac{9}{16}.$$

How to find the greatest common measure is taught before in vulgar fractions, so shall give only one example to refresh the learner's memory.

Let $\frac{5625}{10000}$ be reduced to its lowest or least equivalent fraction.

$$\begin{array}{r} 5625) 10000 \left(1 \\ \underline{4375} \\ 1250 \left(3 \\ \underline{625} \\ 1250 \left(2 \\ \underline{} \\ (0) \end{array}$$

$$5625) \frac{5625}{10000} \left(\frac{9}{16}, \text{ as before.} \right)$$

2d. If the given decimal be a repetend,

R U L E,

The decimal is the numerator of a vulgar fraction, whose denominator consists of as many nines as there are recurring places in the given decimal; both which divide by their greatest common measure (as before) and their quotient will be the least equivalent vulgar fraction.

1. Required the least vulgar fraction equivalent to $0.\dot{6}$?

$$\text{Answer, } 0.\dot{6} = \frac{6}{9} = \frac{2}{3}.$$

2. What is the least vulgar fraction equal to $0.\dot{7}6923\dot{6}$?

The greatest common measure to $\frac{769230}{999999}$ is found to be 76923 .

Therefore, $76923) \frac{769230}{999999} \left(\frac{10}{13}, \text{ the answer required.} \right)$

3. What is the least vulgar fraction equal to $16\dot{2}$?

The greatest common measure to $\frac{162}{999}$ is 27.

Therefore, $27) \frac{162}{999} \left(\frac{6}{37}, \text{ the answer required.} \right)$

3. When the given decimal is part final, and part a circulate,

R U L E,

To as many nines as there are figures in the repetend, annex as many cyphers as there are finite places for a denominator;

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minator; then multiply the nines in the said denominator by the finite part, and to the product add the repeating decimal for a numerator; these divided by their greatest common measure, will give the least equivalent fraction.

What is the least equal vulgar fraction to .53? $\overline{9 \times 5} + 3 = 48$ numerator; .90 being the denominator?

$$\therefore \frac{48}{90} = \frac{8}{15}, \text{ the least vulgar fraction required.}$$

What is the least vulgar fraction equal to .5923?

First, 9990 = denominator, and $\overline{999 \times 5} + 923 = 5920$, numerator.

$$\therefore .5923 = \frac{5920}{9990} = \frac{16}{27}, \text{ as was required.}$$

What is the least vulgar fraction equal to .008497133?

First 999999000 is the denominator.

Likewise $8 \times 999999 + 497133 = 8497125$, numerator.

And 102375 the greatest common measure.

$$\therefore .008497133 = \frac{8497125}{999999000} \left(\frac{83}{9763} \right), \text{ the answer.}$$

A general rule for reducing decimals into vulgar fractions.

Under the given decimal set an unit, with as many cyphers as there are places in the given decimal; then set the finite decimal as a numerator even under the lowest figures of the first numerator, with its proper denominator; lastly, subtract the under numerator from the upper one, and the under denominator from the upper one, the remainder will be a vulgar fraction equivalent to the given decimal, which reduce to its lowest terms.

\ 1. What is the vulgar fraction equivalent to .133?

$$\begin{array}{r} 138 \\ \hline 1000 \\ 13 \\ \hline 100 \\ 125 \\ \hline 900 \end{array} = .133 = \frac{125}{900}$$

2. Required the vulgar fraction equal to .008497133?

$$\text{From } \frac{.008497133}{1000000000}$$

$$\text{Take } - - - \frac{.008}{1000}$$

$$\text{Leaves } - \frac{8497125}{9999000} = .008497133 = \frac{83}{9768}, \text{ as was required.}$$



S E C T. III.

ADDITION of DECIMALS.

WHEN decimal fractions are to be added together, observe that the commas, or separating points in each expression, be placed directly underneath each other; for then primes, seconds, thirds, &c. will fall under those of the same name; and in mixed numbers, units will fall under units, tens under tens, &c.

C A S E I.

To add finite decimals.

R U L E.

Add as in whole numbers, and from the sum or difference, cut off so many places for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

Let .3746 + 137.5 + 1.347 + 375 + 1.85 + .0736285 + 87396.4 + 8.7386429 + 127 + 5.375, be added together.

$$\begin{array}{r}
 .3746 \\
 137.5 \dots \\
 \dots 1.347 \dots \\
 375 \dots \dots \\
 \dots 1.85 \dots \\
 \dots \dots 0736285 \\
 87396.4 \dots \dots \\
 \dots 8.7386429 \\
 127 \dots \dots \\
 5.275 \\
 \hline
 88053.5588714
 \end{array}$$

CASE II.

To add decimals wherein are single repetends.

RULE.

Make every line end at the same place, filling up the vacancies by the repeating digits, and annexing a cypher or cyphers to the finite terms; then add as before, only increase the sum of the right-hand row with as many units as it contains nines; and the figure in the sum, under that place, will be a repetend.

Let $3.\dot{6} + 78.347\dot{6} + 735.\dot{3} + .375 + .2\dot{7} + 187.\dot{4}$, be added together.

$$\begin{array}{r}
 3.\dot{6}66\dot{6} \\
 78.347\dot{6} \\
 735.\dot{3}333 \\
 \dots 3750 \\
 \dots 2777 \\
 187.\dot{4}44\dot{4} \\
 \hline
 1005.444\dot{4}
 \end{array}$$

CASE III.

To add decimals, having compound repetends.

RULE.

Make the repetends similar and conterminous; then add as in Case I. only increase the right-hand figure by as many units

units as are carried from the column of figures, whereĩn all the repetends begin together ; lastly, dash off for a repetend as many places as were so in the numbers added together.

Let $+ 162 + 134.08 + 2.88 + 97.28 + 3.769238 + 99.088 + 1.5 + .814$, be added together.

Made similar and
conterminous,

162.16216216

134.09090909

2.93839383

97.26866666

3.76823076

99.08833333

1.50000000

.81481481

501.62851071.



S E C T. IV.

SUBTRACTION of DECIMALS.

C A S E I.

To subtract finite decimals.

R U L E.

HAVING first set down the greater of the two numbers given (whether it be a whole number, mixed number, or decimal) set down the lesser under it, according to the directions given in addition ; then subtract as in whole numbers, imagining all the vacant places filled with cyphers.

From 375.5 take 86.47284. Also from 87.569245 take 19.87.

Minuend 375.5
Subtrahend 86.47284

Remainder 289.02716

87.569245
19.87

67.699245

From

From 1 take .732594. And from 684 take 9.3275.

From 1.0.....	684.....
Subtr. .732594	9.3275
<hr/>	<hr/>
Rem. .267406	674.6725
<hr/>	<hr/>

Let 375.5 be diminished or made less by .97637387, and shew their difference.

Minuend	375.5.....
Subtrahend	.97637387
<hr/>	<hr/>

Difference 374.52362613, or remainder.

CASE II.

To subtract decimals that have repetends.

RULE.

Make the repetends similar and conterminous, and subtract as in the last case; observing only, if the repetend of the number to be subtracted, be greater than the repetend of the number it is to be taken from, then the right-hand figure of the remainder must be less by unity, than it would be, if the expressions were finite; and the repetend in the remainder will consist of as many places as there are in the other two numbers.

Let 57.73 be lessened by 18.95416, and 51.523 by .66.

From 57.73333	51.523
Take 18.95416	.66
<hr/>	<hr/>
Rem. <u>38.77916</u>	50.863
	<hr/>

Let 47.478173 be made less by 15.56563, and 49.528578 by 38.473600.

From 47.478173	49.528578
Take 15.56563	38.473600
<hr/>	<hr/>
Rem. <u>31.8521613</u>	11.0549785
	<hr/>

From

From 43.8540268 take 20.9252 and from 49.53 take 42.753.

$\begin{array}{r} \text{From } 43.8540268 \\ \text{Subt. } 20.925258 \\ \hline \text{Rem. } 22.9287682 \end{array}$	$\begin{array}{r} 49.5333 \\ 42.7891 \\ \hline 6.7442 \end{array}$
---	--



S E C T. V.

MULTIPLICATION of DECIMALS.

C A S E I.

WHEN both factors are finite decimals, whether they be pure, or joined with integers,

R U L E,

Multiply them as if they were all whole numbers, and from the product (towards the right-hand) cut off so many places for decimal parts in the product, as there were in both the multiplier and multiplicand counted together. But if it so happen that there are not so many places in the product, supply the defect by prefixing cyphers.

$\begin{array}{r} 8.7537 \\ .3275 \\ \hline 437685 \\ 612759 \\ 175074 \\ 262611 \\ \hline 2.86683675 \end{array}$	$\begin{array}{r} 864 \\ .78 \\ \hline 6912 \\ 6048 \\ \hline 673.92 \end{array}$	$\begin{array}{r} 27.576 \\ 6.23 \\ \hline 82728 \\ 55152 \\ 165456 \\ \hline 171.79848 \end{array}$
$\begin{array}{r} .57386 \\ .8237 \\ \hline 401702 \\ 172158 \\ 114772 \\ 459088 \\ \hline .472688482 \end{array}$		$\begin{array}{r} .27345 \\ .273 \\ \hline 82035 \\ 191415 \\ 54690 \\ \hline .07465185 \end{array}$

C A S E

CASE II.

Two decimal fractions being given, to reserve in their product any number of places,

RULE,

Set the unit's place of the multiplier, directly under that figure of the decimal part of the multiplicand, whose place you would reserve in the product, and invert the order of all its other places; that is, write the decimals on the left-hand, and the integers, if any, on the right.

Then in multiplying, always begin at that figure of the multiplicand which stands over the figure wherewith you are then multiplying, setting down the first figure of each particular product directly underneath one another, due regard being had to the increase which would arise out of the two next figures, to the right-hand of that figure in the multiplicand, which you then begin with; carrying one from 5 to 15; two from 15 to 25; three from 25 to 35, &c. and the sum of these lines will give the product.

Let 73.8429753 be multiplied into 4.628754, reserving only five places of decimal parts in the product.

73.8429753, the multiplicand as usual.

457826.4, the multiplier inverted, with the unit's place set under the 5th place in decimals, denoting that there will be five places of parts in the product.

$$\begin{array}{r}
 29537190 \\
 4430579 \\
 147686 \\
 59074 \\
 5169 \\
 369 \\
 30 \\
 \hline
 341.80097
 \end{array}$$

The work at large.

$$\begin{array}{r}
 73.8429753 \\
 4.628754 \\
 \hline
 2953719012 \\
 3692148765 \\
 5169008271 \\
 5907438024 \\
 1476859506 \\
 4430578518 \\
 2953719012 \\
 \hline
 341.8009672917762
 \end{array}$$

Let

Let 843.7527 be multiplied into 8634.875, reserving only the integers in the product.

$$\begin{array}{r}
 843.7527 \\
 578.4368 \\
 \hline
 6750022 \\
 506251 \\
 25313 \\
 3375 \\
 675 \\
 59 \\
 4 \\
 \hline
 7285699
 \end{array}$$

$$\begin{array}{r}
 843.7527 \\
 8634.875 \\
 \hline
 42187635 \\
 59962689 \\
 67500216 \\
 33750108 \\
 25312581 \\
 50625162 \\
 67500216. \\
 \hline
 72856699 \quad 0954125
 \end{array}$$

C A S E III.

If the right-hand figure of the multiplicand be a circulate,

R U L E,

In multiplying, increase the right-hand figure of each resulting line by as many units as there are nines in the product of the first figure in that line, and the right-hand figure of each line will be a circulate; and before you add them together, make them all end at the same place.

$$\begin{array}{r}
 .172\phi \\
 6 \\
 \hline
 1.036\phi
 \end{array}$$

$$\begin{array}{r}
 835.273 \\
 .7484 \\
 \hline
 3341093 \\
 668218\phi \\
 334109333 \\
 5846912333 \\
 \hline
 625.118562\phi
 \end{array}$$

C A S E IV.

If the right-hand figure of the multiplier be a circulate,

R U L E,

Multiply by it as by a finite digit, setting the product one place extraordinary towards the left-hand; then divide
2 that

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that product by 9, continuing the quotient (if needful) till it arrives at a circulate; then beginning at the place under the right-hand figure of the multiplicand, cut off for decimal parts,

63.274	.47375
.1138	.878
<hr/>	<hr/>
9)379644	9)284250
<hr/>	<hr/>
4218228	315838
189822	331625
63274	379000
63274	<hr/>
<hr/>	.41532088
7.19214428	

CASE V.

When the multiplicand and multiplier are each a single circulate,

RULE,

The first line (or that produced by multiplying by the circulate in the multiplier) must be managed as in Case III. only the right-hand figure must be increased by as many units as there are nines in the product of the first figure of that line, the products of the rest must be managed as directed in Case II.

3.728	8574.8
.28	87.8
<hr/>	<hr/>
9) 22348	9) 428718
<hr/>	<hr/>
24822	4763588
74466	600208333
<hr/>	6859486666
.99288	<hr/>
<hr/>	750730.588

CASE VI.

If the multiplier be a compound repetend, and the multiplier a finite number,

RULE,

R U L E,

In multiplying, observe to add to the right-hand place of the product, so many units as there are tens in the product of the left-hand place of the repetend; and the product shall contain a repetend, whose places are equal to those in the multiplicand; and if there are more places of figures in the multiplier than one, make all the several products conterminous towards the right-hand, as in Case II. and IV.

$$\begin{array}{r} 3.878 \\ 8 \end{array}$$

$$\begin{array}{r} 31.897 \\ \hline \end{array}$$

$$\begin{array}{r} 732.888 \\ 43.7 \\ \hline \end{array}$$

$$\begin{array}{r} 5128108 \\ 21971587 \\ 293184834 \\ \hline 32024.0838 \end{array}$$

$$\begin{array}{r} 788.4928 \\ .005 \end{array}$$

$$\begin{array}{r} 3.9914648 \\ \hline \end{array}$$

$$\begin{array}{r} 86.82078 \\ 37.135 \\ \hline \end{array}$$

$$\begin{array}{r} 43180388 \\ 258862189 \\ 868207832 \\ 60414518245 \\ 25862189621 \\ \hline 3205.52042768 \end{array}$$

C A S E VII.

If the multiplier be a compound repetend,

R U L E,

Multiply each figure of the repetend, and add the several products together; then add the result in this manner; set the left-hand figure so many places forwards as exceeds the number of places in the repetend by one, and the rest of the figures in order after it; and thus proceed, till the result last added be carried beyond the first; lastly, add the several results together, beginning under the right-hand place of the first; and from thence dash as many figures for a repetend, as the repetend of the multiplier consists of.

$ \begin{array}{r} 834.75 \\ \underline{2.27} \\ 584325 \\ 166950 \\ \hline 250425 \\ \hline 27296325 \\ 272963 \\ \hline 272 \\ \hline 2732.364\frac{1}{2} \end{array} $	$ \begin{array}{r} 49640.54 \\ \underline{.70508} \\ 14899162 \\ 24820270 \\ \hline 34748378 \\ \hline 349980699162 \\ 34998069 \\ \hline 349 \\ \hline 34998.4169008 \end{array} $
---	---

If the multiplier hath any terminate places joined with the repetend, and if the repetend be small, and these many, multiply and add the products of the repetend first; then multiply by the terminate figures, and add their products to the sum of the products of the repetend; and to this last result, add the said sum of the repetend products.

$$\begin{array}{r}
 874.37 \\
 \underline{1.274} \\
 349748 \\
 612059 \\
 \hline
 \text{Sum } 6470338 \text{ of the products of the repetend.} \\
 174874 \\
 87437 \\
 \hline
 1113.94738 \\
 647033 \\
 6470 \\
 64 \\
 \hline
 1114.60084
 \end{array}$$

But if the terminate figures are few, and the places of the repetend many, subtract the terminate figures from those of the repetend, and multiply by the remainder as a repetend.

$$\begin{array}{r} 1735.8072 \\ 32.4738 \\ \hline 32 \end{array}$$

Remains 324704, the new multiplier.

$$\begin{array}{r} 69432288 \\ 121506504 \\ 69432288 \\ 34716114 \\ 52074216 \\ \hline 5636235410688 \\ 5636235410 \\ 563623 \\ 56 \end{array}$$

$$\hline 56367.9902598$$

CASE VIII.

If both factors have compound repetends,

RULE,

Proceed as in the two last cases; for as the places of the repetend in the product will be uncertain as to their number, they can only be determined (in any manner fit for practice) by continuing and repeating the first product, which will contain a certain repetend equal in places to that of the multiplicand.

$$\begin{array}{r} \text{Multiply } 67.824 \\ \text{into } 5.288 \\ \hline 52 \\ \hline 5.223 \\ 203474 \\ 1358486 \\ 13584864 \\ 339124124 \\ \hline 354.249868 \\ 35424906 \\ 354249 \\ 3542 \\ 35 \end{array}$$

357.827833, &c. Here the fourth place of parts comes out a single repetend, viz. 8.

Again.

Again. Multiply 3.148
into 4.797

$$\begin{array}{r}
 4 \\
 \hline
 4293 \\
 \hline
 9438 \\
 283890 \\
 628090 \\
 12581818 \\
 \hline
 13.503438363636 \\
 1350343636 \\
 135034363 \\
 135034 \\
 135 \\
 \hline
 13.5169533169533
 \end{array}$$

Examples of this kind, though very accurate, yet are more curious than useful; as they may be easier done exact enough for business, by the contracted way of multiplication taught Case II.



SECT. VI.

DIVISION of DECIMALS.

IN any of the following cases in division, if the dividend be greater than the divisor, the quotient will be either a whole or a mixt number; but when the dividend is less than the divisor, the quotient must necessarily be a fraction; for a lesser number is contained in a greater once at the least, but the greater is not contained once in the lesser.

CASE I.

When the divisor and dividend are both finite decimals,

RULE,

Divide as in whole numbers, and from the right-hand of the quotient point off for decimals so many places as the decimal places in the dividend exceed those in the divisor; and those to the left, if any, are integers; but if the places of the quotient are not so many as this rule requires, supply

O

ply the defect by prefixing cyphers to the quotient; but if the decimal places in the divisor be more than those in the dividend, annex cyphers to the dividend to make them equal, and the quotient will be integers until all those cyphers are used.

$$87.364) 714.025972 (8.173$$

$$15.1139$$

$$\begin{array}{r} 637757 \\ 262092 \\ \hline \end{array}$$

$$.7875) 441.0000 (56$$

$$47250$$

.....

.....

$$179) .48624097 (.00271643$$

$$1282 \dots$$

$$294$$

$$1150$$

$$769$$

$$537$$

...

$$.2628) 27.0000 (100.55863$$

$$1500$$

$$15750$$

$$23250$$

$$17700$$

$$15900$$

$$24750$$

$$585$$

CASE II.

To contract the work of division, when the divisor consists of many decimal places.

RULE.

Having determined the value of the quotient figures, let each remainder be a new dividend; and for every such dividend, point off one figure from the right-hand of the divisor; observing at each multiplication to have regard to the increase of the figures so cut off, as in contracted multiplication.

$$.67268479) 56.00000000 (83.2485$$

$$\dots\dots\dots$$

$$218521680$$

$$16716243$$

$$3262547$$

$$571808$$

$$33661$$

$$\dots 27$$

$$384.672158)$$

$$\begin{array}{r}
 384.672158) 14169.206603851 (36.8345 \\
 \underline{2629041863} \\
 3210089158 \\
 \underline{132711894} \\
 17310247 \\
 \underline{1923361} \\
 \dots\dots
 \end{array}$$

$$\begin{array}{r}
 9.365407) 87.076326 (9.2976554 \\
 \underline{2787663} \\
 914582 \\
 \underline{71696} \\
 6138 \\
 \underline{519} \\
 51 \\
 \underline{4}
 \end{array}$$

If any whole, mixed, or decimal number is given to be divided by 10, 100, 1000, &c. only remove the separating point towards the left-hand so many places as there are cyphers in the divisor; also in multiplication the separating point is moved to the right-hand so many places as there are cyphers in the multiplier.

EXAMPLES IN

Multiplication.

Division.

$.7865 \times 10 = 7.865$	$10) 7865 (786.5$
$.7865 \times 100 = 78.65$	$100) 7865 (78.65$
$.7865 \times 1000 = 786.5$	$1000) 7865 (7.865$
$.7865 \times 10000 = 7865$	$10000) 7865 (.7865$
$.7865 \times 100000 = 78650$	$100000) 7865 (.07865$

CASE III.

If the dividend be a repetend.

RULE.

If it be a single repetend, bring down the circulating figure until the quotient either repeats, or is as exact as required; but if the repetend in the dividend be a compound one, then bring down the circulating figures in the same order they stand in; and when you have got through them all, bring down the first figure in the repetend over again &

and so proceed until your quotient either repeats, or be as exact as is necessary.

$$\begin{array}{r}
 .7484) 625118.5628 (835273.3 \\
 \underline{26398} \\
 39465 \\
 \underline{20456} \\
 54882 \\
 \underline{24946} \\
 2494 \\
 \end{array}
 \qquad
 \begin{array}{r}
 137) .586 (.004136253 \\
 \underline{186} \\
 496 \\
 \underline{856} \\
 346 \\
 \underline{726} \\
 416 \\
 \underline{} \\
 58
 \end{array}$$

$$\begin{array}{r}
 41.764) 1761.38407 (42.17, \&c. \text{ ad infinitum.} \\
 \underline{90804} \\
 72760 \\
 \underline{309961} \\
 17613
 \end{array}$$

$$\begin{array}{r}
 764.5) 319.28007117 (.4176222 \\
 \underline{13480} \\
 58350 \\
 \underline{48357} \\
 24871 \dots \\
 \underline{19361} \\
 40712 \\
 \underline{} \\
 2487
 \end{array}
 \left. \begin{array}{l} \dots \\ \dots \\ \dots \end{array} \right\} \text{ ad infinitum.}$$

CASE IV.

If the divisor be a single repetend.

RULE.

If the divisor be only a single repetend, place the dividend under itself, but one place forward towards the right-hand, which subtract from the dividend; the remainder will be a new dividend, which divide by the divisor, in the same manner as if it was a terminate number. But if the divisor consists of terminate numbers, joined to the repetend, subtract those terminate numbers from the divisor, and subtract the dividend as before directed, and the remainder will be a new divisor and dividend.

Divide

Divide 134.28 by .8.

1342

.6) 120.84 (201.4, the true quotient.

Divide 234.8 by .7.

234

.7) 211.2 (301.714285, &c. *ad infinitum*.

Divide 6.251185628 by 875.272.

875.272) 6.251185628

87527 625118562

787.746) 5.626067064 (.00714198

1118450

3307046

1560624

7728780

6390660

88692

Divide 856.988 by 4.84.

4.86) 856.988

48 856988

4.38) 771.2892 (176.0934247

3332

2668

4092

1500

1860

1080

2040

3080

14

CASE V.

If a compound repetend is found in your divisor only, or in both your divisor and dividend,

RULE,

Set the divisor and dividend under themselves, each so many places towards the right-hand, as there are places in the repetend of the divisor; which subtract as in the last case, and the remainder will be a new divisor and dividend. But if the divisor is a compound repetend without any terminate figures, divide by it as a terminate number; first subtracting the dividend from itself, as before directed.

Divide 13.5769538 by 4.797.

$$\begin{array}{r}
 4.797 \overline{) 13.5769538169} \\
 \underline{4 } \\
 135169533 \\
 \hline
 4.293 \overline{) 13.5934363636} \quad (3.14159265358979323846264338327951) \text{ &c. ad infinitum.} \\
 \underline{6244} \\
 19515 \\
 \underline{23416} \\
 1951
 \end{array}$$

Divide 5264.457457938 by 42.7437.

$$\begin{array}{r}
 42.7437 \overline{) 5264.457457938} \\
 \underline{42 } \\
 526445745 \\
 \hline
 42.3394 \overline{) 5263.931007194} \quad (124.32701, \text{ true quotient.}) \\
 \underline{1029991} \\
 1832030 \\
 \underline{1384540} \\
 1143587 \\
 \underline{2967991} \\
 423394 \\
 \hline
 \dots\dots
 \end{array}$$

Divide 395.273714 by .717.

$$\begin{array}{r}
 395.273714 \text{ by } .717 \\
 \underline{395273} \\
 717 \overline{) 394.878341} \quad (1245.673) \\
 \underline{778} \dots\dots \\
 1447 \\
 \underline{1798} \\
 2133 \\
 \underline{2314} \\
 951 \\
 \hline
 \dots
 \end{array}$$

The

The following lemmas and corollaries may be of use in illustrating the different methods and peculiar processes used in the arithmetic of circulating numbers.

L E M M A I.

A series of nines infinitely continued, is equal to unity or one, in the next left-hand place. Thus, $0.9999, \&c. = 1$, and $.0999, \&c. = .1$; also, $.00999, \&c. = .01$; and $.73.999, \&c. = 74$.

DEMONSTRATION. It is evident that $.9 = \frac{9}{10}$ wants only $\frac{1}{10}$ of unity, $.99$ wants only $\frac{1}{100}$, and $.999$ wants only $\frac{1}{1000}$; so that if the series were continued to infinity, the difference between that series of nines and an unit, would be equal to unity divided by infinity; that is, nothing at all. Q. E. D.

L E M M A II.

Any single repetend divided by 10, and the quotient subtracted from the said repetend, the remainder will be the same number complete or terminate.

DEMONSTRATION. Let the given repetend be $3.333, \&c.$
 $3.333 \div 10 = .333$, and $3.333 - .333 = 3$. Also,
 $54.444, \&c. \div 10 = 5.444, \&c.$, and $54.444 - 5.444 = 49$.

C O R O L L A R Y I.

Hence it follows, that if a compound repetend be divided by an unit, with so many cyphers annexed as are equal to the places of the repetend, and the quotient subtracted from the said repetend, the remainder will be the same number complete or terminate, that constituted the repetend. Thus, $325.325 \div 1000 = 325$; and $325.325 - 325 = 325$; and $42.3436 \div 10000$, will be $.00423436$; and $42.3436 - .00423436$, will be 42.3394 .

C O R O L L A R Y II.

Hence also, if any repetend be divided by an unit with as many cyphers as it contains places, and the quotient multiplied by as many nines as the repetend contains places, the result will be the same as before; that is, the same number terminate or complete; for any number divided by 10, and the quotient subtracted, the remainder is the same as the quotient multiplied by nine.

1. Thus, $6.666 \div 10 = .6666$; and $.6666, \&c. \times 9 = 5.999, \&c. = 6$.

2. Again, $325 \times 999 = 324.999, \&c. = 325 = 325.325 - 325$.

3. And $325.325 \div 1000 = .325$.

COROLLARY III.

It is evident from the last corollary, that a single repetend is to the same number terminate or complete, as 10 is to 9; a compound repetend of two places, as 100 to 99; and a compound repetend of three places is to the same number terminate or complete, as 1000 to 999, &c. And by the converse of the said corollary it must follow, that any number multiplied by 1, with as many cyphers as it contains figures, and the product divided by as many nines, will give the same number perpetually circulating.

Thus, $6 \times 10 = 60$, and $60 \div 9 = 6.666, \&c.$

And $325 \times 1000 = 325000$, and $325000 \div 999 = 325$.

COROLLARY IV.

Hence also, if any number be divided by as many nines as it contains figures, and the quotient added to the said number, the result will be the same as before; for any number multiplied by 10, and the product divided by 9, the quotient must be equal to $\frac{1}{9}$ of the same number added to itself.

Thus, the quotient of $6 \div 9 + 6 = 6.666, \&c.$

And the quotient of $325 \div 999$ added to $325 = 325$.

LEMMA III.

Any number divided by 9, 99, 999, &c. will be equal to the sum of the quotients of the same number continually divided by 10, 100, 1000, &c.

Thus, 717 divided by 10, 100, &c.

$$\begin{array}{r}
 71.7 \\
 7.17 \quad 9)717(79\bar{6} \\
 717 \\
 717 \\
 717 \\
 717 \\
 \hline
 79\ 666, \&c.
 \end{array}$$

Also,

Also,

$$236847 \div 1000, \&c.$$

$$\begin{array}{r} 236.847 \\ .236847 \\ 236847 \\ 236 \\ \hline 237.84084 \end{array}$$

$$999) 236847 (237.884$$

$$\begin{array}{r} 3704 \\ 7077 \\ 8400 \\ 4080 \\ \hline 84 \end{array}$$

From these lemmas and corollaries appears the reason of multiplication and division of single as well as compound repetends.

I shall here add the following useful proposition, viz.

To perform the work of multiplication by division, or of division by multiplication.

R U L E.

Divide an unit, with cyphers annexed, by the given multiplier or divisor, the quotient will be the divisor or multiplicator sought.

Let 27576×625 . I would also have a divisor which will give a quotient equal to the product of those numbers.

$$\begin{array}{r} 27576 \\ 625 \\ \hline 138880 \\ 55152 \\ 165456 \\ \hline 17235000 \end{array} \quad \begin{array}{r} 625) 1.000 (.0016 \\ 3750 \\ \hline \dots \end{array} \quad \begin{array}{r} 27576.0000 (17235000 \\ 115 \\ 37 \\ 56 \\ 80 \\ \hline \dots \end{array}$$

Let 67392 be divided by 78, and find a multiplier, which being multiplied into the same number, shall produce a number equal to the quotient.

78) 67392 (864
499
312

$$\begin{array}{r} 78 \overline{) 1.00} \{ .0128205 \\ \underline{220} \\ 640 \\ \underline{160} \\ 400 \\ \underline{} \\ 10 \end{array}$$

$$\begin{array}{r} 67392 \\ \times .0728205 \\ \hline \end{array}$$

336960
134784

539136
134784
67392

863.9991360
8639991360
8639

$$863.9999999999999 = 864.$$

QUESTIONS to exercise DECIMALS.

1. A grocer bought two chefts of sugar, the one weighing net 18 cwt. 3 qrs. 14 lb. at 2 l. 9 s. 8 d. per cwt.; the other weighed net 18 cwt. 1 qr. 21 lb. at 4½ d. per pound, which he mingled together; now I desire to know how much a hundred weight of this mixture is worth?

1.	s.	d.	
2	9	8	per cwt.
		6	

14 18 -
3

$$\begin{array}{r} 44 \ 14 \ - \\ I \ 4 \ 10 \\ - \ 12 \ 5 \\ - \ 6 \ 2\frac{1}{2} \end{array}$$

$$\begin{array}{r} 46 \\ 38 \end{array} \begin{array}{r} 17 \\ 14 \end{array} \begin{array}{r} 5\frac{1}{2} \\ 4\frac{1}{2} \end{array}$$

£ 85 11 10 whole cost.

12	10
20	11.88
<hr/>	
£ 85.5918	

$$\begin{array}{rcl} 1 \text{ qr.} & = & .25 \\ 7 \text{ lb.} & = & .0625 \end{array}$$

$$\begin{array}{r} 37.3125 \quad 12 \quad 12 \quad - \\ \hline 3 \end{array}$$

$$\begin{array}{r} 4\frac{1}{2} \text{ d.} \\ \times 4 \\ \hline 16 \\ \times 7 \\ \hline 106 \\ \times 4 \end{array}$$

2 2 - 2 cwt.
6

$$\begin{array}{r} 37 \text{ } 16 \text{ } - \\ 10 \text{ } 6 \\ \hline 5 \text{ } 3 \\ 2 \text{ } 7\frac{1}{2} \end{array}$$

38 14 4¹/₂
37 cwt.

cwt. qr. lb.
18 3 14
18 1 21

$$\begin{array}{r} 37 \text{ } 1 \text{ } 7 = 37.3125) 85.5918 (2.293914 \\ \underline{1096666} 20 \\ 3504166 5.878280 \\ \underline{146041} 12 \\ 34102 10.539360 \\ \underline{522} 4 \\ 149 2.157440 \end{array}$$

Answer, 2l. 5s. 10 $\frac{1}{2}$ d.

2. What quantity of water will you add to a pipe of mountain wine, value 33l. to reduce the first cost to 4s. 6d. the gallon?

$$\begin{array}{r} 12 \overline{) 6} \\ 20 \overline{) 4.5} \end{array}$$

$$\begin{array}{r} 225) 33.000 (146.8 \\ \underline{1050} 126 \text{ gallons in a pipe.} \\ 1500 \end{array}$$

(150) 20.8 gal. the answer.

3. If a cubic inch of oil olive be .52835 decimal parts of an ounce averdupoise, what quantity of oil, weighing 7 $\frac{1}{2}$ pounds per gallon, will be contained in a cask, allowed to hold 13 $\frac{1}{2}$ gallons of water, each 282 solid inches?

$$13.5 \times 282 = 3760 \text{ cubic inches, content of the cask.}$$

$$3760 \times .52835 = 1986.596 \text{ oz. weight of the oil.}$$

$$16) 1986.596 (124.16225 \text{ lb. averdupoise.}$$

$$\begin{array}{r} 38 \\ 66 \\ 25 \\ 99 \\ 36 \\ 40 \\ 80 \end{array} \quad \begin{array}{r} 7.5) 124.16225 (16.555 \text{ gallons,} \\ \underline{491} \text{the answer.} \\ 416 \\ \underline{412} \\ 372 \end{array}$$

4. A person was possessed of $\frac{3}{4}$ share of a copper-mine, and sold $\frac{1}{4}$ of his interest therein for 1710l.; what was the reputed value of the whole property at the same rate?

$$\frac{1}{4} \text{ of } \frac{3}{4} = \frac{3}{16} = .1875$$

$$.45) 1710.00 (3800 \text{ l. the answer required.}$$

360

5. If

5. If I buy 14 yards of cloth for 10 guineas, how many ells Flemish can I buy for, 283 l. 17 s. 6 d. at the same rate?

	l.	s.
7	10	10
2	1	10
4	15	-d. per yard.
	3	9

11 3 = .5625 per ell Flemish.

By inspection, 283 l. 17 s. 6 d. = 283.875 l.

.5625) 283.875 (504.8 = 504 ells, 2 qrs. the answer.

$$\begin{array}{r} 26250 \\ 37500 \\ 37500 \\ \hline \end{array}$$

6. Goliath of Gath is said to have been six cubits and a half, or a span, high; this answers to 10 feet, 4.592 inches; pray what was the length of the cubit in British measure?

12) 4.592 (.3828
10.3828 feet, the height of Goliath.

6.5) 10.3828 (1.5973241 = 1 foot, 7.168 inches, the answer.

7. A factor bought 84 pieces of stuff, which cost 537 l. 12 s. at 5 s. 4 d. per yard; I demand the number of yards in all, and how many yards in each piece?

$$\begin{array}{r} 12 \overline{) 4} \\ 20 \overline{) 53} \end{array}$$

£ .28 = 5 s. 4 d.

$$\begin{array}{r} 20 \overline{) 12} \end{array}$$

537.6 = 537 l. 12 s.

.28) 537.6 (2016 yards in all, and $7 \times 12 = 84$.

$$\begin{array}{r} 2 \overline{) 5376} \end{array}$$

$$\begin{array}{r} 24 \overline{) 48384} \end{array}$$

$$\begin{array}{r} 144 \overline{) 168} \end{array}$$

$$\begin{array}{r} 12 \overline{) 2016} \\ 7 \overline{) 168} \end{array}$$

24 yards in each piece.

CHAPTER VI. E V O L U T I O N.

OR,

Extracting the ROOTS out of all SINGLE POWERS.

E VOLUTION is, the unravelling or unfolding any proposed number into the parts of which it was made up of or composed.

If any number is multiplied into itself, that product is called a square number.

Thus the square numbers 4, 9, 25, 36, &c. are each of them composed of two equal numbers, viz. $2 \times 2 = 4$, $3 \times 3 = 9$, $4 \times 4 = 16$.

If any number be multiplied into itself, and that product be multiplied into the same number, the second product is called a cube number,

Thus the cube numbers 8, 27, 64, &c. are each composed of $2 \times 2 \times 2 = 8$, $3 \times 3 \times 3 = 27$, $4 \times 4 \times 4 = 64$, &c.

These powers exist in nature, viz. a root is represented by a line or side, having but one dimension, viz. only length; the square is a plain figure of two dimensions, viz. length and breadth; and the cube of three, viz. length, breadth, and thickness.

All the superior powers have no existence in nature, but are composed of a multiplication of any number four or more times into itself.

Thus, $2 \times 2 \times 2 \times 2 = 16$, the biquadrate, whose root is 2.

Or $3 \times 3 \times 3 \times 3 \times 3 = 243$, the sursolid, whose root is 3, have no existence in nature, but may be understood as a series of numbers in geometrical progression.

When any number is proposed to have the root extracted, the first work is to prepare it by points set over (or under) their proper figures, according as the given power, whose root is sought, doth require; which for the square is 2, for the cube is 3, for the biquadrate 4, &c. always beginning those points over the place of unity towards the left-hand; if the given numbers be integers, and descend towards the right-hand in decimal parts.

Thus for the square root 5837429643847

Cube 5837429643847

Biquadrate 5837429643847

Surfoldid 5837429643847

Or in decimals,

Thus for the square root 0.532794384728

Cube 0.532794384728

Biquadrate 0.532794384728

Surfoldid 0.532794384728

A TABLE of POWERS.

	Square or second power.	Cube or third power.	Biquadrate, or square squared, or 4th power.	Surfoldid, or the fifth power.	Cube squared, or the square cubed, or the sixth power.	Second surfoldid, or seventh power.	Biquadrate squared, or the eighth power.	Cube cubed, or the ninth power.
	Index.	Index.	Index.	Index.	Index.	Index.	Index.	Index.
	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

SECT. I.

To Extract the SQUARE ROOT.

RULE.

HAVING pointed the given resolvend as before directed, find the greatest square that is contained in the first period towards the left-hand, setting down the root as a quotient, and subtract that square out of the first period.

2. To the remainder bring down the two figures under the next point for a dividend. *N. B.* This is always to be repeated.

3. Double the quotient for a divisor, enquiring how often it may be had in that dividend (excepting the last figures) and set down the quotient figure, which annex to the divisor. This must also be repeated, as a new divisor must be found to every figure.

4. Then multiply the whole increased divisor, and subtract the product from the dividend. Proceed thus till all the periods are brought down.

5. Instead of doubling the quotient every time for a divisor, always add the last quotient figure to the divisor for a new divisor.

6. If there be a remainder after you have finished your periods, bring down a pair of cyphers for decimals; proceeding as before directed, till the root is as exact as is required.

N. B. You must always, in mixed numbers, cut off as many whole numbers in the root, as there are periods of whole numbers, and as many decimals as there are periods of decimals.

1. Extract the square root of 393129.

$$\begin{array}{r}
 393129 \quad (627 \\
 \underline{36} \\
 122 \quad) \quad 331 \\
 \underline{244} \\
 1247 \quad) \quad 8729 \\
 \underline{8729} \\
 \dots
 \end{array}$$

2. Extract

2. Extract the square root of 75873419337865039195105089.

$$\begin{array}{r}
 75873419337865039195105089 \text{ (8710534962783)} \\
 64 \\
 167 \overline{) 1187} \\
 \underline{1169} \\
 1741 \overline{) 1834} \\
 \underline{1741} \\
 174205 \overline{) 931933} \\
 \underline{871925} \\
 1742103 \overline{) 6090878} \\
 \underline{5226309} \\
 17421064 \overline{) 86456965} \\
 \underline{69684256} \\
 174210689 \overline{) 1677270903} \\
 \underline{1567896201} \\
 174210698 \overline{) 109374702} \\
 \dots\dots \underline{4848283} \\
 \underline{1364070} \\
 \underline{144895} \\
 \underline{5227}
 \end{array}$$

The five last figures in the example above, are found by the contracted method of division.

3. What is the square root of 1850701.764025?

$$\begin{array}{r}
 1850701.764025 \text{ (1360.405, the root required.)} \\
 1 \\
 23 \overline{) 85} \\
 \underline{69} \\
 266 \overline{) 1607} \\
 \underline{1596} \\
 87204 \overline{) 110176} \\
 \underline{108816} \\
 2720805 \overline{) 13604025} \\
 \underline{13604025}
 \end{array}$$

4. Extract

4. Extract the square root of .001234.

.001234 (.0351283362, the root very near.

$$\begin{array}{r}
 9 \\
 \hline
 65 \overline{) 334} \\
 \underline{325} \\
 701 \overline{) 900} \\
 \underline{701} \\
 7022 \overline{) 19900} \\
 \underline{14044} \\
 70248 \overline{) 585600} \\
 \underline{561984} \\
 23616 \\
 \underline{2542} \\
 435 \\
 \underline{14}
 \end{array}$$

5. Extract the square root of 2.

2 (1.414213527, root nearly.

$$\begin{array}{r}
 1 \\
 \hline
 24 \overline{) 100} \\
 \underline{96} \\
 281 \overline{) 400} \\
 \underline{281} \\
 2824 \overline{) 11900} \\
 \underline{11296} \\
 28282 \overline{) 60400} \\
 \underline{56564} \\
 282841 \overline{) 383600} \\
 \underline{282841} \\
 282842 \overline{) 100759} \\
 \underline{15906} \\
 764 \\
 \underline{198}
 \end{array}$$

P

R U L E

RULE II.

First deduct the greatest square, placing the root in the quotient as before.

2. Divide the whole remainder by 2, and point it a-new: this may be called a new dividend.

3. Make the root of the first square a divisor, enquiring how often it may be found in a new dividend, to the next figure, reserving the figure under the next point, for half the square of the quotient figure.

4. Multiply the divisor into it, adding to that product the tens of the half square, if any.

5. Annex the quotient figure to the last divisor for a new divisor, with which proceed as with the last until all be finished.

6. Extract the square root of 3272869681.

$$\begin{array}{r} 3272869681 \text{ (5, first single root.} \\ 25 \end{array}$$

$$2) \overline{772869681}$$

$$5) \overline{386434840.5} \text{ (57209}$$

$$+ 7 \overline{374.5} = 5 \times 7 + \frac{1}{2} \text{ square of 7, viz. } \frac{49}{2} = 24.5$$

$$\begin{array}{r} 57 \quad 1193 \\ + 2) \overline{1142} = 57 \times 3 + \frac{1}{2} \text{ square of 2} \end{array}$$

$$\begin{array}{r} 572 \quad 5148405 \\ + 09 \overline{5148405} = 572 \times 9 + \frac{1}{2} \text{ square 9} = 40.5 \end{array}$$

7. What is the square root of $\frac{7}{8} = .875$?

$$\begin{array}{r} 0.777777 \text{ (.8} \\ 64 \end{array}$$

$$2) \overline{137777} -$$

$$\underline{.068888, \text{ \&c.}}$$

.8)

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$$\begin{array}{r}
 .8) .068884 (.881917103688207 \\
 + 8 \quad 672 \\
 \hline
 .88) \quad 16888 \\
 + 1 \quad 8805 \\
 \hline
 .881) \quad 808388 \\
 + 9 \quad 793305 \\
 \hline
 .8819) \quad 1508388 \\
 + 1 \quad 881905 \\
 \hline
 .88191) \quad 62648388 \\
 + 7 \quad 61733945 \\
 \hline
 .881917) \quad 91444388 \\
 + 1 \quad 88191705 \\
 \hline
 .8819171) \quad 3252683888 \\
 + 03 \quad 2645751345 \\
 \hline
 .881917103) \quad 606932543 \\
 \dots\dots \quad 77792281 \\
 \quad 7238913 \\
 \quad 182576 \\
 \quad 6193
 \end{array}$$



SECT. II.

Some Uses of the SQUARE ROOT.

CASE I.

TO find a mean proportional between any two given numbers.

RULE.

Extract the square root of the product of the two numbers, which root will be the mean proportional sought.

Required a mean proportional between 16 and 256.

First $16 \times 256 = 4096$; also, $\sqrt{4096} = 64$

$\therefore 16 : 64 :: 64 : 256.$ 124) 496

P 2

CASE

CASE II.

To find the side of a square equal in area to any given superficies.

R U L E.

Find the square root of the given superficies, which is the side of the square sought.

Suppose I have a circular elliptical polygonal, or irregular fishpond, containing in surface 9 acres, 2 roods, 15 perches, and would have a square one of the same content; desire you'd tell how many yards each side must be?

A. R. P.

9 2 15

4

38

40

1535 perches.

5.5 x 5.5 = 30.25 square yards in 1 perch.

7675

3070

4605

yds. ft. inch.

46433.75 (215.484918 = 215 1 5.467.

4

Q. E. F.

41) 64

41

425) 2333

2125

4304) 20875

17216

43088) 365900

344704

430964) 2119600

3957447876

356

CASE

CASE III.

Having the area of a circle, to find its diameter.

RULE.

Multiply the square root of the area by 1.12837, the product will be the diameter.

In the midst of a meadow, well stored with grafs,
I took just an acre to tether my horse:
How long must the cord be, that feeding all round,
He may'nt graze less or more than this acre of ground.

4840 square yards in an acre.

$$\begin{array}{r} \sqrt{4840} = 69.57 \\ \begin{array}{r} 36 \\ \hline 129 \overline{) 1240} \\ \underline{1161} \\ 7900 \\ 1385 \overline{) 7900} \\ \underline{6925} \\ 97500 \\ 13907 \overline{) 97500} \\ \underline{97349} \end{array} \end{array}$$

$$\begin{array}{r} \cdot \cdot \cdot 1.12837 \\ \times 75.96 \\ \hline 677022 \\ 101553 \\ 5642 \\ 790 \\ \hline 2 \overline{) 78.5007} \end{array}$$

$39\frac{1}{2}$ yards, length of the tether and horse.

The periphery of any circular figure may be found by multiplying the square root of the area by 3.5449.

Thus $69.57 \times 3.5449 = 246$ yards, 1 foot, $10\frac{1}{2}$ inches, the perimeter of the before-mentioned acre of land.

CASE IV.

Any two sides of a right-angled triangle being given, to find the remaining side.

RULE.

As the square of the hypotenuse, or longest side of a right-angled triangle, is equal to the sum of the squares of the other two sides; consequently the difference of the squares of the hypotenuse, and of either of the other sides, is the square of the remaining side.

I. As I was walking out one day,
Which happen'd on the first of May;

As luck would have it, I did 'spy
 A May-pole raised up on high ;
 The which at first me much surpriz'd,
 Not being before-hand advertiz'd
 Of such a strange uncommon sight ;
 I said I would not stir that night,
 Nor rest content, until I'd found
 Its height exact from off the ground :
 But when these words I just had spoke,
 A blast of wind the May-pole broke ;
 Whose broken piece I found to be,
 Exact in length yards sixty-three ;
 Which by its fall broke up a hole,
 Twice fifteen yards from off the pole ;
 But this being all that I can do,
 The May-pole now being broke in two
 Unequal parts, to aid a friend,
 Ye ladies pray an answer send.

Ladies Diary.

$$\text{First } 63 \times 63 = 3969$$

$$\text{Also } 30 \times 30 = 900$$

$$\sqrt{3069} = 55.3985 \text{ pieces standing.}$$

$$\therefore 63 + 55.3985 = 118.3985 \text{ yards} = 118 \text{ yards, 1 foot, } 2\frac{1}{4} \text{ inches, height of the pole.}$$

2. A castle wall there was, whose height was found
 To be an hundred feet from th' top to th' ground :
 Against the wall a ladder stood upright,
 Of the same length the castle was in height.
 A waggish fellow did the ladder slide,
 (The bottom of it) ten feet from the side.
 Now I would know how far the top did fall,
 By pulling out the ladder from the wall.

$$100 \times 100 = 10000$$

$$10 \times 10 = 100$$

$$\sqrt{9900} = 99.49874$$

$$\therefore 100 - 99.49874 = .50125 = \text{a very little more than 6 inches.}$$

3. I want the length of a shoar, that being to strut 11 feet from the upright of a building, will support a jamb 23 feet, 10 inches, from the ground.

$$11 \times 11 = 121$$

$$23.82 \times 23.82 = 568.024$$

feet. inches.

$$\sqrt{568.024} = 26.29918. \text{ Q. E. F.}$$

4. The height of an elm, growing in the middle of a circular island 30 feet in diameter, plumbs 53 feet; and a line stretched from the top of the tree, straight to the higher edge of the water, 112 feet: what then is the breadth of the moat, supposing the land on either side of the water to be level?

$$112 \times 112 = 12544$$

$$53 \times 53 = 2809$$

$$\sqrt{9735} = 98.666.$$

∴ $30 \div 2 = 15$, and $98\frac{2}{3} - 15 = 83\frac{2}{3}$, breadth of the moat required.

5. Two ships set sail from the same port, one of them goes due east 50 leagues, the other due north 84; how far are they asunder?

$$50 \times 50 = 2500$$

$$84 \times 84 = 7056$$

$$\sqrt{9556} = 97.75, \text{ or } 97\frac{3}{4} \text{ leagues.}$$

6. A line 27 yards long will exactly reach from the top of a fort, on the opposite bank of a river, known to be 23 yards broad: the height of the wall is required.

$$27 \times 27 = 81 \text{ feet, } 23 \times 23 = 69 \text{ feet.}$$

$$81 \times 81 = 6561$$

$$69 \times 69 = 4761$$

$$\sqrt{1800} = 42.426 = 42 \text{ feet, 5 inches.}$$

7. Suppose a light-house built on the top of a rock; the distance between the place of observation, and that part of the rock level with the eye, and directly under the building, is given 330 fathoms; the distance from the top of the rock to the place of observation, is 423 fathoms, and from the top of the building 425: the height of the edifice is required.

$$425 \times 425 = 180625$$

$$310 \times 310 = 96100$$

$$\sqrt{84525} = 290.73187, \text{ light-house and rock.}$$

$$\text{Also } 423 \times 423 = 178929$$

$$- 96100$$

$$\sqrt{82829} = 287.80027, \text{ rock.}$$

$\therefore 290.73187 - 287.80027 = 2.9316 = 17.59 \text{ feet. Q.}$
E. F. the height of the light-house.

8. A ladder 40 feet long may be so planted, that it shall reach a window 33 feet from the ground, on one side the street; and without moving it at the foot, will do the same by a window 21 feet high, on the other side: the breadth of the street is required.

$$40 \times 40 = 1600$$

$$33 \times 33 = 1089$$

$$\sqrt{511} = 22.6$$

$$21 \times 21 = 441$$

$$\sqrt{1189} = 34.48$$

Answer, 57.08 feet. Q. E. F.

9. An ancient bath was found, of a triangular form, the sum of whose three equal sides was 125 feet: the area of the bottom is required.

$$3) 125 (41.6, \text{ each side.}$$

$$2) 41.6 (20.8, \text{ half the side.}$$

$$\text{Then } 41.6 \times 41.6 = 1736.711$$

$$\text{Also } 20.8 \times 20.8 = 434.027$$

$$\sqrt{1302.084} = 36.084, \text{ perpend.}$$

$$\text{Then } 41.6 \times 36.084 = 1503.5.$$

$$\therefore 2) 1503.5 (= 751.75 \text{ feet, the area required.}$$

10. The paving a triangular court, at 18d. per foot, came to 100 l. the longest of the three sides was 88 feet: what then was the sum of the other two equal sides?

18d. = .0751.) 100.000 (1333.3 feet, the area.

$\frac{1}{2} = 44$.) 1333.3 (30.3, perpend.

$44 \times 44 = 1936$

$30.3 \times 30.3 = 918.49$

$\sqrt{2854.79} = 53.425$, one of the two

$\therefore 53.425 \times 2 = 106.85$ feet, the answer. lesser sides.

11. I would plant 10 acres of hop-ground, which must be done, either in the square order, or as the number 4 stands on the dice, or in the quincunx order, as the number 5; the three nearest binds, in both cases, must be set linearly just 6 feet asunder: how many plants more will be required for the last order than for the first, admitting the form of the plat to lay the most advantageous for the plantation in either case?

$6 \times 6 = 36$ square feet each plant in the square order.

$3 \times 3 = 9$

$\sqrt{27} = 5.19615$ perpend.

Also $6 \times 5.19615 = 31.1769$ feet, each in the quincunx order.

In one acre are 43560 square feet.

$10 \times 43560 = 435600$.

31.1769) 435600 (13972 plants in the quincunx } order.
 36) 435600 (12100 plants in the square. }

Difference = 1872. Q. E. F.

12. The quarry of glass is $3\frac{1}{2}$ inches on every side, and as much cross the middle, cost 1d.; the square is $5\frac{1}{2}$ inches by $3\frac{1}{2}$, and cost $1\frac{1}{2}$ d.; what will be saved, glazing 1000 feet, the cheapest of the two ways, suppose the leading of the lights be nearly equal in either kind of work?

$3.75 \times 3.75 = 14.0625$

$1.875 \times 1.875 = 3.515625$

$\sqrt{10.546875} = 3.2476$ perpend.

Then $5.25 \times 3.5 = 18.375$

Also $3.2476 \times 3.75 = 12.1785$ } area of the } square quarry.

$1000 \times 144 = 144000$ square inches.

18.375) 144000.000 (7836.7 squares.

12.1785) 144000.0000 (11824 quarries.

$$\begin{array}{r} \frac{1}{7} \overline{) 7836.7} \text{ d.} \\ \underline{979} \quad 7 \end{array}$$

£ 48 19 7 cost in squares.

$$\begin{array}{r} \frac{1}{12} \overline{) 11824} \text{ d.} \\ \underline{985} \quad 4 \end{array}$$

49 5 4 cost in quarries.

∴ 49 l. 5 s. 4 d. — 48 l. 19 s. 7 d. = 5 s. 9 d. advance in squares.

13. A summer-house is a cube of 10 feet in the clear, the cornice of which projects just 15 inches on a side, and being of timber and stucco, the sides are 6 inches thick, so that the whole front of the roof, from out to out, is $13\frac{1}{2}$ feet. This is hipped from each of the corners to the center, and being truly pediment pitch, it raises $\frac{2}{3}$ of the front, or three feet; I would, by the help of these dimensions, measure the slating, without venturing to climb for more, and compute the cost at $3\frac{1}{2}$ d. per square foot.

First $10 + 2\frac{1}{2} + 1 = 13\frac{1}{2} = \frac{27}{2}$ = whole breadth.

Then $\frac{27}{2} \times \frac{2}{3} = 9$ feet, the rise of the roof.

2) 13.5 (6.75 = half the breadth.

$$\begin{array}{r} \text{Also } 6.75 \times 6.75 = 45.5625 \\ \underline{3 \times 3} \quad = \quad 9 \end{array}$$

$$\sqrt{45.5625} = 7.38664$$

Then $13.5 \times 2 \times 7.38664 = 199.4393$, area of the roof.

$$\begin{array}{r} \frac{1}{10} \overline{) 199.4393} \\ \underline{2.49274} \\ .41545 \end{array}$$

£ 2.90819 = 2 l. 18 s. 2 d. the answer.

14. There are two columns in the ruins of Persepolis left standing upright; one is 64 feet above the plane, the other 50: between these, in a right line, stands an ancient statue, the head whereof is 97 feet from the summit of the higher, and 86 feet from the top of the lower column; the base whereof measures just 76 feet to the center of the figure's base: by these notices the distance of the top of the columns may be, by numbers, easily found.

First

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First $86 \times 86 = 7396$

Also $76 \times 76 = 5776$

$\sqrt{1620}$

$= 40.24922$

Then 50

$= 40.24922$

9.75078, height of the statue.

Also $64 - 9.75078 = 54.24922$

Again, $97 \times 97 = 9409$

$54.24922 \times 54.24922 = 2942.97787$

$\sqrt{6466.02213} = 80.4121$

$80.4121 + 76 = 156.4121$, distance of the columns.

$64 - 50 = 14$, difference of their heights.

$156.4121 \times 156.4121 = 24464.745$

$14 \times 14 = 196$

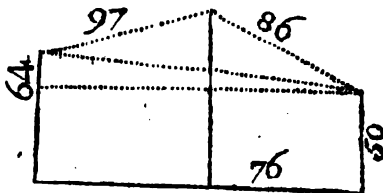
$\sqrt{24660.745} = 157$. Q. E. F.

But if the statue be an enormous Colossus, higher than the towers;

By working as before, the statue will be found 40.29888 feet higher than the lower column.

Also $50 + 40.29888 = 90.29888$, the height of the statue.

$90.29888 - 64 = 26.29888$, higher than the highest column.



$97 \times 97 = 9409$
 $26.29888 \times 26.29888 = 691.6311$

$\sqrt{8717.3689} = 93.3668$

$76 + 93.3668 = 169.3668$, distance of the columns.

$169.3668 \times 169.3668 = 28685.1129$

$14 \times 14 = 196$

$\sqrt{28881.1129} = 169.94$. Q. E. F.

15. The semidiameter of the earth being 3984.58 miles, and the perpendicular height of a mountain three miles; how far will it be seen at sea, or on plain ground, supposing the eye of the spectator to be on the surface of the ground or water?

3984.58

3984.58 femidiameter.
3 height.

$$3987.58 \times 3987.58 = 15900794.2564$$

$$3984.58 \times 3984.58 = 15876877.7764$$

$$\sqrt{23916.4800} = 154.64 \text{ miles.}$$

Q. E. F.

CASE V.

Given three sides of a triangle, to find the area.

RULE.

From half the sum of the three sides, subtract each side severally; let the half sum, and the three differences, be multiplied continually; the square root of the product will be the area required.

1. Having a fish-pond of a triangular form, whose three sides measure 400 yards, 348, and 312; what quantity of ground does it cover?

$$\text{First } 400 + 348 + 312 = 1060.$$

$$\text{Also } \frac{1060}{2} = 530 \text{ yards, half the sum of the three sides.}$$

$$\begin{array}{l} \text{And } 530 - 400 = 130 \\ \quad 530 - 348 = 182 \\ \quad 530 - 312 = 218 \end{array} \left. \vphantom{\begin{array}{l} 530 - 400 \\ 530 - 348 \\ 530 - 312 \end{array}} \right\} \text{differences.}$$

$$\text{Also } 530 \times 130 \times 182 \times 218 = 2733676400.$$

$$\sqrt{2733676400} = 52284.571338 \text{ square yards.}$$

$$4840) 52284.571338 (10.8026 = 10 \text{ a. } 31.8 \text{ p. Q. E. F.}$$

2. A field of a triangular form, whose sides are 380, 420, and 765 yards, lets for 55 s. per acre; how much does the whole bring in per annum?

$$\text{First } 380 + 420 + 765 = 1565.$$

$$\text{Also } \frac{1565}{2} = 782.5 \text{ yards, half the sum of the three sides.}$$

$$\begin{array}{l} \text{And } 782.5 - 380 = 402.5 \\ \quad 782.5 - 420 = 362.5 \\ \quad 782.5 - 765 = 67.5 \end{array} \left. \vphantom{\begin{array}{l} 782.5 - 380 \\ 782.5 - 420 \\ 782.5 - 765 \end{array}} \right\} \text{differences.}$$

$$\text{Also } 782.5 \times 402.5 \times 362.5 \times 67.5 = 1998003710.9375.$$

$$\sqrt{1998003710.9375} = 44699.034 \text{ square yards.}$$

$$4840) 44699.034 (9.2374 = 9 \text{ acres, } 38 \text{ perches.}$$

Answer, 25 l. 8 s. $\frac{3}{4}$ d. per annum.

S E C T.

SECT. III.

To Extract the CUBE ROOT.

RULE I.

HAVING pointed the given resolvend into periods of three figures, as before directed;

Seek the greatest cube in the left-hand period; write the root in the quotient, and the cube under the period; which subtract, and to the remainder bring down the next period; call this a new resolvend, under which draw a line.

2. Under this resolvend write the triple square of the root, so that the units in the latter stand under the place of hundreds in the former; and under the said triple square write the triple root, removed one place to the right; the sum of these is the divisor, under which draw a line.

3. Seek how oft this divisor may be had in the new resolvend (its right-hand place excepted) and write the result in the quotient.

4. Under the divisor write the product of the triple square of the root by the last quotient figure, ~~bring down~~ the units place of this line under that of ~~tens in the~~ divisor; under this line write the product of the triple root by the square of the last quotient figure; let this line be removed one place beyond the right in the former, and under this line, removed one place forward to the right, write down the cube of the last quotient figure, the sum of these three lines call the subtrahend, under which draw a line.

5. Subtract the subtrahend from the new resolvend; to this remainder bring down the next period for another resolvend; the divisor must be the triple square of the quotient, added to the triple thereof, &c. as before directed.

Extract the cube root of 122615327232.

122615327232 (4968

64

58615 new resolvend.

48 triple square of 4.
12 triple of 4.

492 divisor.

432
972
729

53649 subtrahend.

4966327 resolvend.

7203 triple square of 49.
147 triple of 49.

72177 divisor.

43218 triple square of 49 × 6.
5292 triple of 49 × square of 6.
216 cube of 6.

4374936 subtrahend.

591391232 resolvend.

738048 triple square of 496.
1488 triple of 496.

7381968 divisor.

5904384 triple square of 496 × 8.
95232 triple of 496 × square of 8.
512 cube of 8.

591391232 subtrahend = last resolvend; so that
4968 is the true cube root of 122615327232.

R U L E

RULE II.

1. The resolvend being pointed into proper periods, find the nearest less root of the figures of the first punctuation on the left-hand; subtract its cube from the number given; to the remainder annex the next figure for a new resolvend.

2. Take $\frac{1}{3}$ of the resolvend for a dividend.

3. And for a divisor take the square of the root added to half the root (or rather added to the product of the root, and the next quotient figure, leaving out the last figure of the product.)

4. Divide the said dividend by that divisor, the quotient is the second figure of the root.

5. Begin the operation a-new; viz. cube the two figures of the root, and subtract the cube from the given number, annexing another figure for the resolvend.

6. Take the third part of the resolvend for a dividend, and the square of the root added to half the root (or rather added to the product of the root and the next quotient figure, striking off the last figure of the product) for a divisor.

7. The division gives another figure of the root; but the division is to be continued on to two figures, by the contraction in division of decimals, or otherwise.

8. Repeating the operation with four figures in the root, you will get four more by a new division, which gives eight figures in the root; and from 8 to 16; &c. always double.

9. Note, when the cube exceeds the number given, a less figure must be writ in the quotient; and observe every division gives one figure, and the rest are found by continuing the division, and dropping a figure of the divisor every time.

10. If after all the periods, both in whole numbers and decimals, are brought down, the extraction may be continued as far as you please, by still adding ternaries of cyphers. At last cut off as many places of whole numbers as there are points in whole numbers, and the like for decimals.

11. If you desire the last quotient to go true to more places of figures, add half the last quotient to the last root, and square the sum for a divisor, and divide over again.

Extract the cube root out of 92398647506217.

$$\begin{array}{r}
 92398647506217 \quad (45 \\
 \underline{64} \\
 3 \overline{) 283} \quad \begin{array}{l} 16 \text{ square.} \\ 64 \text{ cube.} \\ 4 \times 5 = 20. \end{array} \\
 \underline{16} \quad 94 \quad (5 \\
 \underline{+ 2} \quad 90 \\
 18 \overline{) 4} \\
 \underline{92398} \\
 \underline{91125} \\
 3 \overline{) 12736} \quad \text{square of } 45 = 2025. \\
 \underline{2025} \quad 4245 \quad (208 \quad \begin{array}{l} \text{cube} = 91125. \\ 45 \times 2 = 90. \end{array} \\
 \underline{9} \quad 4068 \\
 2034 \overline{) 177} \quad \begin{array}{l} \text{root } 45208 \text{ in whole numbers,} \\ \text{its square } 2043763264, \\ \text{cube } 92394449638912. \end{array} \\
 \underline{163} \\
 14 \\
 \underline{92398647506217} \quad \begin{array}{l} 45208 \\ 6 \end{array} \\
 \underline{92344449638912} \\
 3 \overline{) 41978673050} \quad 271248 \\
 \underline{2043763264} \quad 1399289101.6 \quad (68465392 \\
 \underline{+ 27124} \quad 12262742328 \\
 2049760388 \overline{) 1730148688} \quad \text{root } 45208.68465. \\
 \underline{1635032310} \\
 \underline{95116378}
 \end{array}$$

R U L E III.

1. After the given resolvend is truly pointed, seek the greatest cube in the left-hand period; write the root in the quotient; subtract the cube from the period, as directed in the other rule; and to the remainder bring down all the remaining periods in the given number, for a new resolvend.

2. To

2. To the root (or quotient) annex as many cyphers as there are remaining periods; multiply this by 3; by this product divide the resolvend, and point the quotient into periods of two places (beginning at units) observing that there be no more points than there were periods brought down to the resolvend.

3. Make the root (found in the first period of the given numbers) a divisor; see how often it may be had in the left-hand period of the quotient (excepting the place under the point) and the figure resulting write in the quotient (to the right-hand of the root first found) and on the right of the divisor; multiply this increased divisor by the last quotient figure; to the remainder bring down the next period; divide this by the last divisor.

Extract the cube root of 8302348000000.

$$\begin{array}{r}
 8302348000000 \\
 \underline{8} \\
 6,0000 \quad 30234800,0000 \\
 \quad 202 \quad 5039133 \quad (\quad 024 \\
 \quad \quad 404 \quad \quad 2000 \\
 \quad \quad \underline{\quad} \\
 2024 \quad 9991 \quad 2024 \\
 \quad \quad 8096
 \end{array}$$

The cube of 2024 = 8291469824
 8302348000000) 20240 \times 3 = 60720, divisor.
 8291469824

$$\begin{array}{r}
 6072,0 \quad 1087817600,0 \quad (\quad 179153,42556 \\
 20248 \quad 179153,425560 \quad (\quad 8.8479 \\
 \quad \quad 161984
 \end{array}$$

$$\begin{array}{r}
 20248.8 \quad 1716942 \\
 \quad \quad 1619904
 \end{array}$$

$$\begin{array}{r}
 20248.84 \quad 9703859 \\
 \quad \quad 8099536
 \end{array}$$

$$\begin{array}{r}
 20240 \\
 + 8.8479
 \end{array}$$

20248.8479, root. Q. E. F.

$$\begin{array}{r}
 20248.847 \quad 160431960 \\
 \quad \quad 141741929 \\
 \quad \quad \underline{\quad} \\
 \quad \quad 18690031
 \end{array}$$

Q

R U L E

RULE IV.

Divide the given resolvend by three times the supposed root, and from the quotient subtract one-twelfth of the square of the supposed root; the square root of the remainder, added to half the supposed root, will give the true root required.

What is the cube root of 146708.483?

Suppose the root 50.

$$\text{Then } 50 \times 3 = 150 \quad 146708.483 \quad (978.8565$$

$$50 \times 50 = 2500; \text{ also } 12 \quad 2500 \quad (208.3333$$

$$\underline{770.5232}$$

$$\sqrt{770.5232} = 27.7$$

$$2 \mid 50 (= 25, \text{ half the supposed root.})$$

52.7, the root.

But for greater exactness I proceed to another operation.

$$\text{Thus, } 52.7 \times 3 = 158.1 \quad 146708.483 \quad (927.94739$$

$$52.7 \times 52.7 = 2777.29 \quad \dots \quad 12 \quad 2777.29 \quad (231.44083$$

$$\underline{696.50656}$$

$$\sqrt{696.50656} = 26.39141$$

$$2 \mid 52.7 \quad (26.35$$

$\therefore 52.74141$, the root more exact.

Extract the cube root of 2.

$$1 \times 3 = 3 \quad 2.0 \quad (.666$$

$$1 \times 1 \text{ and } 12 \quad 1.0 \quad (.083$$

$$\underline{.583}$$

$$\sqrt{.5833} \quad (.7$$

$$2 \mid 1.0 \quad (.5$$

1.2, root.

$$\text{By a second operation } 1.2 \times 3 = 3.6 \quad 2.00 \quad (.355$$

$$1.2 \times 1.2 = 1.44$$

$$12 \mid 1.44 \quad (.12$$

$$\sqrt{4355} \quad (.659$$

$$2 \mid 1.200 \quad (.6..$$

$$\underline{.438}$$

1.259, root.

By

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By a third operation $1.259 \times 3 = 3.777 \} 2.000 (.5295207866.$

$1.259 \times 1.259 = 1.585081 \dots 12) 1.585081 (.1320900833$

$\sqrt{.3974307033 = .63042113$

$2) 1.259 (.6295$

$.3974307033$

1.25992113 , the root. Q. E. F.

What is the cube root of .0001357?

Supposed root .05.

$.05 \times 3 = .15) .0001357 (.0009486$

$.05 \times .05 = .0025$, and $12) .0025 (.0002083$

$\sqrt{.0007383 (.027$

$.0007383$

$2) .05 (.025$

.052 root, which by involution I find too

much. \therefore I take .051 for the supposed root.

$.051 \times 3 = .153) .0001357 (.00088751$

$.051 \times .051 = .002601$, and $12) .002601 (.00021758$

$.00066993$

$\sqrt{.00066993 = .02588$

$2) .051 (.0255$

.05138 root.

Then $.05138 \times 3 = .15414) .000135700 (.0008803685$

$.05138 \times .05138 = .0026399044$

$12) .0026399044 (.0002199920$

$.0006603765$

$\sqrt{.0006603765 = .02569779$

$\frac{.05138}{2} = .02569$

$.05138779$, the root. Q. E. F.

What is the cube root of $13\frac{1}{2}$?

In decimals $13\frac{1}{2} = 13.6$

Supposed root $2 \times 3 = 6) 13.6 (2.27$

$2 \times 2 = 4$; also $12) 4.0 (0.33$

1.94

Q 2

$\sqrt{1.94}$

$$\sqrt{1.94} = 1.39$$

$$\frac{2}{2} = 1.00$$

2.39, root.

$$2.39 \times 3 = 7.17 \quad 13.66 \quad (1.90609019$$

$$2.39 \times 2.39 = 5.7121; \text{ and } \frac{5.7121}{12} = 0.47600833$$

$$\sqrt{1.43008186} = 1.1958$$

$$\frac{2.39}{2} = 1.195$$

2.3908,
root.

$$2.3908 \times 3 = 7.1724 \quad 13.6666 \quad (1.9054454125$$

$$2.3908 \times 2.3908 = 5.71592464$$

$$12 \quad 5.71592464 \quad (0.4763270533$$

$$1.4291183592$$

$$\sqrt{1.4291183592} = 1.195457397$$

$$2 \quad 2.3908 = 1.1954$$

2.390857397, the root. Q. E. F.

The second method of extracting the cube root is that used by that great mathematician Mr. Emerson, in his treatise of arithmetic, and doubles the figures in the root at each operation.

The third is the method Mr. J. Robertson, F.R.S. uses in his mensuration, by which each operation triples the figures in the root.

But the fourth and last I take to be the easiest, as the operations are performed by easy divisions, and an extraction of the square root.

N. B. This method only doubles the figures in the root at each operation.

SECTION IV.

Some Uses of the CUBE ROOT.

THE cube root is of very great use in mathematics, but I shall only exhibit a few cases.

CASE

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CASE I.

To find the side of a cube that shall be equal in solidity to any given solid, as a globe, cylinder, prism, cone, &c.

RULE.

Extract the cube root of the solid content of the given body, which will be the side of a cube of an equal solidity.

Suppose a chest, whose length is 4 feet 7 inches, breadth 2 feet 3 inches, and depth 1 foot 9 inches; required the side of a cube of equal solidity?

F. I.

Length 4 7 = 55 } inches.
Breadth 2 3 = 27 }

385

110

1485

Depth 1 9 = 21

1485

2970

31185 solid inches.

30 × 3 = 90) 31185 (346.5
30 × 30 = 900 . 12) 900 (75

271.5 (16.4
1

271.5

26)

171

156

2) 30 (16.4
15

324) 1550

31.4, root.

For a second operation, 31.4 × 3 = 94.2) 31185 (331.05106

31.4 × 31.4 = 985.96 . . . 12) 985.96 (82.16333

✓ 248.88773 (15.7762

248.88773

2) 31.4 (15.7

31.4762, side of the cube required.

Q 3

CASE

CASE II.

Having the dimensions of any solid body, to find those of a similar solid, any number of times, greater or less than the solid given,

RULE,

Multiply the cube of each of the given dimensions by the difference between the solid given, and that required, if greater (or divide by the difference, if lesser) than the solid given; then extract the cube root of each product or quotient, which will be the dimensions of the solid required.

Suppose the length of a ship's keel be 125 feet, the breadth of the midship beam 25 feet, and the depth of the hold 15 feet; I demand the dimensions of another ship, of the same form, that shall carry three times the burthen?

$$125 \times 125 \times 125 \times 3 = 5859375$$

$$25 \times 25 \times 25 \times 3 = 46875$$

$$15 \times 15 \times 15 \times 3 = 10125$$

$$\therefore \sqrt[3]{5859375} = 180.28, \text{ keel.}$$

$$\text{Also } \sqrt[3]{46875} = 36.05, \text{ midship beam. } \left. \begin{array}{l} \text{And } \sqrt[3]{10125} = 21.6, \text{ depth in the hold.} \end{array} \right\} \text{Q. E. F.}$$

Or suppose the ship was to be but of half the burthen of that whose dimensions are given as above.

$$\frac{125 \times 125 \times 125}{2} = 976562.5$$

$$\frac{25 \times 25 \times 25}{2} = 7812.5$$

$$\frac{15 \times 15 \times 15}{2} = 1687.5$$

$$\therefore \sqrt[3]{976562.5} = 99.202$$

$$\text{Also } \sqrt[3]{7812.5} = 19.84$$

$$\text{And } \sqrt[3]{1687.5} = 11.906$$

CASE III.

Having the dimensions and capacity of a solid, to find the dimensions of a similar solid of a different capacity,

RULE,

Divide the cube of the dimensions given, multiplied into the capacity of the vessel or body required; the cube root of the quotient will be the result.

If

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If a ship of 100 tuns be 44 feet long at the keel, of what length shall the keel of that ship be, whose burthen is 220 tuns?

$$\text{First } 44 \times 44 \times 44 \times 220 = 18740480$$

$$100 \overline{) 18740480} \quad (187404.8$$

$$\sqrt[3]{187404.8} = 57.22592, \text{ the answer required.}$$

CASE IV.

Between two given numbers, to find two mean proportionals,

RULE,

Multiply the less extreme by the cube root of the quotient of the greater extreme, divided by the less; the product is the lesser of the two mean proportionals, which multiplied by the said cube root gives the greater mean sought.

Find two mean proportionals between 7 and 15379.

$$\begin{array}{r} 7 \overline{) 15379} \quad (2197 \\ 11 \times 3 = 33 \overline{) 2197} \quad \left\{ \begin{array}{l} 66.575 \\ 12 \overline{) 121} \quad (10.083 \end{array} \right. \end{array}$$

$$\sqrt{56.492} = 7.5$$

$$\underline{5.5}$$

13. cube root.

$$\begin{array}{l} \therefore 7 \times 13 = 91, \text{ first} \\ \text{and } 91 \times 13 = 1183, \text{ second} \end{array} \left. \vphantom{\begin{array}{l} 7 \times 13 = 91 \\ 91 \times 13 = 1183 \end{array}} \right\} \text{mean propor. Q. E. F.}$$

For as 7 : 91 :: 1183 : 15379.



SECT. V.

To Extract the BIQUADRATE ROOT.

RULE.

EXTRACT the square root of the given resolvent, and the square root of that first root will be the biquadrate root required,

Q 4

Extract

Extract the biquadrate root of 33481581224913441.

33481581224913441 (182979729 (13527, biquadrate
I I root required.

28) 234
224

23) 82
69

362) 1081
724

265 } 1397
1325

3649) 35758
32841

2702) 7297
5404

36587) 291712
256109

27047) 189329
189329

368949) 3560324
329354I

○

3659587) 26678391
25617109

&c.



S E C T. VI.

To Extract the SURSOLID ROOT.

R U L E.

HAVING pointed the given resolvend into periods of five figures, seek such a surfoldid number in the table of powers (or otherwise) as comes nearest to the first period of the resolvend, whether greater or less; and call the respective root, either more than just, or less than just, as it falls out; annexing so many cyphers to it as there are remaining periods of whole numbers in the resolvend.

2. Find the difference between the resolvend and the sur-solid number, so taken, by subtracting the lesser from the greater.

3. Find the cube of the forefaid fursolid root, with its annexed cyphers, which also may be done by the table of powers, and multiply that cube into five, the index of the fursolid, and divide the difference between the resolvend and the

the

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the fursolid number by that product; by which it will be depressed to a square, and when pointed into periods of two figures each, call it the new resolvend.

4. Make the first root without cyphers a divisor, enquiring how often it may be found in the first period of the new resolvend; with this consideration, if the root, now a divisor, be less than just, annex twice the quotient figure to it; but if more than just, subtract twice the quotient figure from a cypher, either annexed, or supposed to be annexed, to that divisor or root, multiplying it, so increased or diminished, with the said quotient figure; setting down the units place of the product under the pointed figure of that period, subtracting it as in division.

Extract the fursolid root of 307682821106715625.

$$\begin{array}{r} 307682821106715625 \text{ (3} \\ 243 \end{array}$$

$$64682821106715625$$

$$3000 \text{ cubed} = 27000000000$$

$$27000000000 \times 5 = 135000000000, \text{ divisor.}$$

$$135000000000) 64682821106715625 \text{ (479132}$$

$$\begin{array}{r} 3) 479132 \text{ (14} \\ + 1 \times 2 = 2 \quad 32 \end{array}$$

$$\begin{array}{r} 32) 1591 \quad 3000 \\ + 4 \times 2 = 8 \quad 1312 \quad 140 \end{array}$$

$$3140$$

By a second operation.

$$\begin{array}{r} 307682821106715625 \\ 3140 \cdot 5 = 305244776182400000 \end{array}$$

$$2438044924315625$$

$$\begin{array}{r} 314 \cdot 3 = 30959144000 \\ 5 \end{array}$$

$$154795720000, \text{ divisor.}$$

$$154795720000) 2438044924315625 \text{ (15750}$$

$$\begin{array}{r} 314) 15750 \text{ (5} \\ 5 \times 2 = 10 \quad 15750 \\ 3150 \quad 0 \end{array}$$

$$\begin{array}{r} \text{First root } 3140 \\ + 5 \end{array}$$

$$\text{True root } 3145 \quad \text{Extract}$$

Extract the surfold root of 9763796029890739602796302988.

$$\begin{array}{r}
 1024 \\
 9763796029890739602796302988 \\
 \hline
 476203970109260397203697012 \\
 400000 \bullet 3 = 640000000000000000 \\
 \hline
 5 \\
 \hline
 32000000000000000000 \\
 32) 47620397010 (1488137406 \\
 400) 1488137406 (037 \\
 \hline
 06 \quad 1182 \quad 400000 \\
 \hline
 394) 29613 \quad 037 \\
 \hline
 14 \quad 27482 \quad 396300, \text{ root.} \\
 \hline
 \end{array}$$

3926

But I only take 396 for a second operation, which I find by involution to be less than just.

$$\begin{array}{r}
 9763796029890739602796302988 \\
 396 \bullet 5 = 9738138110976000000000000000 \\
 \hline
 23657918814739602796302988 \\
 369000 \bullet 3 = 620991360000000000 \\
 62099136) 2365791881479602,7963 (38970160.5871 \\
 396000) 38970160.5871 (098.3889 \\
 18 \quad 356562 \\
 \hline
 39612 \quad 3322060 \\
 16 \quad 3169088 \\
 \hline
 396136 \quad 15297258 \\
 6 \quad 11884098 \\
 \hline
 3961366 \quad 341316071 \\
 16 \quad 316909408 \\
 \hline
 39613676 \quad 34406663 \\
 \quad 31690941 \\
 \hline
 3715722 \\
 396000 \\
 098.3889 \\
 \hline
 396098.3889, \text{ the root sought.} \\
 \hline
 \end{array}$$

S E C T.

SECT. VII.

To *Extract the* ROOT of the SQUARE CUBED; or,
SIXTH POWER.

R U L E.

EXTRACT the square root of the given resolvend;
then extract the cube root of that square root, which
will be the root of the sixth power required.

Or you may first extract the cube root of the resolvend,
and then the square root of that cube root, and that will be
the root required.

Extract the sixth power of 435728381009267809889764416.

435728381009267809889764416 (20874107909304.
4 I find by pointing the square root
— into periods of three figures each,
408) 3572 that there will be five figures in
3264 whole numbers in the root of the
— sixth power.

4167) 30883
29169
—

41744) 171481
166976
—

417481) 450500
417481
—

41748207) 330199267
292237449
—

&c.

2000 × 3 = 60000) 20874107909304 (347901798
12) 4000000000 (33333333

314568465
—

314568465 (17600
10000
—

27600, which by involution I find too much,
therefore take 275.

27500

$$27500 \times 3 = 82500 \quad 20874107909304 \quad (253019489$$

$$275 \times 275 = 75625 \dots 12 \quad 756250000 \quad (63020833$$

$$\underline{\underline{189998656}}$$

$$189998656 \quad (13784$$

$$\begin{array}{r} 1 \quad 2 \end{array} \quad 275 \quad (1375$$

$$\begin{array}{r} 23 \quad 89 \\ \underline{69} \end{array}$$

$\underline{\underline{27534}}$ true root of the sixth power.

$$\begin{array}{r} 267 \quad 2099 \\ \underline{1869} \end{array}$$

$$\begin{array}{r} 2748 \quad 23086 \\ \underline{21984} \end{array}$$

$$\begin{array}{r} 27564 \quad 110256 \\ \underline{110256} \end{array}$$



S E C T. VIII.

To Extract the ROOT of the Second SURSOLID; or, SEVENTH POWER.

R U L E.

HAVING pointed the resolvend into periods of seven figures, seek out such a number of the seventh power, by the table, as comes nearest to the first period of the resolvend, whether greater or lesser, calling its root more than just, or less than just, annexing a proper number of cyphers.

2. Find the difference between the resolvend, and that number of the seventh power, by subtracting the lesser from the greater.

3. Find the sursolid, or fifth power of that root, with its annexed cyphers, by the table of powers; and multiply that sursolid number into seven, the index of the resolvend.

4. Make that product a divisor, by which the foresaid difference must be divided; so that it may be depressed to a square, and pointed as such.

5. Make the first root, without cyphers; a divisor, working with it and the new resolvend, as in the sursolid; only
here

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here you must increase or diminish the divisor with thrice the quotient figure.

Extract the 7th power of 34487717467307513182492153794673.

$$3 \ominus 7 = 2187$$

$$\begin{array}{r} 12617717467307513182492153794673 \\ \text{First root } 30900 \ominus 5 = 24300000000000000000000 \\ 243, \&c. \times 7 = 1701000000000000000000000 \\ \text{Contracted } 1701) 126177174673 (74178233 \\ 3) 74178233 (20 \\ + 2 \times 3 = 6 \quad 72 \qquad \qquad \qquad 30000 \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad 20 \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \hline 360) 21782 \qquad \qquad \qquad 32000 \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \hline \end{array}$$

Second operation.

$$\begin{array}{r} 34487717467307513182492153794673 \\ 32 \ominus 7 = 34359738368 \\ \hline 127979099307513182492153794673 \\ 32 \ominus 5 = 33554432, \text{ which } \times 7 = 234881024 \\ 234881024) 127979099307513 (544868 \\ 320) 544868 (017 \\ + 1 \times 3 = 3 \quad 3203 \\ \hline 3203) 224568 \\ + 7 \times 3 = 21 \quad 224357 \qquad \qquad \qquad 31000 \\ \hline 32051 \qquad 211 \qquad \qquad \qquad + 017 \\ \hline 32017, \text{ true root.} \end{array}$$



S E C T. IX.

To Extract the Root of the BIQUADRATE SQUARED; or, EIGHTH POWER.

R U L E.

EXTRACT the square root of the given resolvend, which will reduce it to a biquadrate number, which call a new resolvend; the square root of which will be a square

square number; of which extract the square root, which root will be the result required.

Let 1121016281320476236246497942460481 be the given resolvend, whereof the root of the eighth power is to be extracted.

$$\begin{array}{r}
 1121016281320476236246497942460481 \\
 \underline{9} \qquad (33481581224913441, \text{biquad. resol.} \\
 63)221 \quad \text{Then } \sqrt{33481581224913441} = 182979729 \\
 \quad \quad \quad \underline{189} \quad \quad \quad \therefore \sqrt{182979729} = 13527, \text{root} \\
 664)3201 \quad \quad \quad \text{of the eighth power. Q. E. F.} \\
 \quad \quad \quad \underline{2656} \\
 6688)54562 \\
 \quad \quad \quad \underline{53504} \\
 66961)105881 \\
 \quad \quad \quad \underline{66961} \\
 669625)3892032 \\
 \quad \quad \quad \underline{3348125} \\
 6696308)54390704 \\
 \quad \quad \quad \underline{53570464} \\
 66963161)82024076 \\
 \quad \quad \quad \underline{66963161} \\
 669631622)1506091523 \\
 \quad \quad \quad \underline{1339263244} \\
 6696316242)16682827962 \\
 \quad \quad \quad \underline{12392632484} \\
 66963162444)329019547846 \\
 \quad \quad \quad \underline{267852649776} \\
 669631624489)6116689807049 \\
 \quad \quad \quad \underline{6026684620401} \\
 6696316244981)9000518664879 \\
 \quad \quad \quad \underline{6696316244981} \\
 66963162449823)230420241989842 \\
 \quad \quad \quad \underline{200889487349469} \\
 669631624498264)2953075464037346 \\
 \quad \quad \quad \underline{2678526497993056} \\
 6696316244982684)27454896604429004 \\
 \quad \quad \quad \underline{26785264979930736} \\
 66963162449826881)6696316244986281 \\
 \quad \quad \quad \underline{6696316244986281}
 \end{array}$$

SECT. X.

To Extract the ROOT of the CUBE CUBED; or,
NINTH POWER.

R U L E.

EXTRACT the cube root of the given resolvend, and the result will be a cubic resolvend; of which extract the cube root also, which will be the root of the ninth power required.

Let 976379602989073960279630298890 be the resolvend given; out of which the root of the ninth power is to be extracted.

$$\begin{array}{r}
 99 \times 3 = 297 \quad) \quad 976379602989073960279 \\
 \underline{99} \qquad \qquad \qquad 32874734107 \\
 \qquad \qquad \qquad 12 \quad) \quad 9805 \quad (\quad 8170833333 \\
 \hline
 891 \\
 891 \\
 \hline
 9805
 \end{array}$$

$$\begin{array}{r}
 \sqrt[9]{247039007740} = 4970000000 \\
 \frac{22}{2} = 495
 \end{array}$$

Root 9920000000

Again,

$$\begin{array}{r}
 9920 \times 3 = 29760 \quad) \quad 976379602989073960279630 \\
 9920 \times 2 = 9840640 \quad (\quad 3280845440151 \\
 \underline{9840640} \qquad \qquad \qquad 820053333333 \\
 \qquad \qquad \qquad 12 \qquad \qquad \qquad \underline{2460792106818}
 \end{array}$$

$$\begin{array}{r}
 \sqrt[9]{24607921068180} = 4960637000 \\
 \frac{22}{2} = 496
 \end{array}$$

Root 9920637000

And,

$$\begin{array}{r}
 9920637 \times 9920637 = 98419038485769000000 \\
 \times 3 \\
 29761911 \quad) \quad 97637960298907396027963028890 \\
 \qquad \qquad \qquad (\quad 32806347784558389421.99 \\
 12 \quad) \quad 98419038485769 \quad (\quad 8201586340480750000 \\
 \hline
 24604761244077639421.99
 \end{array}$$

$$\begin{array}{r}
 \sqrt[9]{24604761244077639421.99} = 4960318663.5616 \\
 \frac{22}{2} = 49603185
 \end{array}$$

Cube root required 9920637163.5616, or new
resolv.
Then

Then,

$$2100 \times 3 = 6300 \quad 9920637163.5616 \quad (1574543.99$$

$$2100 \times 2100 = 4410000. \quad 12 \quad 4410000 \quad (367666.66$$

$$\sqrt{1206877} = 1098$$

$$\frac{2100}{2} = 1050$$

Root 2148 in whole numbers,

Again,

$$2148 \times 2148 = 4613904$$

$$2148 \times 3 = 6444 \quad 9920637163.5616 \quad (1539515.3885$$

$$12 \quad 4613904 \quad (384492$$

$$1155023.3885$$

$$\sqrt{1155023.3885} = 1074.7201$$

$$\frac{2148}{2} = 1074.$$

The root of the 9th power 2148.7201. Q. E. F.

Thus have I endeavoured to make plain the method of extracting the root of all the powers, whose index is any number not exceeding the nine digits, which by a little consideration may be extended to still higher powers. I now conclude this chapter with the following method proposed by Mr. Halliday and others, viz.



S E C T. XI.

TO multiply several figures by several, and the product to be produced in one line only.

R U L E.

Multiply the units of the multiplicand by the units of the multiplier, setting down the units of the product, and carry the tens; next multiply the tens in the multiplicand by the units of the multiplier, to which add the product of the units of the multiplicand, multiplied by the tens in the multiplier, and the tens carried; then multiply the hundreds in the multiplicand by the units of the multiplier, adding the product of the tens in the multiplicand multiplied by the tens in the multiplier, and the units of the multiplicand by the hundreds in the multiplier; and so proceed till you have

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have multiplied the multiplicand all through, by every figure in the multiplier.

$$\begin{array}{r} 321432 \\ 213213 \\ \hline 68533481016 \end{array}$$

EXPLANATION.

First, $3 \times 2 = 6$; secondly, $3 \times 3 + 1 \times 2 = 11$, *i. e.* 1 and carry 1.

Thirdly, $3 \times 4 + 2 \times 2 + 1 \times 3 + 1 = 20$, that is, 0 and go 2.

Fourthly, $3 \times 1 + 3 \times 2 + 1 \times 4 + 2 \times 3 + 2 = 21$, *i. e.* 1 and go 2.

Fifthly, $3 \times 2 + 1 \times 2 + 1 \times 1 + 3 \times 3 + 2 \times 4 + 2 = 28$, *i. e.* 8 and go 2.

Sixthly, $3 \times 3 + 2 \times 2 + 1 \times 2 + 1 \times 3 + 2 \times 1 + 3 \times 4 + 2 = 34$, *i. e.* 4 and go 3.

Seventhly, $1 \times 3 + 2 \times 3 + 2 \times 2 + 1 \times 4 + 3 \times 1 + 3 = 23$, *i. e.* 3 and go 2.

Eighthly, $2 \times 3 + 2 \times 4 + 1 \times 1 + 3 \times 2 + 2 = 23$, *i. e.* 3 and go 2.

Ninthly, $3 \times 3 + 2 \times 1 + 1 \times 2 = 15$, *i. e.* 5 and go 1.

Tenthly, $1 \times 3 + 2 \times 2 + 1 = 8$, to set down.

Lastly, $2 \times 3 = 6$, which finishes the work.

$$\begin{array}{r} 35234 \\ 52424 \\ \hline 187107216 \end{array}$$

First, $4 \times 4 = 16$, that is, 6 and go 1.

$3 \times 4 + 4 \times 2 + 1 = 21$, that is, 1 and go 2.

$2 \times 4 + 3 \times 2 + 4 \times 4 + 2 = 32$, *i. e.* 2 and go 3.

$5 \times 4 + 2 \times 2 + 3 \times 4 + 4 \times 2 + 3 = 47$, *i. e.* 7 and go 4.

$3 \times 4 + 5 \times 2 + 2 \times 4 + 3 \times 2 + 4 \times 5 + 4 = 60$.

$3 \times 2 + 5 \times 4 + 2 \times 2 + 3 \times 5 + 6 = 51$.

$3 \times 4 + 5 \times 2 + 2 \times 5 + 5 = 37$.

R

3×2

$$\overline{3 \times 2} + \overline{5 \times 5} + 3 = 34.$$

Lastly, $\overline{3 \times 5} + 3 = 18.$

Mr. Halliday says, that this is not only performed very expeditiously in small figures, but also in great figures may be done readily enough by any person who can add one number to another, not exceeding 81; but I for my part think it a hazardous puzzling operation, and only fit for the practice of another Jedidiah Buxton.

The END of the FIRST BOOK.



Arithmetical Collections

AND

IMPROVEMENTS.



B O O K II.

Containing PROPORTION, with its Use ; also the
Use of the RULES of PRACTICE, in various
Branches of MERCHANDIZE and TRADE.



CHAPTER I.

PROPORTION DISJUNCT,

CALLED THE

GOLDEN RULE; *or*, RULE of THREE.

PROPORTION Disjunct, or the Golden Rule,
are either direct or reciprocal, called Inverse, and those
are both single and compound.



SECT. I.

DIRECT PROPORTION.

DIRECT proportion is when of four numbers the first
beareth the same ratio, or proportion to the second, as
the third doth to the fourth ; as in these :

5 : 35 :: 17 : 119 ; *or*, 65 : 13 :: 20 : 4.

By ratio is here meant the common multiplier or divisor ;
and it shews the habitude or relation one number hath to
another, viz. whether it be double, triple, quadruple, &c.
so that proportionality is a similitude of ratios.

R 2

That

That is, the greater or less the second term is in respect to the first, the greater or less will the fourth be in respect to the third.

Thus the ratio or common multiplier is 7 in the first four proportional numbers, viz. 35; the second term in the proportion is 7 times greater than 5, the first term; so is 119, the fourth term, 7 times greater than 17, the third term.

Also 5 is the ratio or common divisor in the second four proportional numbers; for 13, the second term in the proportion, is 5 times less than 65, the first term; so is 4, the fourth term, 5 times less than 20, the third term.

If four numbers are in direct proportion, the product of the two extremes will always be equal to the product of the two means, viz. $5 \times 119 = 35 \times 17$, each being equal to 595, and $65 \times 4 = 13 \times 20 = 260$.

If four numbers are proportional, they will also be so in alternation, inversion, composition, subtraction, conversion, and mixtly. Euclid 5. Def. 12, 13, 14, 15, 16.

That is, if $65 : 13 :: 20 : 4$ be in direct proportion.

Then $65 : 20 :: 13 : 4$ alternate.

And $13 : 65 :: 4 : 20$ inverted.

Also $65 + 13 : 13 :: 20 + 4 : 4$ compounded.

Or $65 + 20 : 20 :: 13 + 4 : 4$ alternately compound.

Again, $65 - 13 : 13 :: 20 - 4 : 4$ subtracted.

Or $65 - 20 : 20 :: 13 - 4 : 4$ alternately subtracted.

And $65 : 13 + 65 :: 20 : 4 + 20$ converted.

Lastly, $65 + 13 : 65 - 13 :: 20 + 4 : 20 - 4$ mixtly.

When three numbers are given, and a fourth proportional is required, in order to state the question right, observe the following directions; viz.

First, That always two of the three given terms are only supposed, and assign or limit the ratio or proportion; the third moves the question, and the fourth gives the answer.

Secondly, The term which moves the question, hath generally some of these words before it, viz. What will? How many? How long? How far? Or how much? &c.

Thirdly, That the first term in the supposition be of the same kind and denomination with that term which moves the question, and the term sought will be of the same kind and denomination with the second term in the supposition.

All questions thus prepared may be answered by three several rules, but the first is most commonly used.

R U L E I .

Multiply the second and third terms together, and divide their product by the first term; the quotient will be the answer required.

R U L E II .

Divide the second term by the first, then multiply the quotient into the third term, and their product will be the answer required.

R U L E III .

Divide the third term by the first, then multiply that quotient into the second term, and their product will be the answer.

1. If $3\frac{1}{4}$ yards of kersey cost 8 s. 9 d. what will $257\frac{1}{2}$ yards cost at that rate?

By R E D U C T I O N .

$$\begin{array}{r}
 \begin{array}{ccc}
 \text{yds.} & \text{s.} & \text{d.} \\
 \text{As } 3\frac{1}{4} : 8 & 9 : : 257\frac{1}{2} \\
 \hline
 4 & 12 & 4 \\
 \hline
 15 & 105 & 1030 \\
 & & 105 \\
 & & \hline
 & & 515 \\
 & & 103 \\
 & & \hline
 & & 12) \\
 & 15) 108150 & (7210 \text{ pence.} \\
 & 31 & 20) 600 \text{ 10 d.} \\
 & 15 & \hline
 & 30 & - 10 d. \\
 & 0 &
 \end{array}
 \end{array}$$

By D E C I M A L S .

$$\begin{array}{l}
 \begin{array}{l}
 4 \overline{) 3.75} = 3\frac{3}{4} \text{ yards.} \\
 \hline
 \end{array}
 \quad
 \begin{array}{l}
 \begin{array}{r}
 12 \overline{) 9} \\
 20 \overline{) 8.75} \\
 \hline
 \text{£ } .4375
 \end{array}
 \end{array}
 \quad
 \begin{array}{l}
 \begin{array}{l}
 4 \overline{) 2} \\
 \hline
 257.5 \text{ yards.}
 \end{array}
 \end{array}
 \end{array}$$

R 3

As

As 3.75 : .4375 :: 257.5

$$\begin{array}{r}
 257.5 \\
 21875 \\
 30625 \\
 21875 \\
 \hline
 8750
 \end{array}$$

3.75) 112.65625 (30.0416 = 30 l. -s. 10 d. as before.

$$\begin{array}{r}
 \dots 1562 \\
 \dots 625 \\
 2500 \\
 250, \&c.
 \end{array}$$

2. If $1\frac{1}{4}$ ounce of silver plate cost 10 s. $11\frac{1}{4}$ d. what will a service, weighing 327 oz. 12 pwt. 9 gr. cost at that rate?

oz. pwt. s. d. oz. pwt. gr.
As 1 15 : 10 $11\frac{1}{4}$:: 327 12 9

20 12 20

$$\begin{array}{r}
 35 \qquad 131 \qquad 6552 \\
 24 \qquad \qquad 4 \qquad 24 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 140 \qquad 525 \text{ farth. } 26208 \\
 70 \qquad \qquad \qquad 13104 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 840 \text{ gr.} \qquad 157257 \\
 \hline
 525
 \end{array}$$

$$\begin{array}{r}
 786285 \\
 314514 \\
 \hline
 786285
 \end{array}$$

4)
840) 82559925 (98285 farthings.

695 12) 24571 $\frac{1}{2}$ pence.

239 20) 2047 $\frac{7}{8}$

719 £ 102 7 $\frac{7}{8}$ answer.

525

By DECIMALS.

$$\begin{array}{r}
 20 \overline{) 15} \\
 \text{oz. } 1.75
 \end{array}$$

$$\begin{array}{r}
 4 \overline{) 11.25} \\
 12 \overline{) 10.9375} \\
 \hline
 £ 0.546875
 \end{array}$$

$$\begin{array}{r}
 4 \times 6 = 24 \\
 20 \overline{) 9.225} \\
 \hline
 \text{oz. } 327.61875
 \end{array}$$

As

$$\text{As } 1.75 : .546875 :: 327.61875$$

$$\begin{array}{r} .546875 \\ \hline 163809375 \\ 229333125 \\ 262095000 \\ 196571250 \\ 131047500 \\ 163809375 \end{array}$$

$$\begin{array}{r} 1.75) 179.16650390625 (102.380859 = \\ 416 \quad 102 \text{ l. } 7 \text{ s. } 7\frac{1}{4} \text{ d. as before.} \\ 666 \\ 1415 \\ 1503 \\ 1039 \\ 1640 \\ \hline 65 \end{array}$$

3. If 2 cwt. 3 qrs. 21 lb. of sugar cost 6l. 1s. 8d. what will 12 cwt. 2 qrs. cost at that rate?

By DECIMALS.

$$\begin{array}{r|l} 7 \times 421.3. & 12 \overline{) 8} \\ & 20 \overline{) 1.8} \\ \hline \text{Cwt. } 2.9375 & \text{£ } 6.083 \end{array} \quad \begin{array}{r|l} & 4 \overline{) 2} \\ \hline & 12.5 \end{array}$$

$$\begin{array}{r} \text{Cwt. } \text{£} \\ 2.9375 : 6.083 :: 12.5 \end{array}$$

$$\begin{array}{r} 12.5 \\ \hline 30418 \\ 73000 \end{array}$$

$$\begin{array}{r} 2.9375) 76.0418 (25.8865 = 25 \text{ l. } 17 \text{ s. } 8\frac{1}{4} \text{ d. the answer.} \\ 172916 \\ 26041 \\ 2591 \\ 198 \\ 15 \end{array}$$

By REDUCTION.

cwt. qrs. lb.	l. s. d.	cwt. qrs.
As 2 3 21	: 6 1 8	:: 12 2
<u>4</u>	<u>20</u>	<u>4</u>
11	121	50
<u>28</u>	<u>12</u>	<u>28</u>
329 lb.	1460	1400
	<u>1400</u>	

$$\begin{array}{r}
 \text{12)} \\
 329) 2044000 \text{ (6212}\frac{1}{2}\text{)} \\
 \underline{700} \quad 20) 517 - 8\frac{1}{2} \\
 420 \\
 \underline{910} \quad \text{£25 17 8}\frac{1}{2}, \text{ as before,} \\
 252 \\
 \underline{4} \\
 1008 \\
 \hline
 \end{array}$$

By VULGAR FRACTIONS.

$$\text{cwt. qr. lb.} \quad \text{cwt. l. s. d.} \quad \text{l.} \quad \text{cwt. qr. cwt.} \\
 2 \ 3 \ 21 = \frac{47}{16}, \quad 6 \ 1 \ 8 = \frac{73}{12}; \text{ and } 12 \ 2 = \frac{25}{2}.$$

$$\text{As } \frac{47}{16} : \frac{73}{12} :: \frac{25}{2}. \text{ First, } \frac{73}{12} \times \frac{25}{2} = \frac{1825}{24}.$$

$$\text{Then } \frac{47}{16} \left(\frac{1825}{24} \right) \left(\frac{3650}{141} \right) = 25 \text{ l. } 17 \text{ s. } 8\frac{1}{2} \text{ d. answer as before.}$$

When any one term in the proportion is an unit, the answer will sometimes be most readily obtained by practice, as in the two following examples.

4. If I give 5 s. 4 d. for one ounce of silver, what must I pay for $32\frac{1}{2}$ ounces at that rate?

$$\begin{array}{r} \text{s. d. s.} \\ 5 \text{ } 4 = 5.8; \text{ and } 32\frac{1}{2} \text{ oz.} = 32.5 \text{ oz.} \\ \text{oz. s. oz.} \\ 1 : 5.8 :: 32.5 \\ \hline 5.8 \\ 9) 975 \\ \hline 1088 \\ 1625 \\ \hline 173.38 \\ \hline \end{array} = 8 \text{ l. } 13 \text{ s. } 4 \text{ d.}$$

By PRACTICE.

$$\begin{array}{r} \text{s. d.} \\ 1\frac{1}{2}) 5 \text{ } 4 \\ \hline 4 \\ \hline 1 \text{ } 4 \\ \hline 8 \text{ } 10 \text{ } 8 \\ \hline 2 \text{ } 8 \\ \hline \end{array}$$

Answer, £ 8 13 4

5. If a silver tankard, weighing 21 ounces, cost 5l. 19s. what is that an ounce?

$$\begin{array}{r} \text{oz. l. s. oz.} \\ 21 : 5 \text{ } 19 :: 1 \\ \hline 20 \\ 21) 119(5 \text{ s. } 8 \text{ d. the answer.} \\ \hline 14 \\ 12 \\ \hline 168 \end{array}$$

As before, £ - 5 8

6. If a piece of cloth cost 10l. 16 s. 8 d. I demand how many yards it contains, the ell English being worth 8s. 4 d.?

$$\begin{array}{r} 12 \text{ } 4 \\ 20 \text{ } 8.8 \\ \hline \text{£ } .418 = 8 \text{ } 4 \text{ } 1\frac{1}{4} = 1.25 \end{array}$$

$$\begin{array}{r} 12 \text{ } 8 \\ 20 \text{ } 16.8 \\ \hline \text{£ } 10.88 = 10 \text{ } 16 \text{ } 8 \end{array}$$

As .418 : 1.25 :: 10.88

$$\begin{array}{r} 1.25 \\ 5416 \\ 21666 \\ \hline 108333 \text{ yds.} \\ .418) 13.5416 (32.5 = 32\frac{1}{2} \text{ yds. the answer.} \\ \hline 41 \text{ } 13.541 \\ .375) 12.1875 \\ \hline 937 \\ \hline 1875 \\ \hline \end{array}$$

It is to be observed, that there may be superfluous terms in a question, which must be omitted; as the 12 months in the next question.

7. If 100 l. in 12 months gain 4 l. 10 s. what will 74 l. 10 s. gain in the same time, at the same rate of interest?

l.	l. s.	l. s.
100	: 4 10	:: 74 10
20	20	20
<hr/>		
2000	90	1490
<hr/>	<hr/>	90

2000) 134100 ($67.0\frac{1}{2} = 3\text{ l. } 7\text{ s. } -\frac{1}{2}\text{ d. } \frac{2}{3}$, answer.

$$\begin{array}{r}
 100 \\
 12 \\
 \hline
 1200 \\
 4 \\
 \hline
 4800 \\
 \hline
 800
 \end{array}$$

Sometimes the analogy or proportion will not bear, until some operation in addition, subtraction, multiplication, or division, are performed; or, perhaps, an operation in one or more of those rules may be required after the proportion, in order to find out the number sought, as in some of the following examples.

8. If 19 yards of yard-wide stuff exactly line 14 yards of silk of another breadth; how many yards of the latter will line 184 pieces of the former, each piece holding $28\frac{1}{2}$ yards?

$$\begin{array}{r}
 184 \\
 28\frac{1}{2} \\
 \hline
 1472 \\
 368 \\
 92
 \end{array}$$

19 : 5244 :: 14

19) 73416 (3864 yards, the answer.

$$\begin{array}{r}
 164 \\
 121 \\
 76 \\
 0
 \end{array}$$

9. If

9. If $24\frac{1}{2}$ lb. of raisins cost 9s/ $2\frac{1}{4}$ d. what will 18 fraills cost, each weighing 3 qrs. $19\frac{1}{4}$ lb.?

4 2	4 1	4 1
4 x 7 24.5 = 6.125	12 2.25	28 19.25
4 .875	20 9.1875	4 3.6875
Cwt. .21875 = $24\frac{1}{2}$ lb.	£ .459375	C. .921875

.921875
18

7375000
921875

1.

.21875 : .459375 ::

16.59375
573954

6637500
829688
149344
4978
1161
83

.21875) 7.622754 (38.846875

106025	20
185254	16.937500
10254	12
1504	11.250000
191	4
16	1.000000

Answer, 34l. 16s. $11\frac{1}{4}$ d.

10. The globe of the earth, under the line, is 360 degrees in circumference; each degree $69\frac{1}{2}$ miles; and this body being turned on its own axis, in the syderal day, or 23 hours, 56 minutes; at what rate an hour are the inhabitants of Bencoolen, situate in the midst of the burning zone, carried from west to east, by this rotation?

$$\begin{array}{r}
 360 \\
 69\frac{1}{2} \\
 \hline
 \text{h. } 3240 \\
 23 \text{ } 56 \text{ } 216 \\
 60 \text{ } 180 \\
 \hline
 1436 : 25020 :: 60 \\
 60 \\
 \hline
 \text{Miles. F. P.} \\
 1436) 1501200 (1045 \text{ } 3 \text{ } 9\frac{167}{11}, \text{ the answer,} \\
 6520 \\
 7760 \\
 \hline
 580 \\
 \times 8 \\
 \hline
 4640 \\
 .332 \\
 \times 40 \\
 \hline
 13280 \\
 \hline
 354
 \end{array}$$

11. A factor bought 72 pieces of Holland, which cost 537 l. 12 s. at 5 s. 4 d. per ell Flemish; I demand how many yards there were in all, and how many ells English in each piece?

$$537 \text{ l. } 12 \text{ s.} = 537.6 \text{ l.} \dots 5 \text{ s. } 4 \text{ d.} = .28 \text{ l. and } \frac{1}{4} \text{ yd.} = .75 \text{ yd.}$$

$$\begin{array}{r}
 \text{l. } \text{yd.} \text{ } \text{l.} \\
 .28 : .75 :: 537.6 \\
 75
 \end{array}$$

$$\begin{array}{r}
 26880 \\
 37632 \\
 \hline
 .28) 403.200 \left| \begin{array}{l} 72) \\ 1512 \text{ yards in all.} \\ 21 \text{ yards in a piece.} \end{array} \right. \\
 2 \quad 40320 \\
 \hline
 .24) 362.88 \\
 122 \\
 28 \\
 48 \\
 \hline
 1
 \end{array}$$

$$1.25) 21.00 (16 \text{ ells English, and } 1 \text{ yd. in a piece.}$$

12. A factor bought a certain quantity of tabby and brocade, which together cost him 126l. 14s. 10d.: the quantity of tabby he bought was 48 yards, at 4s. 4d. per yard, and for every two yards of tabby he had five of brocade; how many yards of brocade had he, and what did it cost him a yard?

$\frac{1}{3}$	48				
$\frac{1}{12}$	9	12		l.	s.
	-	16		126	14
				10	8
£					-
	10	8	price of the tabby.	116	16
				10,	price of brocade.

Then as 2 : 5 :: 48

$$\begin{array}{r} 5 \\ 2 \overline{) 240} \end{array}$$

120 yards of brocade.

120) 116l. 6s. 10d. (19s. 4 $\frac{1}{2}$, $\frac{1}{3}$ qr. the answer.

$$\begin{array}{r} \times 20 \\ \hline 2326 \\ 46 \\ \times 12 \\ \hline 552 \\ 82 \\ \times 4 \\ \hline 328 \\ 88 \\ \hline \end{array}$$

13. If I sell 24 yards of Holland for 10l. 10s.; how many ells Flemish shall I sell for 283l. 17s. 6d. at that rate?

$$\begin{array}{r} 3 \overline{) 24} \\ 8 \end{array}$$

32 ells Flemish = 24 yards.

As

l. s. ell F. l. s. d.
As 10 10 : 32 :: 283 17 6

20	20
<hr/> 210	<hr/> 5677
12	12
<hr/> 2520	<hr/> 68130

32

13626

20439

2520) 2180160 (865 $\frac{1}{2}$ ells Flemish, the answer.

1641

1296

360

14. There are two numbers, 75 is the less, to which the greater is in proportion as 8 to 5; what is their sum, and the product of their sum and difference, the difference and product of their squares, and the sum of the squares of their two quotes, the greater divided by the less, and again the less by the greater?

As 5 : 8 :: 75 : 120, the greater number.

75 + 120 = 195, their sum.

120 - 75 = 45, difference.

195 × 45 = 8755, product of their sum and differ.

14400 - 5625 = 8775, diff. of the sq. of their sum and dif.

14400 + 5625 = 20025, sum of those squares.

75) 120 (1.6, quote of the greater divided by the less.

1.6 × 1.6 = 2.56, its square.

120) 75.00 (.625, quote of the less divided by the greater.

.625 × .625 = .390625, its square.

2.56 × .390625 = 2.950625. Q. E. F.

15. There are two numbers more, the greater is 224, bearing proportion to the other, as 8 to 7; what is the square of their sum, difference, and either quote; what is the result of the square of the sum of their difference, added to the product of their sum and difference?

8 : 7 :: 224 : 196, lesser number.

$224 + 196 = 420 \times 420 = 176400$, square of their sum;

$224 - 196 = 28 \times 28 = 784$, square of their difference.

$\frac{224}{28} \times \frac{196}{28} = \frac{2}{1} \times \frac{14}{1} = 1 \frac{1}{2}$, sq. of the quote of the gr. ÷ less.

$\frac{224}{28} \times \frac{196}{28} = \frac{2}{1} \times \frac{14}{1}$, sq. of the quote of the less divided by the gr.

$420 \times 28 + 28 = 11788$, product of their sum, and difference added to their difference.

Lastly, $11788 \times 11788 = 138956944$. Q. E. F.

16. In a series of proportional numbers, the first is 5, the third 8, and the product of the second and third 78.4; what is the difference of the second and fourth?

8) 78.4 (9.8, second.

Then $5 : 9.8 :: 8 : 15.68$, fourth.

$\therefore 15.68 - 9.8 = 5.88$, the answer.

17. A May-pole 30 feet 11 inches long, at a certain time of day, will cast a shadow 98 feet 6 inches long; I would hereby find the breadth of a river, that running within $20\frac{1}{2}$ feet of the foot of a steeple 300 feet 8 inches high, will at the same time throw the extremity of its shadow 30 feet 9 inches beyond the stream?

F. In. Feet. Shad. Shadow.

50 11 = 50.918 : 98.5 :: 30.9 : 581.6515.

$20.5 + 30.75 = 51.25$.

Therefore $581.6515 - 51.25 = 530.4015 = 530$ f. 4.818 in. the answer required.

18. Suppose the sea allowance for the common men to be 5 pounds of beef, and 3 pounds of biscuit a day, for a mess of four people; and that the price of the first barrelled be to the king $2\frac{1}{4}$ d. per pound, and of the second $1\frac{1}{2}$ d; such was the ship's company, that their flesh cost the government 12 l. 12 s. per day; pray what did they pay for their bread a week?

lb.

d.

5 beef value $11\frac{1}{4} = .046875$ } per day, { .328125 } per week.
3 biscuit $4\frac{1}{2} = .01875$ } or { .13125 } per week.

12 l. 12 s. = 12.6 l. per day, or 88.2 l. per week.

lb. worth lb. worth

beef. biscuit. beef. biscuit.

$.328125 : .13125 :: 88.2 : 35.28 = 35$ l. 5 s. $7\frac{1}{4}$ d. answer.

19. In

19. In the year 1582, pope Gregory reformed the Julian kalendar; ordaining, that as the year is found to consist only of 365 days, 5 hours, and about 49 minutes, in order to prevent the inconveniencies of carrying the account of time too forward, by taking the solar year at 365 days and 6 hours full, which in a series of years must bring Lady-day to Michaelmas, that the christian states for the future should drop 3 days in account every 400 years; that is to say, for each of the first three centuries in that space of time, the intercalary day in February should be omitted; but retained as formerly in the last or fourth century, beginning with the year 1600, when 10 whole days were sunk at once: by which artifice the variation of time will not, at least for a long space, be very considerable. According to this regulation, it is required to know in what year of Christ the new stile, as it is called, will be 20 days, as now it is only 11, before the old stile, which makes no such allowance?

20 — 11 = 9 days to be sunk.

D. Y. D.

As 3 : 400 :: 9 days : 1200 years to come.

∴ 1200 + 1700 = 2900, the year required.

20. If the scavenger's rate, at $1\frac{1}{2}$ d. in the pound comes to 6s. $7\frac{1}{2}$ d. where they ordinarily assess $\frac{2}{3}$ of the rent; what will the king's tax for that house be, at 4s. the pound, rated at the full rent?

6s. $7\frac{1}{2}$ d. = $\frac{13}{8}$ l. ∴ $1\frac{1}{2}$ d. = $\frac{1}{160}$ l. ∴ 4s. = $\frac{1}{10}$ l.

Then $\frac{2}{3}$ of $\frac{1}{160}$ ($\frac{1}{120}$).

As $\frac{1}{160}$: $\frac{1}{120}$:: $\frac{1}{3}$: $\frac{1}{2}$ = 13 l. 5 s. the answer required.

21. Agreed for the carriage of $2\frac{1}{2}$ tons of goods, three miles wanting $\frac{1}{10}$, for $\frac{1}{3}$ of $\frac{3}{4}$ of a guinea; what is that per hundred for a mile?

$\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{3}{10}$ = $\frac{9}{400}$; and $2\frac{1}{2}$ miles = $\frac{5}{2}$.

Then as $\frac{5}{2}$ tons : $\frac{9}{400}$:: $\frac{1}{10}$: $\frac{9}{40000}$ l.

And $\frac{9}{40000}$ ($\frac{9}{40000}$) l. = $\frac{37}{100}$ of a farthing, or little more than $\frac{1}{2}$ a farthing, the answer required.

22. A father dying left his son a fortune, $\frac{1}{8}$ of which he ran through in six months; $\frac{2}{3}$ of the remainder held him a twelvemonth longer, at which time he had bare 348 l. left; pray what did his father bequeath him?

$$\frac{16}{16} - \frac{3}{16} = \frac{13}{16} \text{ remaining at the end of six months.}$$

$$\frac{2}{3} \text{ of } \frac{13}{16} = \frac{13}{24} = 696 - -$$

$$\text{For } \frac{1}{4} \text{ of } \frac{13}{16} = \frac{13}{64} = 348 - - \text{ by the question.}$$

$$\frac{13}{24} : \frac{3}{16} :: \frac{696}{1} : 240 \text{ } 18 \text{ } 5\frac{1}{2}$$

$$\underline{1284 \text{ } 18 \text{ } 5\frac{1}{2}}, \text{ the answer required.}$$

23. A person dying, left his wife with child, and making his will, ordered, that if she went with a son, $\frac{2}{3}$ of the estate should belong to him, and the remainder to his mother; and if she went with a daughter, he appointed the mother $\frac{2}{3}$, and the girl $\frac{1}{3}$; but it happened that she was delivered both of a son and daughter, by which she lost in equity 2000 l. more than if it had been only a girl; what would her dowry have been, had she only had a son?

As the son was to have twice as much as the mother; and the mother twice as much as the daughter, let the estate be divided as follows, viz. $4 + 2 + 1 = 7$, the whole estate, so that as she had both a son and a daughter, the mother must have but $\frac{2}{7}$ of the whole estate; whereas, had it been only a daughter, she would have had $\frac{2}{3}$.

$$\frac{2}{7} = \frac{6}{21}, \text{ and } \frac{2}{3} = \frac{14}{21}. \therefore \frac{14}{21} - \frac{6}{21} = \frac{8}{21} = 2000 \text{ l.}$$

$$\frac{8}{21} : \frac{2000}{1} :: \frac{1}{3} : \frac{14000}{8} = 1750 \text{ l. the answer.}$$

24. A younger brother received 2200 l. which was just $\frac{5}{12}$ of his elder brother's fortune; and $3\frac{1}{2}$ times the elder's money was half as much again as the father was worth; what was that?

$$\frac{5}{12} \Big) \frac{2200}{1} \Big(\frac{26400}{5} = 5280 \text{ l. = elder brother's fortune.}$$

$$5280 \times 3\frac{1}{2} = 16500.$$

$$1\frac{1}{2} = \frac{3}{2} : \frac{16500}{1} :: \frac{1}{4} : \frac{33000}{5} = 11000 \text{ l. father's fortune.}$$

25. A person making his will, gave to one child $\frac{1}{3}$ of his estate, to another $\frac{1}{4}$; and when these legacies came to be

be paid, one turned out 540l. 10s. more than the other: what did the testator die worth?

$$\frac{11}{39} = \frac{330}{1170}, \text{ and } \frac{19}{30} = \frac{741}{1170}. \text{ Then } \frac{741}{1170} - \frac{330}{1170} = \frac{411}{1170}$$

$$= 540 \text{ l. } 10 \text{ s.} = \frac{1081}{2} \text{ l. } \frac{411}{1170} : \frac{1081}{2} :: \frac{1264770}{822}$$

$$= 1538 \text{ l. } 12 \text{ s. } 11 \frac{1}{4} \text{ d. } + \frac{89}{137} \text{ q, the answer.}$$

26. If $\frac{1}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of a ship be worth $\frac{1}{13}$ of $\frac{6}{7}$ of $\frac{11}{13}$ of the cargo, valued at 12000l.; what did both ship and cargo stand the owner in?

$$\frac{3}{7} \text{ of } \frac{4}{5} \text{ of } \frac{7}{8} = \frac{3}{10}, \text{ and } \frac{1}{9} \text{ of } \frac{6}{7} \text{ of } \frac{11}{13} = \frac{22}{273}$$

$$\text{Then } \frac{3}{10} : \frac{22}{273} :: \frac{12000}{1} : \frac{880000}{273} = 3223 \text{ l. } 8 \text{ s. } 10 \frac{1}{2} \text{ d. } \frac{45}{91}$$

$$\therefore 3223 \text{ l. } 8 \text{ s. } 10 \frac{1}{2} \text{ d. } \frac{45}{91} + 12000 \text{ l.} = 15223 \text{ l. } 8 \text{ s. } 10 \frac{1}{2} \text{ d.}$$

$$\frac{45}{91} \text{ the answer.}$$

27. In some parishes in the country they take off 3l. a year in 17 from the rents, in assessing the farmers; what will the landlord receive net out of a farm of 140l. a year in those places, when the king's tax is as now 4s. in the pound?

$$4 \text{ s.} = .2 \text{ l. As } 17 : 3 :: 140 : 24.706 \text{ l., abatement.}$$

$$\text{Then } 140 \text{ l.} - 24.706 \text{ l.} = 115.294 \text{ l.}$$

$$\text{Also } 115.294 \text{ l.} \times .2 = 23.0588 \text{ l., tax.}$$

$$\therefore 140 \text{ l.} - 23.0588 \text{ l.} = 116.9412 \text{ l.} = 116 \text{ l. } 18 \text{ s. } 10 \text{ d. and.}$$

28. If I leave Exeter at ten o'clock on Tuesday morning for London, and ride at the rate of two miles an hour without intermission; you set off from London for Exeter at six the same evening, and ride three miles an hour constantly; the question is, whereabouts on the road you and I shall meet, if the distance of the two cities be 130 miles?

$$8 \times 2 = 16 \text{ miles, I had travelled before you set out.}$$

$$130 - 16 = 114; \text{ and } 2 + 3 = 5 \text{ miles; both go in 1 hour.}$$

$$\text{Then } 5 : 1 :: 114 : 22 \frac{2}{3} \text{ hours, they will meet.}$$

$$22 \frac{2}{3} \times 3 = 68 \frac{2}{3}$$

$$\text{And } 28 \frac{1}{2} \times 2 + 16 = 61 \frac{1}{2} \text{ miles } \left\{ \begin{array}{l} \text{distant from London.} \\ \text{distant from Exeter.} \end{array} \right.$$

29. A sets out from London to Lincoln, at the very same time that B sets forward for London from Lincoln, distant

100 miles; at eight hours end they meet on the road, and it then appeared that A had ~~rid~~ ^{travell'd} ~~a~~ ⁷/₈ miles an hour more than B; at what rate an hour did each of them travel?

hours. miles. hour. miles.

8 : 100 :: 1 : $12\frac{1}{2}$, both travelled,

Then $12\frac{1}{2} - 2\frac{1}{2} = 10$, and 2) 10 (5 miles, B rode.

And $5 + 2\frac{1}{2} = 7\frac{1}{2}$ miles, A rode an hour.

30. A reservoir for water has two cocks to supply it, by the first it may be filled alone in 44 minutes, by the second in just an hour; and it hath a discharging cock, by which it may, when full, be emptied in half an hour: now suppose these three cocks, by accident, should all of them be left open, and the water should chance to come in; what time, supposing the influx and efflux of the water to be always alike, would this cistern be in filling?

In one minute is filled by $\left\{ \begin{array}{l} 1st \frac{1}{44} = \frac{15}{1552} \\ 2d \frac{1}{60} = \frac{16}{1552} \end{array} \right\}$ of the cistern.

Also $\frac{1}{60} + \frac{1}{44} = \frac{16}{1552}$ of the cistern filled in a minute, both running.

In one minute runs out $\frac{1}{30} = \frac{22}{1552}$ of the whole cistern.

And $\frac{16}{1552} - \frac{22}{1552} = \frac{14}{1552} = \frac{1}{112}$ fills in one minute, all being open.

$\frac{1}{112} : 1 :: 1 : 112$ minutes = $2\frac{1}{2}$ hours, the answer required.

31. A can do a piece of work in 10 days, B alone in 12; set them both about it together, in what time will it be finished?

A $\frac{1}{10} = \frac{12}{120}$, B $\frac{1}{12} = \frac{10}{120}$ of the work in 1 day.

Then $\frac{12}{120} + \frac{10}{120} = \frac{22}{120}$ 1 day :: 1 work : $\frac{120}{22} = 5\frac{5}{11}$ days, the answer.

32. B and C together can build a boat in 18 days; with the assistance of A, they can do it in 11 days; in what time will A do it by himself?

B + C can perform $\frac{1}{18} = \frac{10}{180}$ of the work in one

A + B + C can perform $\frac{1}{11} = \frac{18}{198}$ day.

Then $\frac{18}{198} - \frac{10}{180} = \frac{1}{198}$, A can perform of the whole work in one day.

$\frac{1}{198} : 1 :: 1 : 198$ days, the answer.

33. If A alone can do a piece of work in 10 days, A and B together in seven days; in what time can B do it alone?

A + B can do $\frac{1}{7} = \frac{10}{70}$; A alone $\frac{1}{10} = \frac{7}{70}$ of the work in one day.

Then $\frac{1}{70} - \frac{7}{70} = \frac{3}{70}$, B's day's work.

$\therefore \frac{3}{70} : 1 \text{ day} :: 1 \text{ work} : \frac{70}{3} = 23\frac{1}{3} \text{ days}$, the answer required.

34. X, Y and Z can, working together, compleat a staircase in 12 days; Z is man enough to do it alone in 24 days, and X in 34; in what time could Y get it done by himself?

X $\frac{1}{34} = \frac{12}{408}$, Z $\frac{1}{24} = \frac{17}{408}$, and X + Y + Z do $\frac{1}{12} = \frac{34}{408}$, all working one day.

Then $\frac{12}{408} + \frac{17}{408} = \frac{29}{408}$, done in one day by X and Z working together.

And $\frac{34}{408} - \frac{29}{408} = \frac{5}{408}$, done in one day by Y alone.

$\therefore \frac{5}{408} : 1 \text{ day} :: 1 \text{ work} : \frac{408}{5} = 81\frac{3}{5} \text{ days}$, the answer.

35. Three workmen can do a piece of work in certain times, viz. A can do it in three weeks, B can do thrice the work in eight weeks, and C five times in 12 weeks; in what time can they finish it jointly?

Newton's Universal Arithmetic.

A can do $\frac{1}{3} = \frac{12}{36}$
 B - - $\frac{3}{8} = \frac{16}{36}$
 C - - $\frac{5}{12} = \frac{15}{36}$ } their sum $\frac{43}{36} = \frac{2}{1}$ work, all working together one week.

1 week = 6 working days, and 1 day = 12 working hours.

$\therefore \frac{2}{1} \text{ work} : 6 \text{ days} :: 1 \text{ work} : \frac{43}{2} \text{ days} = 21\frac{1}{2} \text{ days}$, the answer.

36. If a cardinal can pray a soul out of purgatory, by himself, in an hour, a bishop in three, a priest in five, and a friar in seven; in what time can they pray out three souls, all praying together?

Palladium.

While the cardinal prays 1 = $\frac{105}{105}$
 The bishop - - - $\frac{1}{3} = \frac{35}{105}$
 The priest - - - $\frac{1}{5} = \frac{21}{105}$
 And the friar - - - $\frac{1}{7} = \frac{15}{105}$ } their sum $\frac{176}{105}$, in an hour together.

fouls. hour. fouls. hours. hour. / //

$\therefore \frac{176}{105} : 1 :: 3 : \frac{315}{176} = 1\frac{47}{23} = 23\frac{2}{23}$, the answer.

37. I am dispatched on a commission from London to Edinburgh, distant by computation say 350 miles, and my rout is settled at 22 miles a day; you four days after are sent after me with fresh orders, and are to travel 32 miles a day; whereabouts on the road shall I be overtaken by you?

$$22 \times 4 = 88 \text{ miles you have travelled before I set out.}$$

$$32 - 22 = 10 \text{ miles I gain each day of you.}$$

$$10 : 1 :: 88 : 8.8 \text{ days.}$$

$$\text{Then } 8.8 \times 32 = 281.6.$$

$\therefore 350 - 281.6 = 68.4 = 68 \text{ miles, } 3 \text{ furlongs, } 8 \text{ poles, on this side Edinburgh.}$

38. If the sun moves every day one degree, and the moon thirteen; and at a certain time the sun be at the beginning of Cancer, and in three days after the moon in the beginning of Aries; the place of their next following conjunction is required? *Newton's Universal Arithmetic.*

$$30 \times 3 = 90 \text{ degrees, from the first of Aries to Cancer.}$$

$$90 + 3 = 93 \text{ degrees, the sun before the moon.}$$

$$13 - 1 = 12 \text{ degrees, the moon gains in one day.}$$

$$12 : 1 :: 93 : 7\frac{1}{2} \text{ days, in which time the sun will be overtaken.}$$

$$\therefore 7\frac{1}{2} + 3 = 10\frac{1}{2} \text{ degrees of Cancer, the answer.}$$

39. If the half of fifteen be seven,
What is the fourth of eleven?

$$\text{As } \frac{15}{2} : 7 :: \frac{11}{4} : \frac{77}{30} = 2\frac{1}{6}, \text{ the answer required.}$$

In mechanics, a lever of the second order is, when the power acts at one end, the prop fixed directly at the other, and the weight somewhere between them.

In this order of levers, their force are in a contra-proportion to their lengths.

40. If a lever be 100 inches long, what weight, lying $7\frac{1}{2}$ inches from the end, resting on a pavement, may be moved with the force of 168 lb, lifting at the other end of the lever?

$$100 - 7.5 = 92.5, \text{ longest end.}$$

$$\text{inches. lb. inches. lb.}$$

$$\therefore 168 : 92.5 :: 2072, \text{ the answer.}$$

In a lever of the third order, the prop is planted at one end of the bar, the weight at the other end, and the moving force somewhere between.

41. A water-wheel turns a crank, working three pump-rods, fixed just six feet from the joint or pin; by which their several levers, each nine feet in length, are fastened, for the sake of the intended motion, at one end; the suckers of the pumps being worked by the other, shews them to be levers of the third order: now I would know what the length of the stroke in each of the barrels will be, if the crank be made to play just nine inches round its center?

$9 \times 2 = 18$ inches, the diameter of the crank.
 feet. inch. feet. inch.

$6 : 18 :: 9 : 27$, the length of the stroke.

42. With what force ought that water-wheel to be driven, which, circumstanced as in the last question, raises three cubic feet of water at every revolution of the wheel, each experimentally weighing $62\frac{1}{4}$ lb. averdupoise; the friction of the machine rejected?

$62\frac{1}{4} \text{ lb.} \times 3 = 187\frac{1}{4} \text{ lb.} =$ weight of 3 solid feet of water.

Therefore $6 : 187\frac{1}{4} :: 9 : 281\frac{1}{4} \text{ lb.} =$ force required, rejecting the friction. Q. E. F.

The magnitude of spheres are directly in proportion to the cubes of their diameters.

43. If the diameter of the earth is 7970 miles, of the moon 2170 miles, supposing them to be exact spheres, as they are not, what comparison is there between them in point of magnitude?

Cube of the earth's diameter = 506261573000.

Cube of that of the moon = 10218313000.

$\therefore 10218313 : 506261573 :: 1 : 49.5445$. Q. E. F.

The less porous a body is, the greater is its density.

44. The compactness or density of the moon is to that of the earth, as $132\frac{1}{2}$ is to 100; what proportion then is there between the quantity of matter in the earth, and that in the moon?

The earth in the foregoing question is found to be 49.5445 times bigger than the moon.

1235 : 100 :: 49.5445 : 40.117. Q. E. F.

That is, the earth contains 40.117 times more matter.

The velocity of sound is found by experiment to be uniform; viz. about 1150 feet in one second of time, if it meets with nothing to retard or obstruct its motion.

45. If I see the flash of a piece of ordnance fired by a vessel in distress at sea, which happens, we will suppose, nearly at the instant of its going off, and hear the report a minute and three seconds afterwards; how far is she off, reckoning for the passage of sound as before?

1 minute 3 seconds = 63 seconds.

As 1 second : 1150 feet :: 63 seconds : 72450 feet = 13 miles, 5 furlongs, 30 poles, 5 yards, the answer required.

46. How long after firing the warning-gun in Hyde-Park, may the same be heard at Highgate, taking the distance at $5\frac{1}{2}$ miles?

$5\frac{1}{2}$ miles = 29920 feet.

Then 1150 feet : 1 second :: 29920 feet : 26 seconds, $1\frac{1}{3}$ third, the answer required.

47. Suppose a maid carrying apples to market was met by three boys, and that the first took half that she had, but returned 10; that the second took one-third that she then had, but returned two; lastly, the third took away half that she had left, but returned her one; and when she had got clear, she had 12 apples left; what number of apples had she at first?

Emerson's Arithmetic.

First $12 - 1 = 11$; and $11 \times 2 = 22$, before she met the last boy.

Also $22 - 2 = 20$; and $\frac{2}{3} : 20 :: \frac{1}{3} : 30$, the number she had before she met with the second boy; and before the first boy returned her 10, she had but 20, equal to what the boy took.

$20 \times 2 = 40$ apples, at the first. Q. E. F.

Proof $40 \div 2 = 20$; also $20 + 10 = 30$, when she met the second boy.

Likewise $30 \div 3 = 10$; and $30 + 2 = 32$, when met by the last.

Lastly, $32 \div 2 = 16$; and $16 + 1 = 17$ left, per question.

48. A tradesman begins the world with 1000 l. and finds that he can gain 1000 l. in 5 years by land trade alone; and that he can gain 1000 l. in 8 years by sea trade alone; and likewise that he spends 1000 l. in $2\frac{1}{2}$ years by gaming; how long will his estate last, if he follows all three?

Emerson's Arithmetic.

$$\frac{1000}{5} = 200 \text{ l. gain by land trade in one year.}$$

$$\frac{1000}{8} = 125 \text{ l. gain by sea trade in one year.}$$

$$\underline{325 \text{ l. his whole gain.}}$$

$$\frac{1000}{2\frac{1}{2}} = 400 \text{ l. lost by gaming in one year.}$$

Difference 75 l. loss at the year's end.

$\therefore 75 \text{ l.} : 1 \text{ year} :: 1000 \text{ l.} : 13\frac{1}{3} \text{ years, the answer.}$

49. A clock hath two hands or pointers; the first, A, goes round once in 12 hours; the second, B, once in an hour; now, if they both set forward together, in what time will they meet again?

Emerson's Arithmetic.

As A goes only $\frac{1}{12}$ of the circumference in an hour,

And B goes the whole, or $\frac{12}{12}$;

Then $\frac{1}{12} - \frac{1}{12} = \frac{1}{12}$, B gains in an hour.

$$\therefore \frac{1}{12} : 1 :: 1 : \frac{1}{12} = 1 \text{ } 5 \text{ } 27\frac{1}{12}$$

The velocity acquired by heavy bodies falling near the surface of the earth, is $16\frac{1}{2}$ feet in the first second; and as $16\frac{1}{2}$ feet are to the square of one second, or 1; so is the given distance, to the square of the seconds required.

Or by multiplying $16\frac{1}{2}$, the descent of a heavy body in one second of time, by as many of the odd numbers, beginning from unity, as there are seconds in the given time; viz, by 1 for the first, 3 for the second, 5 for the third, 7 for the fourth, &c. the sum total will give the space it hath passed.

50. Suppose a stone let go into an abyss, should be stopped at the end of the eleventh second after its delivery, what space would it have gone through?

$$1^2 : 16.083 :: 11 \times 11 = 121 : 1946.083. \text{ Q. E. F.}$$

Or,

Or,

about 16.083	1 = 16.083	} in the	1st	} seconds of time.
16.083 ×	3 = 48.250		2d	
	5 = 80.416		3d	
	7 = 112.583		4th	
	9 = 144.750		5th	
	11 = 176.916		6th	
	13 = 209.083		7th	
	15 = 241.250		8th	
	17 = 273.416		9th	
	19 = 305.583		10th	
	21 = 337.75		11th	

1946.083, as before.

51. If a stone be $19\frac{1}{2}$ seconds in descending from the top of a precipice to the bottom, what is the height of the same?

$$1^2 : 16.083 :: 19.5 \times 19.5 = 380.25 : 6115.6875.$$

∴ 6) 6115.6875 (= 1019 fathom, 1 foot, $8\frac{1}{2}$ inches. Q.E.F.)

52. If a hole could be bored through the center of the earth, in what time, after the delivery of a heavy body on its surface, would it arrive at its center?

The semidiameter of the earth 3980 miles = 21014400 feet.

$$16.083 : 1^2 :: 21014400 : 1306594.82.$$

seconds, min. " "

$$\sqrt{1306594.82} = 1135.554 = 18\ 55\ 33. \text{ Q. E. F.}$$

1. If the quantities of matter in any two or more bodies, put in motion, be equal, the forces wherewith they are moved will be in proportion to their velocities.

2. If the velocities of these bodies be equal, their forces will be directly as the quantities of matter contained in them.

3. If both the quantities of matter and the velocities be unequal, the forces with which bodies are moved, will be in a proportion compounded of the quantities of matter they contain, and of the velocities wherewith they move.

53. There are two bodies, the one contains 25 times the matter of the other (or is 25 times heavier) but the lesser moves with 1000 times the swiftness of the greater; in what proportion are the forces by which they are moved?

As

As 25 : 1000 :: 1 : 40, the less is moved with a force so much greater than the other.

54. There are two bodies, one of which weighs 100 lb. the other 60; but the lesser body is impelled by a force 8 times greater than the other; the proportion of the velocities, wherewith these bodies move, is required?

As 60 : 100 :: 1 : $1\frac{2}{3}$ = $\frac{5}{3}$.

So that the velocity of the less to the greater will be $\frac{3}{5} \times \frac{8}{1} = \frac{24}{5} = 4\frac{4}{5} : 1$.

So the velocity of the less to the greater will be, as $13\frac{1}{5}$ to 1, or as 40 to 3.

55. There are two bodies, the greater contains 8 times the quantity of the matter in the less, and is moved with a force 48 times greater; the ratio of the velocity of these two bodies is required?

If the forces were equal, the velocity of the lesser would be 8 times that of the greater.

But as the force the greater is moved with is 48 times that which moves the less,

As 8 : 48 :: 1 : 6; so the velocity of the less to that of the greater is as 1 to 6.

1. In comparing the motions of bodies, if their velocities be equal, the spaces described by them shall be in the direct proportion of the times in which they are described.

2. If the times be equal, then the spaces described will be as their velocities.

3. If the times and the velocities be unequal, the spaces will be in a proportion compounded of the times and velocities.

56. There are two bodies, one of which moves 40 times swifter than the other; but the swifter body has moved but one minute; whereas the other has been in motion two hours; the ratio of the spaces described by these two bodies is required?

In two hours are 120 minutes.

As 40 : 120 :: 1 : 3, so is the space the swifter hath moved to that of the slower.

57. Suppose one body to move 30 times swifter than another; as also the swifter to move 12 minutes, the other only

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only 1; what difference will there be between the spaces by them inscribed, supposing the last has moved 60 inches?

60 inches = 5 feet, moved by the second.

And $1 : 5 :: 30 \times 12 = 360 : 1800$, by the first.

$\therefore 1800 - 5 = 1795$ feet, the answer.

58. There are two bodies, one whereof has described 50 miles, the other only 5; but the first hath moved with five times the velocity of the second; what is the ratio then of the times they have been describing those spaces?

As $5 : \frac{50}{5} :: 1 : 2$; so that the first body hath been in motion double the time of the second.



SECTION II.

RECIPROCAL PROPORTION,

CALLED, THE

GOLDEN RULE; or, RULE OF THREE INVERSE.

RECIPROCAL proportion is, when of four numbers the third (viz. that which moves the question) beareth the same ratio or proportion to the first, as the second does to the fourth.

Therefore the less the third term is, in respect to the first, the greater will the fourth be in respect to the second.

And the greater the third term is, in respect to the first, the less will the fourth term be in respect to the second.

Therefore, observe that in any question in proportion, when MORE requires MORE, or LESS requires LESS, the terms are in direct proportion.

But if MORE requires LESS, or LESS requires MORE, then the terms will be in reciprocal proportion.

The same directions for stating the question are to be observed here as in direct proportion.

The question being truly stated, observe this general rule.

RULE.

R U L E.

Multiply the first and second terms together, and divide their product by the third term, the quotient will be the answer required.

1. If a penny white loaf ought to weigh 6 ounces and 12 drams ~~averdupoise~~, when wheat is sold at 6 s. 6 d. per bushel; what must it weigh, when wheat is sold at 4 s. the bushel?

$$\begin{array}{rcll} \text{s. d.} & \text{l.} & \text{oz. dr.} & \text{s. l.} \\ 6 & 6 = .325 & - & 6 \text{ } 12 = 6.75, \text{ and } 4 = .2. \\ \text{Then as } .325 & : 6.75 & :: .2 & \end{array}$$

$$\begin{array}{r} .325 \\ \hline 3375 \\ 1350 \\ \hline 2025 \end{array}$$

$$\text{.2) } 2.19375 \text{ (10.96875} = 10 \text{ } 15\frac{1}{2}, \text{ the ansr.}$$

Here it is plain, that the less the price of wheat, the bigger the loaf ought to be.

2. A general is besieged in a town, in which are 1569 soldiers, with provision of victuals for three months; how many must depart the garison, that the same victuals may last the remaining soldiers $7\frac{1}{2}$ months?

$$\begin{array}{rcll} & \text{mon. fold.} & \text{mon.} & \\ \text{Reciprocally } 3 & : 1569 & :: 7.5 & \end{array}$$

$$\begin{array}{r} 3 \\ \hline 7.5) 4707 \text{ (627, or 628, may stay.} \\ 207 \\ \hline 570 \\ \hline 45 \end{array}$$

Consequently $1569 - 627 = 942$, or at the least 941 must depart.

3. How many yards of ell-wide flannel is sufficient to line a cloak, containing $18\frac{1}{2}$ yards of camblet, $\frac{1}{4}$ yard wide?

$$\frac{3}{4} =$$

$\frac{3}{4} = .75$ yd. $18\frac{1}{4} = 18.75$ yds. $1\frac{1}{4} = 1.25$ yd.
Reciprocally, $.75 : 18.75 :: 1.25$

$$\begin{array}{r} .75 \\ \hline 94375 \\ 132125 \\ \hline \end{array}$$

yds. yds. qr. n.

1.25) 1415625 (11.325 = 11 1 $\frac{1}{2}$, the answer,

$$\begin{array}{r} 165 \\ 406 \\ 312 \\ 625 \\ \hline \dots \\ \hline \end{array}$$

4. How many yards of matting, that is $2\frac{1}{2}$ feet wide, will cover a floor that is 17 feet long, and 15 feet 3 inches broad?

$$\begin{array}{ccccccc} \text{feet.} & \text{in.} & & \text{yds.} & & \text{feet.} & \text{yds.} & & \text{feet.} & & \text{yd.} \\ 15 & 3 & = & 5.08\bar{3} & - - & 17 & = & 5.0 & - & 2\frac{1}{2} & = & .8\bar{3} \\ & & & \text{yds.} & & & & & & & & \end{array}$$

Reciprocally, $5.08\bar{3} : 5.0 :: .8\bar{3}$
 5.0

g) 30500

3488\$
2541\$6

$\frac{28.905}{8} = 3.613125$ yds. yds. ft. in.
 $.83 \cdot 28.905 \quad (34.56 = 34 \text{ I } 8\frac{2}{3}, \text{ the answer.})$
 $\quad \quad \quad 8 \cdot 28905$

.75) 26.0150

5. A borrowed of his friend B 250 l. for 7 months, promising to do him the like favour; some time after B hath an occasion for 300 l.; how long may he keep it to be made full amends for the favour?

Reciprocally,

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1. mon. 1.
Reciprocally, 250 : 7 :: 300

7

300) 1750 (5 months and 25 days, the ans. required.

250

× 30, days in a month.

75

6. A regiment of soldiers; consisting of 976 men, are to be new clothed, each coat to contain $2\frac{1}{2}$ yards of cloth, that is, $1\frac{1}{2}$ yards wide, and lined with shalloon $\frac{1}{2}$ yard wide; how many yards of shalloon will line them?

976
2 $\frac{1}{2}$

1952
488

Reciprocally, 2440 : 1.625 :: .875

1.625

1220
488

1464

244

.875) 3965.000 (4531.42857 = 4531 yds. qr. nls.
4650 the
2750 answer.

1250

3750

2500

7500

5000

6250

125

7. If a tailor can make a coat and waistcoat with three yards and three quarters of broad-cloth, of one yard and a half's

half's breadth: how many yards of stuff, of $\frac{1}{4}$ yard's breadth, will he require to fit the same person?

yd. br.

$$\frac{1}{2} = \frac{1}{4} : 3\frac{1}{2} = \frac{15}{4} :: \frac{1}{4} \text{ First } \frac{1}{4} \times \frac{15}{4} = \frac{15}{8}$$

$$\left(\frac{15}{8}\right) \frac{45}{3} = 9 \text{ yards, the answer required.}$$

There being three orders of levers, or three varieties, wherein the weights, props, or moving powers, may be differently applied to the vertic, or inflexible bar, in order to effect mechanical operations in a convenient manner.

A lever of the first order hath the power placed at one of its ends, and the weights to be raised is put at the other, and the fulcrum or prop somewhere between them.

In this order, the power applied at one end will be reciprocally proportional to the distances of those ends from the fulcrum, or point supported; or in the steelyards, as the distance of the weight from the point of suspension.

8. What weight will a fellow be able to raise, who presses with the force of a hundred and a half on the end of an equipoised handspike 100 inches long, which is to meet with a convenient prop exactly $7\frac{1}{2}$ inches above the other end of the machine?

$100 \rightarrow 7.5 = 92.5$, the longest end of the lever from the fulcrum.

inch. lb. inch. lb. cwt.

Reciprocally, $92.5 : 168 :: 7.5 : 207\frac{1}{2} = 18\frac{1}{2}$, the answer.

9. What weight, hung at 70 inches distance from the fulcrum of a steelyard, will equipoise a hogshhead of tobacco $9\frac{1}{2}$ cwt. freely suspended at two inches distance on the contrary side?

$$9\frac{1}{2} \text{ cwt.} = 1064 \text{ lb.}$$

in. lb. in. lb.

Reciprocally, $2 : 1064 :: 70 : 307\frac{1}{2}$, the answer.

The effects or degrees of light, heat, and attraction, are reciprocally proportional to the squares of their distances from the center whence they are propagated.

10. Suppose that in a room where two men, A and B are sitting there is a fire, from which A is three feet, and B six feet distant; it is required to find how much hotter it is at A's feet, than at B's?

Reciprocally,

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Reciprocally, $6 \times 6 = 36 : 1 :: 3 \times 3 = 9 : 4$; so that A's place is four times as hot as B's.

11. Supposing the earth to be 81000000 miles distant from the sun; I would know at what distance from him another body must be placed, so as to receive light and heat double to that of the earth?

$$81000000 \times 81000000 = 6561000000000000.$$

Recip. $1 : 6561000000000000 :: 2 : 3280500000000000$.
 $\therefore \sqrt{3280500000000000} = 57275849$ miles, the answer.

12. Mercury, the nearest of the planets to the source of heat, light and life, in our system, the sun is about 32 million of miles from him; and Saturn, the remotest of the planets, is usually distant about 777 millions of miles; what comparison or proportion is there between the solar influences on these two bodies?

$32 \times 32 = 1024$, and $777 \times 777 = 603729$, squares of distance, cyphers omitted.

Saturn. Mercury.

Recip. $603729 : 1 :: 1024 : 589\frac{59}{1024}$. The solar influence on Mercury to that of Saturn, is as $589\frac{59}{1024}$ to 1.

13. The distance between the earth and sun is accounted 81000000 of miles; the distance between Jupiter and the sun 424000000 of miles; the degrees of light and heat received by Jupiter, compared with that of the earth, is required?

$81 \times 81 = 6561$, and $424 \times 424 = 179776$, squares of their distances, the cyphers being omitted.

Recip. $179776 : 1 :: 6561 : 27.4$; so that the sun's influence on the earth to that on the planet Jupiter, is 27.4 to 1.

14. A certain body on the surface of the earth weighs 112 lb.; the question is, whether this body must be carried, that it may weigh but 10 lb.?

lb. sq. r. lb.

Recip. $112 : 1 :: 10 : 11.2$, square semidiameter.

Then $\sqrt{11.2} = 3.34664$, semidiameter of the earth from its center; or 9351 $\frac{1}{2}$ miles from its surface.

15. A GEOGRAPHICAL PARADOX.

There is a vast country in Ethiopia superior, to whose inhabitants the moon doth always appear to be most enlightened; when she is least enlightened; and to be least, when most; admitting the mean distance of the earth and moon's centers 240000 miles, in what proportion is this illumination?

Sun from the earth $81000000 + 240000 = 81240000$,
sun from a full moon.

$81000000 - 240000 = 80760000$ miles, the sun from a new moon.

$8124 \times 8124 = 65999376$ } squares of their different dis-
 $8076 \times 8076 = 65221776$ } tances, the cyphers omitted.

Recip: $65221776 : 1 :: 65999376 : .9882$, so that the proportion of light and heat a new moon hath to that of a full one is,

As 1 to .9882, or as 458329 to 452929, in whole numbers.

16. If a body weighs 16 ounces upon the surface of the earth, what will its weight be 50 miles above it, taking the earth's diameter at 7970 English miles?

$7970 \div 2 = 3985$ miles, the earth's semidiameter.

$3985 \times 3985 = 15880225$, its square; also $3985 + 50 = 4035$.

And $4035 \times 4035 = 16281225$.

Recip: $15880225 : 16 :: 16281225 : 15.02$, $9\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$ lbs.

It hath been found by experiment, that a pendulum 39.2 inches long, in our latitude, vibrates 60 times in one minute; and that the length of pendulums are to one another, as the square of the number of their vibrations made in the same space of time.

17. What is the length of that pendulum which swings half seconds, or vibrates 120 times in a minute?

Recip. $3600 : 39.2 :: 14400 : 9\frac{1}{2}$ inches. Q. E. F.

18. What difference will there be in the number of vibrations made by a pendulum of 6 inches long, and another of 12 inches long, in an hour's time?

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Reciprocally, $39.2 : 3600 :: \begin{cases} 12 : 11760 \\ 6 : 23520 \end{cases}$

$\sqrt{11760} = 108.444$ $\sqrt{23520} = 153.362$

Then $153.362 \times 60 = 9201.72$

And $108.444 \times 60 = 6506.64$

2695.08 Q. E. F.

In comparing the motions of bodies, the ratio or proportion between their velocities will be compounded of the direct ratio of the forces wherewith they are moved, and the reciprocal of the quantities of matter they contain.

19. The battering ram of Vespasian weighed, suppose 100000 pounds, and was moved, let us admit, with such a velocity, by strength of hands, as to pass through 20 feet in one second of time, and this was found sufficient to demolish the walls of Jerusalem; with what velocity must a bullet that weighs but 30 lb. be moved, in order to do the same execution?

Recip. $100000 : 20 :: 30 : 66666\frac{2}{3}$ feet, in one second.

20. A body weighing 20 lb. is impelled by such a force, as to send it 100 feet in a second; with what velocity would a body of 8 lb. weight move, if it were impelled by the same force?

Reciprocally, $20 : 100 :: 8 : 250$ feet. Q. E. F.



SECT. III.

COMPOUND PROPORTION;

OR, THE

RULE OF FIVE.

THE rule of five is so called, from having five numbers given to find a sixth; three of which five given numbers, are only conditional, or supposed: and the other two move the question.

I

All

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All questions in this rule include two in the rule of three, either both direct, or one in direct, and the other in reciprocal proportion; which so depend upon each other, that the answer of the first being made the middle term of the second, the fourth term of the second will be the final answer of the question.

Yet here observe, that many questions, though they may be wrought by two (or more) operations in the rule of three, cannot be answered by the rule of five.

In order to solve any question in the rule of five, observe the following directions.

Always place the three conditional terms in this order, let that number which is the principal cause of gain, loss, or action, &c. be put in the first place; that number which denotes the space of time, or distance of place, &c. be put in the second place; and that number which is the gain, loss, or action, &c. be put in the third place: that done, place the other two terms, which move the question, under those of the same name.

Then if the blank or term sought fall under the third place,

R U L E,

Multiply the three last terms together for a dividend, and the two first together for a divisor; the quotient arising from them will be the sixth term.

But if the blank or term sought fall under the first or second place,

R U L E II.

Multiply the first, second, and last terms together for a dividend, and the other two together for a divisor; the quotient arising from them will be the sixth term.

1. If the carriage of 5 cwt. 3 qrs. weight, 150 miles, cost 3l. 7s. 4d.; what must be paid for the carriage of 7 cwt. 2 qrs. 25 lb. weight, 64 miles, at the same rate?

cwt.	qrs.	lb.	l.	s.	d.	d.	cwt.	qrs.	lb.	lb.
3	3	= 644	3	7	4 = 808,	and	7	2	25 = 865.	
		lb.		miles.	d.					
		644		150	808					
		865		64						

808 × 865 × 64 = 44730880, dividend.

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$$644 \times 150 = 96600 \quad 447 \overset{12}{8} 80 (463.$$

$$\begin{array}{r} 6090 \quad \text{d.} \\ 2948 \ 20 \ 38 \ 7 \\ \hline 5080 \ \text{£} \ 1 \ 18 \ 7, \text{ the ans.} \end{array}$$

2. If 2 men can do $12\frac{1}{2}$ rods of ditching in $6\frac{1}{2}$ days; how many rods may be done by 18 men in 14 days?

men.	days.	rods.
2	6.5	12.75
18	14.	

$$12.75 \times 18 \times 14 = 3150, \text{ dividend.}$$

$$2 \times 6.5 = 13 \quad 3150 (242\frac{1}{13} \text{ rods, the answer.}$$

3. If a regiment of soldiers, consisting of 939, can eat up 351 quarters of wheat in 7 months; how many soldiers will eat up 1464 quarters in 5 months, at that rate?

soldiers.	months.	qrs. wheat.
939	7	351
	5	1464

$$939 \times 7 \times 1464 = 9622872, \text{ dividend.}$$

$$351 \times 5 = 1755 \quad 9622872 (5483\frac{2}{3} \text{ soldiers, the answer.}$$

$$\begin{array}{r} 8478 \\ 14587 \\ 5472 \\ \hline 9) 17737 (1993 \end{array}$$

4. If 30 men can perform a piece of work in 11 days; how many will accomplish another, four times as big, in one-fifth of the time?

men.	days.	work.
30	11	1
	11	4

$$30 \times 11 \times 4 = 1320, \text{ dividend.}$$

$$\frac{11}{5} \Big) \frac{1320}{1} \Big(\frac{660}{11} = 600 \text{ men, the answer.}$$

5. If 9 men in 21 days mow 108 acres of ground; in how many days will 5 men mow 72 acres, at the same rate of working?

9 men.

men.	days.	acres.
9	21	108
5		72

$9 \times 21 \times 72 = 13608$, dividend.
 $108 \times 5 = 540$ 13608 (25 $\frac{1}{2}$ days, the answer.

When the terms in proportion are more than 5, as may sometimes happen, the following rule of Mr. Emerson's may be useful.

R U L E.

1. Here, as in the single rule of three, put that term in to the second place, which is of the same denomination with that sought; and the terms of supposition one above another in the first place; also the terms of demand in the same order, one above another, in the third place; then the first and third of every row will be of one name, and must be reduced to the same denomination, viz. the lowest concerned.

2. Then proceed with each row, as with so many separate questions in the single rule of three, in order to find out the several divisors, using the second term in common for each of them; that is, in any row, say, If the first term gives the second, does the third require more or less? if more, mark the lesser extreme; if less, the greater for a divisor.

3. Multiply all these divisors together for a divisor, and all the rest of the numbers together for a dividend; the quotient is the answer, and of the same name with the second term.

4. To contract the work, when the same numbers are concerned in both divisor and dividend, throw them out of both; or divide any numbers by their greatest common divisor, and take the quotients instead of them.

6. If the carriage of 150 feet of wood, that weighs 3 stone a foot, comes to 31. for 40 miles; how much will the carriage of 54 feet of freestone, that weighs 8 stone a foot, cost for 25 miles?

• 150 feet.	31.	54 feet.
• 3 stone.		8 stone.
• 40 miles.		25 miles.

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$$\frac{54 \times 8 \times 25 \times 3}{150 \times 3 \times 40} = \frac{54 \times 1 \times 25 \times 1}{150 \times 1 \times 5} = \frac{84 \times 5}{150} = \frac{54}{30} = \frac{9}{5}$$

5) 9 (1 l. 16 s. the answer.

$$\begin{array}{r} 4 \\ \times 20 \\ \hline 80 \end{array}$$

Or by an arithmetical equation further infixed upon and explained in exchange.

• 180 feet.	3 l. . 14 feet.
• 3 flocs.	• 3 flocs.
• 40 miles.	• 78 miles.

Divide both the divisors and dividends by their greatest common measure, cancelling as you have done with them, and setting down the quotients, till you have brought the divisor and dividend to their lowest terms.

$$\begin{array}{r} 30 \\ 8 \\ 5 \\ \cdot \cdot \cdot 5) 9 (1 l. 16 s. the answer. \\ \hline 4 \\ \times 20 \\ \hline 80 \end{array}$$

7. If 248 men in $5\frac{1}{2}$ days, of 11 hours each, dig a trench of 7 degrees of hardness, and 232 $\frac{1}{2}$ yards long, 3 $\frac{1}{2}$ wide, and 2 $\frac{1}{2}$ deep; in how many days of 9 hours will 24 men dig a trench of 4 degrees of hardness, and 337 $\frac{1}{2}$ yards long, 5 $\frac{1}{2}$ wide, and 3 $\frac{1}{2}$ deep?

248 men.	$5\frac{1}{2}$ days.	24 • men.
11 hours.		9 • hours.
7 degrees.		4 • degrees.
• £ 232 $\frac{1}{2}$		£ 337 $\frac{1}{2}$
• 3 $\frac{1}{2}$ wide.		5 $\frac{1}{2}$ wide.
• 2 $\frac{1}{2}$ deep.		3 $\frac{1}{2}$ deep.

Now

Now I transpose all the divisors to the left, and all the dividends to the right-hand, freeing all the terms from fractions, by multiplying each side by the denominators.

$$\begin{array}{r}
 24 \\
 8 \\
 4 \\
 48 \\
 33 \\
 14 \\
 2 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 248 \\
 12 \\
 7 \\
 848 \\
 44 \\
 12 \\
 11 \\
 \hline
 \end{array}$$

Then dividing each side by all their common measures, and cancelling the numbers done withal.

$$\begin{array}{r}
 3 \\
 8 \\
 83 \\
 3 \\
 2 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 31 \\
 138 \\
 11 \\
 3 \\
 7 \\
 21 \\
 \hline
 \end{array}$$

Then $2 \times 2 = 4$, divisor; and $11 \times 7 \times 21 = 1617$, dividend.
 $\therefore 4) 1617 (404\frac{1}{4}$ days, the answer.

Those who want a further explanation of this method, may find it more fully treated upon in arbitration of exchange, which I had written some time before I had the perusal of Mr. Emerson's book.



S E C T. IV. COMPOUND PROPORTION;

OR, THE RULE OF THREE REPEATED.

ALL questions in the foregoing rule of five (as hath been before observed) may be resolved by two or more operations in the rule of three repeated; a few examples whereof we shall give: also several questions that cannot be

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solved by the rule of five, may be answered by two or more repetitions of the rule of three; variety whereof followeth.

1. If 2 men can do $12\frac{1}{2}$ rods of ditching in $6\frac{1}{2}$ days; how many rods may be done by 18 men in 14 days?

men. rods. men.

As 2 : $12\frac{1}{2}$:: 18

$12\frac{1}{2}$

2) 225 (112 $\frac{1}{2}$ rods.

Also $6\frac{1}{2}$ days = $\frac{13}{2}$: 112 $\frac{1}{2}$ rods = $\frac{225}{2}$:: $\frac{14}{1}$ days.

$\frac{225}{2} \times \frac{14}{1} = \frac{1575}{1}$. And $\frac{13}{2} \left(\frac{1575}{2} \right) \left(\frac{3150}{13} \right) = 242\frac{4}{11}$ rods, the answer.

2. If a regiment of soldiers, consisting of 939, can eat up 351 quarters of wheat in 7 months; how many soldiers will eat up 1464 quarters in 5 months, at that rate?

gra. wh. fold. qrs. wh.

Directly, 351 : 939 :: 1464 : $\frac{152744}{39}$.

Reciprocally, $\frac{7}{1}$: $\frac{152744}{39}$:: $\frac{5}{1}$: 5483 $\frac{2}{3}$ soldiers, the answer as before in the rule of five.

3. If 9 men in 21 days mow 108 acres of ground; in how many days will 5 men mow 72 acres, at the same rate of working?

acres. days. acres. days.

Directly, 108 : 21 :: 72 : 14

men. days. men.

Reciprocally, 9 : 14 :: 5 : 25 $\frac{1}{2}$ days, the answer.

N. B. The first question is what is generally called by authors the double rule of three direct; and the second and third, the double of three inverse.

4. By selling 240 oranges at five for 2d. half of which cost two a penny, and the other half three a penny, I evidently lost a groat; pray how comes that about?

ora. d. ora. d. s. ora. d. ora. d. s. d.

As 2 : 1 :: 120 : 60 = 5. Again, 3 : 1 :: 120 : 40 = 3 4.

Then 5 s. + 3 s. 4d. = 8 s. 4d. the cost.

ora. d. ora. d. s.

And 5 : 2 :: 240 : 96 = 8, consequently lost 4d.

5. If

5. If 12 apples be worth 21 pears, and 3 pears cost a halfpenny; what will be the price of fourcore and four apples?

$$\begin{array}{rcl} \text{pears. d.} & \text{pears. d.} & \\ 3 : .5 :: 21 : 3.5, & \text{price of 12 apples.} & \\ \text{apples. d.} & \text{apples. d.} & \text{s. d.} \\ 12 : 3.5 :: 84 : 24.5 = 2 \frac{1}{2}, & \text{the answer.} & \end{array}$$

6. A gay young fellow had 18200 l. left him by an old uncle, to whose memory he expended 3 per cent. of his whole fortune in a sumptuous funeral and monument; 9 per cent. of the remainder he made a present of to his cousins, forgotten for his sake by the old man; with $\frac{2}{7}$ of what was left he bought a fine seat; with $\frac{1}{7}$ of the residue a stud of horses; he squandered away 550 l. upon one mistress; and after he had lived after the rate of 2000 l. a year for 19 months together, he had both ruined his health, and impaired his fortune; pray at his death what was there left for his sister, who was his heir at law?

$$100 : 3 :: 18200 : 546, \text{ funeral and monument.}$$

$$18200 - 546 = 17654.$$

$$100 : 9 :: 17654 : 1588.86, \text{ cousins.}$$

$$17654 - 1588.86 = 16065.14.$$

$$16065.14 \times \frac{2}{7} = 4590.04, \text{ seat.}$$

$$16065.14 - 4590.04 = 11475.1.$$

$$8) 11475.1 (1434.3875, \text{ horses.}$$

$$11475.1 - 1434.3875 = 10040.7125.$$

$$12 : 2000 :: 19 : 3166.6, \text{ riotous living.}$$

$$+ 550, \text{ mistress.}$$

$$10040.7125 - 3716.6 = 6324.04583.$$

$$6324.04583 \text{ l.} = 6324 \text{ l. } - \text{s. 11 d. the answer.}$$

7. If a sack of coals be the allowance of 7 poor people for a week; how many poor belonged to that parish, which, when coals were 1 l. 16 s. per chaldron, had 41 l. to pay in 6 weeks on that account?

$$3 \text{ s. } 1 \text{ chal. } 1 \text{ chal.}$$

$$1 \text{ } 16 = \frac{9}{5} : \frac{1}{1} :: \frac{41}{1} : \frac{205}{9} = 22\frac{7}{9} \text{ chaldron.}$$

Here 12 sacks of 3 bushel each are accounted 1 chaldron.

Then $\frac{1}{12} \times \frac{9}{5} = \frac{3}{20}$ chaldron burnt by 7 in 6 weeks.

$$\text{As } \frac{1}{2} : \frac{7}{1} :: \frac{205}{9} : \frac{2870}{9} = 318\frac{2}{3} \text{ poor, the answer.}$$

8. It

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8. It is a rule in some parishes to assess the inhabitants in proportion to $\frac{3}{10}$ of their rent; what is the yearly rent pay of that house, which pays 8 l. 10 s. to the king under this limitation, at 5 s. in the pound?

tax. rent. tax. rent.

$$.25 : 1 :: 8.5 : 34.$$

$$.8 : 1 :: 34 : 42.5 = 42 \text{ l. } 10 \text{ s.}, \text{ the answer.}$$

9. A and B on opposite sides of a wood 134 toises about, they begin to go round it both the same way at the same instant of time; A goes 11 toises in 2 minutes, and B 17 in 3: the question is, how many times will they surround the wood before the nimbler overtakes the slower?

min. toises. m.

$$2 : 11 :: 3 : 16\frac{1}{2} \text{ toises.}$$

Then $17 - 16\frac{1}{2} = \frac{1}{2}$ toise B gains of A in going 17. toise. toises. round.

$$\therefore \frac{1}{2} : 17 :: \frac{1}{2} : 17 \text{ rounds gone by A, and } 16\frac{1}{2} \text{ B.}$$

10. A cistern holds 103 gallons, and being brimful, has 2 cocks to run off the water; by the first of which a pail of 3 gallons will be filled in 1 minute, by the other in 1 minute and 15 seconds; in what time will this cistern be emptied through both these apertures together, supposing the efflux of water all along the same?

First cock runs off 3 gallons $= \frac{3}{103} = \frac{15}{515}$ of the cistern in 1 minute.

As 1 min. 15 sec. $= \frac{5}{4} : \frac{3}{103} :: 1 : \frac{12}{515}$, run off by the second cock in 1 minute.

And $\frac{15}{515} + \frac{12}{515} = \frac{27}{515}$, run off by both in 1 minute.

$$\therefore \frac{27}{515} : 1 :: 1 : \frac{515}{27} = 19 \text{ min. } 4\frac{1}{3} \text{ seconds, the answer.}$$

11. If, when Port wine is 17 guineas the hoghead, a company of 45 people will spend 30 l. therein, in a certain time; what is wine a pipe when 13 persons more will spend 63 l. in twice the time, drinking with equal moderation?

45 men

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45 men : 20 l. :: 58 men : 25 $\frac{7}{8}$.
 And $25 \frac{7}{8} \times 2 = 51 \frac{1}{4}$ l's. worth, at 17 guineas per hoghead.
 $51 \frac{1}{4} : 17 \cdot 85 :: 63 : 21 \cdot 8125$ l. per hoghead.
 $\therefore 21 \cdot 8125 \text{ l.} \times 2 = 43 \cdot 625 \text{ l.} = 43 \text{ l. } 12 \text{ s. } 6 \text{ d. the answer.}$

12. In distress at sea they threw out 17 hogheads of sugar, worth 34 l. per hoghead, the worth of which came up to but $\frac{2}{3}$ of the indico they cast overboard; besides which they threw out 13 iron guns, worth 18 l. 10 s. 2 piece; the value of all these amounted to $\frac{2}{3}$ of $\frac{2}{3}$ of the ship and lading; pray what of this value came into port?

	l.	s.	d.
17 hogheads of sugar, at 34 l. per hoghead,	578	-	-
As $\frac{4}{7} : \frac{578}{1} :: \frac{2}{7} : \frac{2023}{2}$ indico, value	-	1011	10
13 iron guns, at 18 l. 10 s. each, is	-	240	10
Value of the whole cast overboard	-	1830	-

Then $1830 \text{ l.} = \frac{2}{3}$ of $\frac{2}{3}$ of the ship and lading, or $\frac{4}{9}$ of the whole.
 $\therefore \frac{27}{91} : \frac{1830}{1} :: \frac{64}{91} : \frac{39040}{9}$, arrived at port 4337 l. 15 s. 6 $\frac{2}{3}$ d.

13. A, B and C will trench a field in 12 days; B, C and D in 14; C, D and A will do it in 15; and D, A and B in 18; in what time will it be done by all of them; and each of them singly?

First A, B, C }
 B, C, D } can do { $\frac{1}{12} = .0833333$
 A, C, D } $\frac{1}{14} = .0714285$ } part of the
 A, B, D } $\frac{1}{15} = .0666666$ } whole work.
 A, B, C } $\frac{1}{18} = .0555555$

All working three days will do .276984 part of the work.
 Then, .276984 : 3 days :: 1 work : 10.8309505 days,
 all working.

B, C, D

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B, C, D 14

— 10.83095

days.

3.16905 : 10.83095 :: 14 : 47.848 by A

A, C, D 15

— 10.83095

4.16905 : 10.83095 :: 15 : 38.969 by B

A, B, D 18

— 10.83095

7.16905 : 10.83095 :: 18 : 27.194 by C

A, B, C 12

— 10.83095

1.16905 : 10.83095 :: 12 : 111.176 by D

alone.

14. If during the tide of ebb a wherry sets out from London westward, and at the same instant another should put off at Chertsey for London, taking the distance by water 34 miles; the stream forwards this, and retards the other, $2\frac{1}{2}$ miles in an hour; the boats are equally laden, the rowers equally good, and the ordinary way of working in still water, would proceed at the rate of five miles an hour: the question is, where in the river the two boats would meet?

It is plain from the question, that he that rows towards } London goes $\left\{ 7\frac{1}{2} \right\}$ miles in an hour.
from } $\left\{ 2\frac{1}{2} \right\}$

h. m. h.
Sum 10 : 1 :: 34 : 3.4

h. mil.s.

∴ 1 : 2.5 :: 3.4 : $8\frac{1}{2}$ } miles from } London.
And 1 : 7.5 :: 3.4 : $25\frac{1}{2}$ } miles from } Chertsey.

15. A young hare starts 5 rods before a greyhound, and is not perceived by him till she has been up 34 seconds; she scuds away at the rate of 12 miles an hour, and the dog in view makes after her at the rate of 20; how long will the course hold, and what ground will be run, beginning with the out-setting of the dog?

5 rods or poles = 82.5 feet - - 1 hour = 3600 seconds.
12 miles = 63360 feet; and 20 miles = 105600 feet.

Then

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Then $3600'' : 63360 \text{ feet} : 34'' : 598.4 \text{ feet.}$

$82.5 + 598.4 = 680.9 \text{ feet, the hare had start.}$

$8 : 20 :: 680.9 : 1702\frac{1}{2}$, run by the greyhound.

$105600 - 63360 = 42240.$

$42240 : 3600 :: 680.9 : 58\frac{1}{11}''$, run by the greyhound.

16. A lent his friend B fourscore and eleven guineas from the 11th of December to the 10th of May following; B, on another occasion, let A have a 100 marks from September the 3d to Christmas following; quere, how long ought the person obliged to let his friend use 40l. fully, to retaliate the favour?

First, 91 guineas $= 95 \text{ l. } 11 \text{ s.}$ for 150 days.

And 100 marks $= 66 \text{ l. } 13 \text{ s. } 4 \text{ d.}$ 113 days.

Reciprocally, $95.55 : 150 \text{ days} :: 66.8 : 215 \text{ days.}$

$215 \text{ l.} - 113 = 102.$

Recip. $66.8 \text{ l.} : 102 :: 40 : 170 \text{ days nearly. Q. E. F.}$

17. There are two pieces of clock-work, moving with a *gy*, will each of them lower a weight uniformly to the depth of 35 feet; the first weight, or A, descends $\frac{13}{10}$ inches in an hour; and when it is let down 12 feet, the second, or B, is put off; and the train of wheels belonging to this machine is so ordered, that the weights will be, in the same level, 100 inches before they come to the bottom; the velocity of B's descent is required?

As $\frac{13}{10}$ inches : 1 hour $:: 12 \text{ feet} = 144 \text{ inches} : \frac{1440}{13}$

$35 \text{ feet} = 420 \text{ inches} \leftrightarrow 100 \text{ inches} = 320 \text{ inches}$, where the weights will be level.

As $\frac{13}{10}$ inches : 1 hour $:: 320 \text{ inches} : \frac{3200}{14}$, time A descends 320 inches.

Then $\frac{3200}{13} \rightarrow \frac{1440}{13} = \frac{1760}{13}$ hours, time B descends to a level with A.

$\therefore \frac{1760}{13} \text{ hours} : 320 \text{ inches} :: 1 \text{ hour} : \frac{416}{176} = 2\frac{4}{11} \text{ inches, the answer.}$

18. My water-tub holds 147 gallons, the pipe usually brings in 14 gallons in 9 minutes, the tap discharges, at a medium, 40 gallons in 31 minutes; supposing these both carelessly to be left open, and the water to be turned on at two o'clock in

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in the morning; the servants at five, finding the water running, shuts the tap, and is solicitous in what time the tub will be filled after this accident, in case the water continues flowing from the main?

First, 9 minutes : 14 gallons :: 31 minutes : $48\frac{2}{3}$ gallons, fills in 31 minutes.

Then $48\frac{2}{3} - 40 = 8\frac{2}{3}$ gal. in the tub at the end of 31 min.

Also 31 min. : $8\frac{2}{3}$ gal. :: $3 \times 60 = 180$: $47\frac{3}{4}$ gal. in three hours.

Further, $147 - 47\frac{3}{4} = 99\frac{1}{4}$ gal. the tub wants of being full.

And 14 gal. : 9 min. :: $99\frac{1}{4}$: 63 min. $48\frac{2}{3}$ seconds, the tub will be full.

∴ The tub will be full at 3 minutes $48\frac{2}{3}$ seconds after 6.

19. One being asked what hour of the day it was, answered, the day at this time is 16 hours long; if now $\frac{1}{2}$ of the hours past be added to $\frac{2}{3}$ of the remainder, you will have the hour desired, reckoning from sun-rising.

First, $\frac{1}{2}$ of the hours past + $\frac{2}{3}$ of those to come = hours past.

∴ $\frac{1}{2}$ hours past = $\frac{2}{3}$ of those to come.

And $\frac{2}{3} + 2 = \frac{8}{3}$ of the hours to come = hours past, or time of the day; consequently, the ratio of the hours past are to those to come,

As $\frac{2}{3}$ to $\frac{8}{3}$, or as $1\frac{1}{2}$ to 1.

Then $\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$, the sum of those ratios.

∴ $\frac{2}{3} : \frac{4}{3} :: 16 : 9\frac{1}{2}$, hours from sun-rising. } Q. E. F.
And $\frac{2}{3} : \frac{4}{3} :: 16 : 6\frac{2}{3}$, hours to sun-setting. }

20. A triangular bath 6 feet deep, is exactly inclosed by 3 square pavilions, and rectangular; the sum of whose plans together make just 50 poles; the area of A, the left, is to that of B, the middle one, as $4\frac{1}{2}$ to 8; and the sum of the areas of A and C, the biggest, is to that of B, as $8\frac{1}{2}$ to 4; how many wine hogheads of water will this bath receive?

As $8B : 4\frac{1}{2}A :: 4B : 2\frac{1}{2}A$.

$8\frac{1}{2} - 2\frac{1}{2} = 6\frac{1}{2} = C$'s proportional part.

Also $A 2\frac{1}{2} + B 4 + C 6\frac{1}{2} = 12\frac{1}{2}$.

As $12.5 : 50 :: \left\{ \begin{array}{l} 2.25 : 9 \\ 4 : 16 \\ 6.25 : 25 \end{array} \right\}$ poles, area of the $\left\{ \begin{array}{l} A. \\ B. \\ C. \end{array} \right.$ pavilion.

And

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And by inspection the sides of the triangle $A = 3$, $B = 4$, $C = 5$.

And as 16 feet make a pole, $16.5 \times 4 = 9.5$ feet, the perpendicular.

Also $16.5 \times 4 = 66$ feet - - - $\frac{66}{2} = 33$, half the base.

$49.5 \times 33 = 1633.5$ feet = area of the triangle.

And $1633.5 \times 6 = 9801$ cubic feet, solidity of the bath.

Also $9801 \times 1728 = 16936128$ cubic inches.

$\therefore 231) 16936128 (73316\frac{2}{3} \div 63 = 1163$ hogheads,
47 $\frac{1}{2}$ gallons. Q. E. F.

21. A certain man hires a labourer on this condition, that for every day he worked he should receive 12 d. but for every day he was idle he should be mulcted 8 d. when 390 days were past, neither of them were indebted to one another; how many days did he work, and how many days was he idle?

As for every day he worked he received - - - 12 d.

So for every day he played he paid - - - 8

their sum 20 d.

And as his idle days came to the same money as those he worked, therefore the proportion will be contra;

via. 20 : 8 :: 390 : 156 } days he { worked. } Q. E. F.
and 20 : 12 :: 390 : 234 } days he { played. }

22. A man hired a labourer for 40 days, on condition that he should have 20 d. for every day he wrought, and forfeit 10 d. for every day he idled, at last he received 41 s. 8 d. for his labour; how many days did he work, and how many was he idle?

41 s. 8 d. = 500 pence.

20) 500 (25 days wages.

Then 40 - 25 = 15 days more.

For every of which days he worked he had 20 d.

And for every day he played he paid - - - 10

30

By contra proportion,

As 30 : 10 :: 15 : 5

And 30 : 20 :: 15 : 10

$\therefore 10 =$ days he was idle.

And 25 + 5 = 30 days he worked.

22. There

23. There is an island 73 miles round, and three footmen all start together, to travel the same way about it; A travels 5 miles a day, B 8, and C 10; when will they all come together again? *Emerson's Arithmetic.*

First, $8 - 5 = 3$ miles B } gains of A in one day.
 Also $10 - 5 = 5$ miles C }
 m. d. m.

Then $3 : 1 :: 73 : 24\frac{1}{3}$ days, when A and B meet.

And $5 : 1 :: 73 : 14\frac{2}{3}$ days, when A and C meet.

And B nor C can never meet with A, but at the end of these periods.

Then $24\frac{1}{3} = \frac{73}{3}$, and $14\frac{2}{3} = \frac{73}{5}$.

$\frac{73}{5} : \frac{73}{3} :: 219 : 365$.

$\therefore 219 \times 24\frac{1}{3}$ }
 or $365 \times 14\frac{2}{3}$ } 5329 days. Q. E. F.

The following machine being accounted a lever of the second order, whose force is directly, and its pressure in a contra-proportion to the length.

24. In giving directions for making an Italian chair, the shafts whereof were settled at 11 feet between the axletree, whereon the principal bearing is, and the backband, by means of which the weight is partly thrown upon the horse. A dispute arose whereabouts on the shafts the center of the body of this machine should be fixed; the coachmaker advised this to be done at 30 inches from the axletree; others were of opinion, that at 24 it would be a sufficient incumbrance to the horse. Now, admitting the two passengers, with their baggage, ordinarily to weigh 2 cwt. a-piece, and the body of the vehicle to be about 70 lb. more, pray what will the beast in both these cases be made to bear more than his harness?

First, $11 - 2.5 = 8.5$; also $11 - 2 = 9$, and 4 cwt. 70 lb. = 518 lb.

Directly, $11 : 518 :: 8.5 : 400\frac{1}{11}$, force.

Contra, $8.5 : 400\frac{1}{11} :: 2.5 : 117\frac{2}{11}$, pressure in the former.

Also $11 : 518 :: 9 : 423\frac{2}{11}$, force } in the 2d case.
 $\therefore 9 : 423 :: 2 : 94\frac{2}{11}$, pressure }

Answer, the beast bears $117\frac{2}{11}$ lb. in the former, and $94\frac{2}{11}$ lb. in the second case.

25. If a lever 40 effective inches long will, by a certain power thrown successively thereon, in 13 hours raise a weight 104 feet; in what time will two other levers, each 18 effective inches long, raise an equal weight 73 feet?

As 40 inches : 104 feet :: 18×2 : 93.6.

Then 93.6 feet : 13 hours :: 73 feet : 10 hours 8 $\frac{1}{2}$ minutes, the answer.

26. A weight of 1 $\frac{1}{2}$ lb. laid on the shoulders of a man, is no greater a burden to him than its absolute weight, or 24 ounces; what difference will he feel between the said weight applied near his elbow, at 12 inches from the shoulder, and in the palm of his hand, 28 inches therefrom; and how much more must his muscles then draw, to support it at right-angles, that is, having his arm extended right out?

As 1 : 1.5 lb. :: 12 in. : 18 lb. weight 12 in. } from the
And 1 : 1.5 lb. :: 28 in. : 42 lb. weight 28 in. } shoulder.
∴ 42 — 18 = 24 lb. the answer required.

27. A ball weighing four pounds upon the surface of the earth, to what height in the air must it be carried to weigh but three pounds, and how long would it be falling to the ground? *Ladies Diary.*

Taking the earth's semidiameter at 4000 miles.

Then $4000 \times 4000 = 16000000$.

As the weights of bodies decrease as the square of their distance from the earth's center, we have,

∴ Recip. $4 : 16000000 :: 3 : 21333333.3$.

Then $\sqrt{21333333.3} = 4618.8021$.

Then $4618.8021 - 4000 = 618.8321$ miles = 3267275 feet, or 618 miles, 6 feet, 16 poles, 3 yards, 1 foot, the height of the ball must be carried.

Again, $16.083 : 1$ square second :: $3267275 : 203837.48$.
 $\sqrt{203837.48} = 451.48'' = 7' 31'' 29'''$, the time of falling.

But at that great distance from the earth, when the ball will have lost $\frac{1}{4}$ of its weight, its velocity will also be diminished.

Viz. $16.083 \times .75 = 12.0625$.

Then as $12.0625 : 1 :: 3267275 : 270862$.

$\sqrt{270862} = 520'' = 8' 40''$, the time of falling.

U

28. A

28. A ball descending by the force of gravity from the top of a tower, was observed to fall half the way in the last second of time; required the tower's height, and the whole time of descent?

Ladies Diary, 1755.

The square roots of the distances being as the times, viz. as the $\sqrt{1} : \sqrt{2} ::$ is the time of falling through the first half, to the time of falling through the whole required height.

\therefore As $\sqrt{2} - 1 = .4142 : \sqrt{2} = 1.4142 :: 1 : 3.414$ seconds, the time of descent.

And $1^2 : 16.083 :: \square 3.4142 = 11.6574 : 187.48$ feet, the tower's height.

29. Suppose that in every single revolution of the upper stone of a water-mill, it evacuates or grinds one-eighth of a pint of meal; and supposing there be eight standards or pinions in the rounds that turn it once, and that these rounds are driven by a wheel of 45 teeth; also the mean circumference of the water-wheel on the same axis be 40 feet, which requires one hundred and a half to move it, or put it into motion: now, if a floodgate, whose breadth is a foot, and height half a foot, and the height of the water be three feet above the surface of the hole, be let run directly against the upper surface of the wheel, it is required to find the quantity of meal ground in an hour by the said mill?

Gentlemen's Diary, 1751.

First, $1 \times .5 \times 3 = 1.5$; also 62.5 lb. weight of a cubic foot of water. Then $62.5 \times 1.5 = 93.75$ lb. the instantaneous pressure of the water.

Also, $16\frac{1}{2} = \frac{193}{12} :: 1^2 : 3 \text{ feet} : \frac{36}{193} = \text{square of the time.}$

And $\sqrt{\frac{6}{193}} = \text{the time.}$

$\therefore \sqrt{\frac{6}{193}} : 6 :: 1 : \sqrt{193} = 13.89244$ feet, velocity per second.

Also $1\frac{1}{2} \text{ cwt.} = 168 \text{ lb.}$

$93.75 \times 13.8924 \div 168 = 7.75248$ feet, any point of the wheel moves in one second.

$7.75248 : 1 :: 40 : 5.1595$ seconds, moving round.

$5.1595 : 1 :: 3600 : 697.742$ rounds, the wheel moves in an hour.

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$\frac{45}{8} = 5.625$. As 1 : 5.625 :: 697.742 : 3924.799 rounds,
the stone moves in an hour.

∴ 8) 3924.799 (490.6 pints = 7 bushels, 2 pecks, 10 $\frac{1}{2}$ pints, the mill grinds in an hour. Q. E. F.

30. Observed, that while a stone was descending to measure the depth of a well; a string and plummet (that from the point of suspension, or the place where it was held, to the center of oscillation) or that part of the bob, which being divided by a circular line, struck from the center abovesaid, would divide it into two parts of equal weight measured just 18 inches, had made eight vibrations; pray what was the depth, allowing 1150 feet per second, for the return of sound to the ear?

$$39.2 : 3600 :: 18 : 7840.$$

✓ 7840 = 88.54378 vibrations in one minute.

60) 88.54378 (1.475729 vibrations in a second.

1.475729) 8.000000 (5.421 seconds in eight vibrations.

12 : 16.083 :: □ 5.421 = 29.387 : 472.640918 feet.

As 1150 feet : 1 :: 472.640918 : 41099, time the sound was returning.

5.421 + 41099 = 5.01, true time of the body's descent.

∴ 5.01 × 5.01 = 25.1001 × 16.083 = 403.6932 feet, the depth of the well. Q. E. F.

31. Aspire my genius ! help my rhiming muse,
In themes I in my native country chuse :
Whilst others plow the waves, and tread the strands
Of distant oceans, and of foreign lands ;
To fill the mouth of fame with something new
(No matter 'tis how much of it is true)
From Alps or mountains, stories strange they bring,
Of deserts, caves, or horrid monsters sing.
Tell how Vesuvius' sulph'rous darts do fly,
Or Ætna's smoke obscures the azure sky ;
Or magnify the hazards they have run,
Scylla's and Charybdis' pointed rocks to shun.
Such tales we take on trust, from those who rove,
Tho' none give rules, by which the truth to prove.

But this by numbers may explained be,
By those who never did the cavern see :
In Derbyshire, a wonder of the Peak,
Is Eldon-hole, as poets often speak ;

Whose depth exactly none could e'er descry,
 Tho' atheist Hobbes his utmost skill did try,
 And wrote *De Mirabilibus Pecci*. }

And burlesque Cotton, does strange tales rehearse,
 In rustic words, and Hudibrastic verse,
 How he this mighty orifice did plumb,
 But could not at the bottom of it come,
 With sixteen hundred yards of rope let loose;
 And tells a story of a woman's goose:
 Fab'lous the one, so must the other be
 Erroneous too, without philosophy;
 Extension of the rope might him deceive,
 Or small proportion which the plumb would have
 To such a length; and part in water drown'd,
 When in this vast abyss, within the ground.

But I the depth have found exactly true,
 By gravity, a method something new.
 As heavy bodies do accelerate,
 In spaces known first to our Newton great,
 Four pond'rous stones into the well let fall,
 In measur'd time, agreed in numbers all.
 A pendulum, sixty-one inches long,
 By which the time I measured (was not wrong)
 Vibrated freely (whilst that each stone fell)
 Eight times; by which the depth I'd have you tell,
 Allowing rightly for the approach of sound,
 That your own works may not themselves confound.

Ladies Diary, 1722.

First, $39.2 : 3600 :: 61 : 2313.4426$.

$\sqrt{2313.4426} = 48.09826$ vibrations in one minute.

60) 48.09826 (.8016377 vibrations in one second.

.8016377) 8.0000000 (9.97957 seconds at eight vibrations.

Also $1^2 : 16.083 :: \square 9.97957 = 99.59182 : 1601.7584$.

$1150 \text{ feet} : 1'' :: 1601.7584 \text{ feet} : 1.39283$, time of the sound's ascent.

Then $9.97957 - 1.39283 = 8.58674$, true time of the stone's descent.

$\therefore 1^2 : 16.083 :: \square 8.58674'' = 73.7321 : 1185.88 \text{ feet}$,
 the depth of Elden-hole. Q. E. F.

CHAPTER II.

SECT. I.

SIMPLE INTEREST.

INTEREST is a small sum of money paid for the use of a larger sum, at any rate agreed upon; which according to law must not exceed 5*l.* for the interest of 100 *l.* principal, for one year.

CASE I.

The principal, rate of interest, and time, being given; to find the interest.

First, When the yearly interest of any sum is required,

RULE,

Multiply the principal by the rate of interest per cent. per annum, dividing the product by 100; which is done by cutting off the two right-hand figures, those to the left being the interest in pounds. Then multiply the remainder (if any) by 20, cutting off as before for shillings; and that remainder by 12, cutting off as before directed for pence; and find the farthings (if any) after the same manner; the figures to the left of those cut off being the interest.

1. What is the interest of 873*l.* 16*s.* 8*d.* for a year, at 5 per cent.?

$$\begin{array}{r}
 \begin{array}{rcl}
 \text{l.} & \text{s.} & \text{d.} \\
 873 & 16 & 8 \\
 \hline
 & 5 & \\
 \hline
 \pounds 43.69 & 3 & 4 \\
 \hline
 & 20 & \\
 \hline
 & 13.83 & \text{s.} \\
 \hline
 & & 12 \\
 \hline
 & 10.00 & \text{d.} \\
 \hline
 \end{array}
 \end{array}$$

By PRACTICE.

$$\begin{array}{r}
 \begin{array}{rcl}
 \text{l.} & \text{s.} & \text{d.} \\
 5 \text{ per cent} = \frac{1}{20} \times 873 & 16 & 8 \\
 \hline
 \pounds 43 & 13 & 10
 \end{array}
 \end{array}$$

Answer, 43*l.* 13*s.* 10*d.*

U 3

2. What

2. What is the interest of 1437 l. 17 s. for a year, at $4\frac{1}{2}$ per cent?

l.	s.	d.
1437	17	—
		$4\frac{1}{2}$
575	8	—
718	18	6
£ 64.70	6	6
	20	
	14.06 s.	
	12	
	.78	
	4	
	3.12 qrs.	

Answer, 64 l. 14 s. $-\frac{1}{2}$ d.

By PRACTICE.

	l.	s.	d.
20 per cent. $= \frac{1}{5}$	1437	17	—
4 s. $= \frac{1}{2}$	287	11	4
$\frac{1}{8}$	57	10	$3\frac{1}{2}$
	7	3	$9\frac{1}{2}$
	£ 64	14	$-\frac{1}{2}$

3. What is the interest of 178 l. 16 s. for a year, at $3\frac{1}{2}$ per cent.?

l.	s.
178	16
	$3\frac{1}{2}$
538	18
133	14 6 d.
£ 6.68	12 6
	20
	13.72 s.
	12
	8.70 d.
	4
	2.80 qrs.

By PRACTICE.

	l.	s.	d.
20 per cent. $= \frac{1}{5}$	178	16	—
$\frac{1}{10}$	35	13	$2\frac{1}{2}$
$\frac{1}{20}$	17	6	$1\frac{1}{4}$
$\frac{1}{40}$	8	8	$3\frac{1}{2}$
$\frac{1}{80}$	4	4	$1\frac{3}{4}$
$\frac{1}{160}$	2	2	$9\frac{1}{8}$
	£ 6	13	8

Secondly, When the interest of any sum is required for several years,

R U L E,

R U L E,

After the yearly interest is found, multiply that by the number of years, the product will be the answer.

4. What is the interest of 74l. 15s. for five years, at $4\frac{1}{2}$ l. per cent. per annum?

By the RULE OF FIVE.

100 . 1 . 4.375
74.75 . 5 .
 $4.375 \times 74.75 \times 5 = 1635.15625$ dividend, and $100 \times 1 = 100$ divisor.
 $\therefore 100) 1635.15625 (16.3515625 = 16l. 7s. -\frac{1}{4}d.$ the answer.

By the RULE.

l.	s.
$\frac{1}{4}$ 74	15
	$4\frac{1}{2}$
<hr/>	
299	- -
18	13 9
9	6 $10\frac{1}{2}$
<hr/>	
£ 3.27	- $4\frac{1}{2}$
<hr/>	
× 20	
<hr/>	
5.40s.	
<hr/>	
× 12	
<hr/>	
4.84 d.	
<hr/>	
4	
<hr/>	
3.38 qrs.	

By PRACTICE.

l.	s.	d.
20 per cent. = $\frac{1}{5}$	$\frac{1}{5}$ 74	15 -
	<hr/>	
$\frac{1}{10}$	14	19 -
	<hr/>	
	1	9 $10\frac{3}{4}$
$\frac{1}{10}$	1	9 $10\frac{3}{4}$
	<hr/>	
	3	8 $\frac{3}{4}$
	<hr/>	
	1	$10\frac{3}{4}$
<hr/>		
Interest for 1 year	3	5 $4\frac{3}{4}$
		5
<hr/>		
Answer, £	16	6 $11\frac{3}{4}$

The small defect in the practical methods, is owing to the parts of a farthing omitted.

5. What is the interest of 963l. 7s. 6d. at $3\frac{1}{2}$ per cent. for 13 years?

l.	s.	d.
$\frac{1}{5}$ 963	7	6
		$3\frac{1}{2}$
2890	2	6
481	13	9
120	8	$5\frac{1}{4}$
£ 34.92	4	$8\frac{1}{4}$
20		
£ 18.44		
12		
5.36 s.		
4		
1.44 d.		

By PRACTICE.

l.	s.	d.
20 per cent. = $\frac{1}{5}$	963	7 6
$\frac{1}{10}$	192	13 6
$\frac{1}{20}$	19	5 4
$\frac{1}{40}$	9	12 8
$\frac{1}{80}$	4	16 4
$\frac{1}{160}$	1	4 1

Year's interest £ 34 18 5

Year's int. by the 1st method £ 34 18 $5\frac{1}{4}$
 $\times 13$ Answer, £ 453 19 $8\frac{1}{4}$

By FIVE NUMBERS.

100 . . . 1 . . 3.625
 963.375 . 13 .
 $963.375 \times 3.625 \times 13 = 45399.046875$
 100) 45399.046875 (453.99046875 = 453l. 19s. $9\frac{1}{4}$ d.
 answer.

Thirdly, When the interest of any sum is required for years and months,

R U L E,

First find the yearly interest, which multiply by the number of years, as before; then for the months divide the yearly interest by the part or parts the given months are of a year, which add together with the rest, and their sum will be the answer.

6. What is the interest of 56l. 10s. for 7 months, at $4\frac{1}{2}$ per cent. per annum?

$$\begin{array}{r}
 \text{l. s.} \\
 56 \text{ } 10 \\
 \underline{4\frac{1}{2}} \\
 226 \text{ } - \text{ } - \\
 14 \text{ } 2 \text{ } 6 \\
 \hline
 2.40 \text{ } 2 \text{ } 6 \\
 20 \\
 \hline
 \text{£ } 8.02\text{ } 12 \\
 \hline
 .30 \\
 4 \\
 \hline
 1.20 \text{ qrs.}
 \end{array}$$

$$\begin{array}{r}
 \text{l. s. d.} \\
 \frac{1}{2} 2 \text{ } 8 \text{ } -\frac{1}{2} \text{ per annum.} \\
 \hline
 \frac{1}{2} 1 \text{ } 4 \text{ } - \\
 - \text{ } 4 \text{ } - \\
 \hline
 \text{£ } 1 \text{ } 8 \text{ } - \text{ the answer.}
 \end{array}$$

By FIVE NUMBERS.

P.	T.	G.
100	. 12	. 4.25
56.5	. 7	

$56.5 \times 4.25 \times 7 = 1680.875$, dividend.
 $100 \times 12 = 1200$ $1680.875 (1.400729 = 11. 8 \text{ s. the answer.}$

7. What is the interest of 759 l. 16 s. 7 d. for 12 years, 4 months, at $5\frac{1}{2}$ l. per cent. per annum?

$$\begin{array}{r}
 \text{l. s. d.} \\
 \frac{1}{4} 759 \text{ } 16 \text{ } 7 \\
 \hline
 5\frac{1}{2} \\
 3799 \text{ } 2 \text{ } 11 \\
 94 \text{ } 19 \text{ } 6\frac{1}{2} \\
 \hline
 \text{£ } 38.94 \text{ } 2 \text{ } 5\frac{1}{2} \\
 20
 \end{array}$$

$$\begin{array}{r}
 \text{l. s. d.} \\
 \text{Year's interest } 38 \text{ } 18 \text{ } 9\frac{1}{2} \\
 \hline
 12\frac{1}{2} \\
 467 \text{ } 5 \text{ } 9 \\
 12 \text{ } 19 \text{ } 7\frac{1}{2} \\
 \hline
 \text{Answer, £ } 480 \text{ } 5 \text{ } 4\frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 18.82 \text{ s.} \\
 12
 \end{array}$$

$$\begin{array}{r}
 9.89 \text{ d.} \\
 4
 \end{array}$$

$$\begin{array}{r}
 3.59 \text{ qrs.}
 \end{array}$$

By FIVE NUMBERS.

P.	T.	G.
100	. 1	. 5.125
759.82916	. 12.8	

$5.125 \times 759.82916 \times 12.8 = 48027.534232\text{ } 8$
 $100) 48027.534 (480.27534 = 480 \text{ l. } 5 \text{ s. } 5\frac{1}{2} \text{ d. the answer.}$

8. Lent, at Christmas 1760, the sum of 5000l at $4\frac{1}{2}$ per cent. ; after which time I lent several sums at the same rate, and drew upon the borrower, as business required; viz. on Lady-day 1761, I drew for 185 guineas; on Midsummer-day 1761, I lent 500 moldores, and drew for 700l. ; on Michaelmas-day 1761, I lent 569l. 17s. : I demand what cash the borrower owed me at that time?

5 l. = $\frac{1}{100}$ | 5000 l. principal, at $4\frac{1}{2}$ per cent.

250
— 25

225, interest for 1 year at $4\frac{1}{2}$ per cent.

4) 225

s.

56

5, interest due at Lady-day 1761.

5000 —, principal.

5056

5, amount at Lady-day.

194 5 drawn = 185 guineas.

1.

5 = $\frac{1}{100}$ | 4862 —, new principal,

$\frac{1}{2} = \frac{1}{100}$

243

2

— 24

6

24

6

24

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6

4) 218 15 $9\frac{1}{2}$, interest for 1 year at $4\frac{1}{2}$.

54

13

$11\frac{1}{2}$, interest due at Midsummer.

4862 — —, last principal.

4916

13

$11\frac{1}{2}$

— 25

—

— = 700 l. — 500 moldores, or 675 l.

1.

5 = $\frac{1}{100}$ | 4891 13 $11\frac{1}{2}$, new principal at $4\frac{1}{2}$ per cent,

$\frac{1}{2} = \frac{1}{100}$

244

11

— 24

9

24

9

24

9

24

9

24

9

24

9

24

9

24

9

24

9

24

9

24

9

24

9

24

9

4) 220 2 $6\frac{1}{2}$, interest for 1 year.

55

—

$7\frac{1}{2}$, interest due at Michaelmas.

4891 13 $11\frac{1}{2}$ last principal.

569

17

—

—

—

—

—

—

—

—

—

—

—

—

—

—

£ 5516 11 $6\frac{1}{2}$, the answer.

Fourthly,

Chap. II. SIMPLE INTEREST. 299

Fourthly, When the interest of any sum is required for days,

R U L E,

First find the interest for a year, then by the rule of three direct, viz. 365 : one year's interest :: the days the money is at interest : interest required.

9. What is the interest of 375 l. 15 s. for 127 days, at $4\frac{1}{2}$ per cent. per annum?

l. s.	days.	l. s. d.	days.
$\frac{1}{2}$ 375 15	As 365 : 17 16 10 $\frac{1}{2}$:: 127		
4 $\frac{1}{2}$	12		
<hr/>		<hr/>	
1503 - -		214 2 3	
187 17 6		10	
93 18 9		<hr/>	
<hr/>		2141 2 6	
£ 17.84 16 3		124 17 11 $\frac{1}{2}$	
20		<hr/>	
<hr/>		365) 2266 - 5 $\frac{1}{2}$ (6l. 4s. 1 $\frac{1}{2}$ d. anf.	
16.86 s.		76	
12		× 20	
<hr/>		<hr/>	
10.35 d.		1520	
4		60	
<hr/>		× 12	
1.40 qrs.		<hr/>	
<hr/>		725	
		360	
		× 4	
		<hr/>	
		1443	
		348	
		<hr/>	

10. What is the interest of 284 l. 10 s. for two years, four months, and 25 days, at $3\frac{1}{2}$ per cent. per annum?

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$$\begin{array}{r}
 \text{l. s.} \\
 284 \ 10 \\
 \underline{3\frac{1}{2}} \\
 853 \ 10 \\
 142 \ 5 \\
 \hline
 \text{£} 9.95 \ 15 \\
 20
 \end{array}$$

$$\begin{array}{r}
 19.15 \text{ s.} \\
 12 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 1.80 \text{ d.} \\
 4 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 3.20 \text{ qrs.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{l. s. d.} \\
 \frac{1}{2} \ 9 \ 19 \ 1\frac{3}{4}, \text{ year's interest.} \\
 2
 \end{array}$$

$$\begin{array}{r}
 19 \ 18 \ 3\frac{1}{2}, \text{ two years.} \\
 3 \ 6 \ 4\frac{1}{2}, \text{ four months.} \\
 - 13 \ 7\frac{1}{2}, \text{ twenty-five days.} \\
 \hline
 \end{array}$$

$$\text{£} 23 \ 18 \ 3\frac{1}{2}, \text{ the answer required.}$$

$$\begin{array}{r}
 \text{days. l. s. d. days.} \\
 365 : 9 \ 19 \ 1\frac{3}{4} :: 25 \\
 \underline{5} \\
 49 \ 15 \ 8\frac{1}{2} \\
 \underline{5} \\
 365) 248 \ 18 \ 7\frac{1}{2} (13 \text{ s. } 7\frac{1}{2} \text{ d.} \\
 20
 \end{array}$$

$$\begin{array}{r}
 4978 \\
 1328 \\
 233 \\
 \times 12 \\
 \hline
 2796 \\
 221 \\
 \times 4 \\
 \hline
 884
 \end{array}$$

Or as five per cent. is statute interest, multiply the given sum by the number of days, and divide the product by 7300 (viz. $\frac{100 \times 365}{5}$); the quotient will give the interest at five per cent. but if the interest at a higher or lower rate is required, take aliquot parts of the quotient for a difference, which add or subtract accordingly.

11. What is the interest of 547l. 15 s. at five per cent. for 320 days?

$$\begin{array}{r}
 547.75 \\
 320 \\
 \hline
 109550 \\
 164325 \\
 \hline
 \end{array}$$

$$7300) 1752.8000 (24.0109 = 24 \text{ l. } -\text{s. } 2\frac{1}{2} \text{ d.}$$

12. What

12. What is the interest of 248 l. 19 s. for 175 days, at $4\frac{1}{2}$ per cent. ?

$$\begin{array}{r}
 248.95 \\
 175 \\
 \hline
 124475 \\
 174265 \\
 24895 \\
 \hline
 \end{array}$$

73) 435.6625 (5.96798 , at five per cent.
 $\begin{array}{r} 706 \\ 496 \\ 582 \\ 715 \\ 580 \end{array}$ $\begin{array}{r} 596798 \\ \hline 5.371182 \end{array}$ $= 5 \text{ l. } 7 \text{ s. } 5 \text{ d. at } 4\frac{1}{2} \text{ per cent.}$

13. What is the interest of 713 l. 17 s. 6 d. for 193 days, at $3\frac{1}{4}$ per cent. ?

$$\begin{array}{r}
 713.875 \\
 193 \\
 \hline
 2141625 \\
 6424875 \\
 713875 \\
 \hline
 \end{array}$$

73) 1377.77875 (18.87368 , at 5 per cent.
 $\begin{array}{r} 647 \\ 637 \\ 537 \\ 268 \\ 497 \\ 595 \end{array}$ $\begin{array}{r} 4 \\ 4.71842, \text{ at } 1\frac{1}{2} \text{ per cent.} \\ \hline 14.15526 \end{array}$ $= 14 \text{ l. } 3 \text{ s. } 1\frac{1}{2} \text{ d. the ans.}$

It being looked upon as inaccurate in the calculation of interest to take aliquot parts for months, because the year is divided into months confisting of an unequal number of days; I have therefore, to find the number of days from one time to another, inserted the following TABLE.

Days	January	February	March	April	May	June	July	August	September	October	November	December
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

Chap. H. SIMPLE INTEREST. 303

The Use of the foregoing TABLE.

First, To find the number of days from the end of the year to any given day in any month in the year following.

Opposite the given day in the margin look under the given month, which will shew the number of days required.

Thus, from December 31, till August 18 following, are 230.

And till October 30, are 303 days, &c.

Secondly, To find the days from any given day of any month, to the end of the year. - - - Suppose July 27.

From 365 days in a year,

Take the number answering July 27, viz. 208

Rem. 157, days required.

Thirdly, To find the number of days between a given day in any given month, and any given day of any other month in the same year.

Suppose the days between April 5, and November 28, be required.

The number answering November 28 - 332

Subtract that answering April 5 - - 95

Rem. 237, days fought.

Fourthly, To find the number of days from any given day of any month in one year, to any given day of any month in the next year.

Suppose the number of days from the 21st of August 1758, till the 27th of May 1759, were required.

From - - - - - 365, days in a year.

Take the No answering Aug. 21 - 233

Rem. 132, to the end of the

Add the No answering May 27 - 147 year.

279, days required.

But

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But in the biffextile, or leap-year, if one of the given days be before the 29th of February, and the other after, one day must be added on that account. Thus, if the number of days in the last example had been from the 21st of August 1759, till the 27th of May 1760, it would have been 280 days.

13. What is the interest of 150l. from the 18th day of January, to the 11th of November, at $4\frac{1}{2}$ per cent. ?

$$\begin{array}{r}
 314 \text{ November 11.} \\
 18 \text{ January.} \\
 \hline
 297 \\
 150 \\
 \hline
 30) \\
 73) 445.50 (6.1027 \\
 \quad .3051 \\
 \hline
 5.7976 = 5 \text{ l. } 15 \text{ s. } 4\frac{1}{4} \text{ d.}
 \end{array}$$

14. What is the interest of 384l. 16 s. from the 7th of May, to the 11th of December, at $5\frac{1}{2}$ per cent. ?

$$\begin{array}{r}
 345 \text{ December 11.} \\
 127 \text{ May 7.} \\
 \hline
 218 \\
 3848 \\
 \hline
 30784 \\
 3848 \\
 \hline
 7696 \\
 \hline
 73) 838.864 (11.49128, \text{ at } 5 \text{ per cent.} \\
 \quad 1.14912, \text{ at } \frac{1}{2} \text{ per cent.} \\
 \hline
 12.6404 = 12 \text{ l. } 12 \text{ s. } 9\frac{1}{4} \text{ d.}
 \end{array}$$

15. What is the interest of 537l. 15 s. from November the 11th, 1764, till June the 5th, 1765, at $3\frac{1}{2}$ per cent. ?

$$\begin{array}{r}
 537.75 \\
 \underline{206} \\
 322650 \\
 \underline{107550} \\
 73)1107.765 \quad 5) \\
 \hline
 15.17486, \text{ at } 5 \text{ p. cent. } 206, \text{ days in all.}
 \end{array}$$

$$\begin{array}{r}
 305 \\
 \underline{315} \\
 50 \\
 156, \text{ to June the 5th.}
 \end{array}$$

$$\begin{array}{r}
 \frac{3}{4} \quad 3.03497 = 1 \text{ per cent.} \\
 \frac{1}{2} \quad .75874 = \frac{1}{2} \text{ per cent.} \\
 \frac{1}{4} \quad .37947 = \frac{1}{4} \text{ per cent.} \\
 \hline
 4.17318 = 1\frac{3}{4} \text{ per cent.} \\
 \hline
 11.00168 = 11 \text{ l. } 4 \text{ s. } \frac{1}{4} \text{ d. the answer.}
 \end{array}$$

Interest is to be calculated in the same manner on cash accounts, accounts current, &c. where partial payments are made, and partial debts contracted.

16. On the 1st of May I lent Ralph Newlands, per bill, at one day's date, 500l., which I received back in the following partial payments; viz. on the 13th of May 50l.; on the 4th of June 56l.; on the 14th of July 44l.; on the 23d ditto 50l.; on the 18th of August 87l.; on the 30th ditto 13l.; on the 21st of September 30l.; on the 18th of October 30l.; on the 29th ditto 40l.; on the 11th of November 50l.; on the 28th of December 50l.: what interest is due, at five per cent.?

Mr. Ralph Newlands. - - - Debtor.		Days.	Prod.
	1.		
May 1. Lent per bill at one day's date	500	13	6500
13. Received in part	50		
	<u>450</u>		
	Bal. 450	22	9900
June 4. Received in part	56		
	<u>394</u>		
	Bal. 394	40	15760
July 14. Received in part	44		
	<u>350</u>		
	Bal. 350	9	3150
23. Received in part	50		
	<u>300</u>		
	Bal. 300	26	7800
Aug. 18. Received in part	87		
	<u>213</u>		
	Bal. 213	12	2556
30. Received in part	13		
	<u>200</u>		
	Bal. 200	22	4400
Sept. 21. Received in part	30		
	<u>170</u>		
	Bal. 170	27	4590
Oct. 18. Received in part	30		
	<u>140</u>		
	Bal. 140	11	1540
29. Received in part	40		
	<u>100</u>		
	Bal. 100	13	1300
Nov. 11. Received in part	50		
	<u>50</u>		
	Bal. 50	47	2350
Dec. 28. Received in full of principal	50		
	<u>598.46</u>		

73) 598.46 (81. 3 s. 11½ d. interest on this account.

17. Lent John Jamefon, per bill, dated 18th of January, payable one day after date, 878 l. 19 s. 10 d., which I received back in the following partial payments; viz. on the 27th of February 57 l. 15 s. 7 d.; on the 18th of March 37 l. 14 s.; on the 29th of April 34 l. 11 s.; on the 12th of May 136 l. 15 s. 7 d.; on the 19th of June 67 l. 13 s. 4 d.; on the 15th of

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of July 15l. 15s. 6d.; on the 25th ditto 111l. 11s. 11d.; on the 3d of October 78l. 7s. 4d.; on the 19th of November 100l.; on the 23d ditto 100l.; and on the 30th of December received the balance of the principal: how much interest ought I to claim, at five per cent.?

Cash transactions with JOHN JAMISON.

	Extended.				Products.			
	l.	s.	d.		l.	s.	d.	
Jan. 18. Lent per bill at one day's date	878	19	10	40	351	59	13	4
Feb. 27. Received in part	37	15	7					
Bal.	841	4	3	19	156	3	-	9
Mar. 18. Received in part	37	14						
Bal.	783	10	3	42	329	07	10	6
April 29. Received in part	34	11						
Bal.	748	19	3	13	973	6	10	3
May 12. Received in part	136	15	7					
Bal.	612	3	8	38	232	62	19	4
June 29. Received in part	67	13	4					
Bal.	544	10	4	26	141	57	8	8
July 15. Received in part	15	15	6					
Bal.	528	14	10	10	528	7	8	4
Ditto 25. Received in part	111	11	11					
Bal.	417	2	11	70	292	00	4	2
Oct. 3. Received in part	78	7	4					
Bal.	338	15	7	47	159	22	12	5
Nov. 19. Received in part	100							
Bal.	238	15	7	4	955	2	4	
Ditto 23. Received in part	100							
Bal.	138	15	7	37	513	4	16	7
Dec. 30. Received in full of principal	138	15	7					
					187	327	6	8

COMPUTATION of EXAMPLE 17.

			l.	s.	d.				l.	s.	d.
By the Table.			878	19	10	19 Nov.	323		338	15	7
						3 Oct.	276				6
					5		47		2032	13	6
25 Feb.	58		4394	19	2						7
18 Jan.	18				8				14228	14	6
	40		35159	13	4				1693	17	11
18 March	77		821	4	3				15922	12	5
27 Feb.	58				6	23 Nov.	4		238	15	7
	19		4927	5	6						4
					3				955	2	4
			14781	16	6	30 Dec.	364		1138	15	7
			821	4	3		327				6
			15603	-	9		37		832	13	6
29 April	119		783	10	3						6
18 March	77				6				4996	1	-
	42		4701	1	6				138	15	7
					7				5134	116	7
			32907	10	6	The sum of all these multiplied into their respective times are 187327 l. 6 s. 8 d.					
12 May	132		748	19	3	Then					
29 April	119				13	l. s. d. l. s. d.					
	13		9736	10	3	73)1873.27 6 8(25 13 2½					
19 June	170		612	3	8	413 interest.					
12 May	132				12						
	38		7346	4	0						
					3						
			22038	12	0						
			1224	7	4						
			23262	19	4						
15 July	196		544	10	4						
19 June	170				13						
	20		7078	14	4						
					2						
			14157	8	8						
25 July	10		528	14	10						
					10						
			5287	8	4						
3 Oct.	276		417	2	11						
25 July	206				7						
	70		2920	-	5						
					10						
			29200	4	2						

48.27	
20	
965.46	
235	
16.46	
12	
197.60	
51.60	
4	
206.40	

CASE

CASE

CASE II.

The amount, rate per cent. and time given; to find the principal.

RULE.

As the amount of 100 l. at the rate and time given : is to 100 l. :: so is the amount given : to the principal required.

1. What principal sum, being put out to interest at three per cent. per annum, will amount to 3998 l. 12 s. 10 $\frac{1}{4}$ d. in 3 $\frac{1}{4}$ years, and 54 days ?

Time 3.3979452 = 3 $\frac{1}{4}$ years. days.
 3 interest of 100 l. for a year.

10.1938356 = interest of 100 l. for 3 $\frac{1}{4}$ years, 54 days.
 + 100

110.1938356, amount of 100 l. for the said time.
 As 110.1938356 : 100 :: 3998.6447918 : 3628.737275.
 ∴ 3628.737275 = 3628 l. 14 s. 9 d. the answer.
 And 3998 l. 12 s. 10 $\frac{1}{4}$ d. — 3628 l. 14 s. 9 d. = 369 l. 18 s. 1 $\frac{1}{4}$ d. discount.

2. What is 309 l. 16 s. 10 d. due three years, one quarter, two months, 18 days hence, worth in ready money, abating or discounting 4 $\frac{1}{4}$ per cent. per annum ?

3.465982, time.
 573.4, rate reverfed.

1386393
 103979
 24262
 1733

15.16367 + 100 = 115.16367.
 115.16367 : 100 :: 309.8418 : 269.044627.
 269.044627 l. = 269 l. — s. 10 $\frac{1}{4}$ d. the answer.
 And 309 l. 16 s. 10 d. — 269 l. — s. 10 $\frac{1}{4}$ d. = 40 l. 15 s. 11 $\frac{1}{4}$ d. discount.

CASE III.

The account, principal, and time given; to find the rate of interest.

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R U L E.

As the principal multiplied into the time : is to the whole interest :: so is 100 l. : to the rate per cent. per annum?

1. At what rate of interest, per cent. per annum, will 3628 l. 14 s. 9 d. amount to 3998 l. 12 s. 10 d. in 3 years, and 54 days?

<p>3628.737275, principal. <u>549793.3, time inverted.</u> 10886211825 1088621183 326586355 25401168 3265863 145150 <u>18144</u></p>	<p>3998.6447929, amount. <u>3628.737275, principal.</u> <u>369.907518</u> interest.</p>
--	---

$12330.249681 : 369.907518 :: 100 : 3 \text{ per cent.}$
 $369.907518 \times 100 = 36990.7518.$
 $12330.2497 \div 36990.7518 \{ 3 \text{ per cent. per annum, the answer.}$

2. At what rate of interest, per cent. per annum, will 269 l. — s. 10 d. amount to 309 l. 16 s. 10 d. in 3 years, two months, and 18 days.

<p>269.044627, principal. <u>2895643</u> 8071339 1076178 161427 13452 2421 215 <u>5</u></p>	<p>309.841666, amount. <u>269.044627</u> <u>40.797039</u>, interest.</p>
---	--

$932.5037 : 40.797039 \text{ interest} :: 100 : 4.375 = 4\frac{1}{2}$
 per cent. the answer.

C A S E I V.

The principal, amount, and rate of interest being given, to find the time.

R U L E.

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R U L E.

As the interest of the whole principal for one year, at the given rate : is to one year :: so is the whole : to the time required.

1. In what time will 3628 l. 14s. 9d. amount to 3998 l.
 12s. 10 $\frac{1}{2}$ d. at three per cent. per annum?

$$\begin{array}{r} 3628.7375 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 3998.6348, \text{ amount.} \\ - 3628.7373, \text{ principal.} \\ \hline \end{array}$$

$$\begin{array}{r} 108.862125, \text{ year's interest.} \\ \hline \end{array} \quad \begin{array}{r} 369.9075, \text{ interest.} \\ \hline \end{array}$$

$$\begin{array}{r} 108.862 : 1 :: 369.9073 \\ 108.862 \mid 369.9073 \quad (3.397938 = 3\frac{1}{4} \text{ years, } 54 \text{ days.} \\ \underline{326586} \quad - 25 \end{array}$$

$$\begin{array}{r} 433213 \quad 147938 \\ \underline{326586} \quad \underline{365} \end{array}$$

$$\begin{array}{r} 106627 \quad 739690 \\ \underline{97976} \quad \underline{887628} \end{array}$$

$$\begin{array}{r} 8651 \quad 443814 \\ \underline{7620} \quad \underline{5399737} = 54 \text{ days.} \end{array}$$

$$\begin{array}{r} 1021 \\ \underline{979} \end{array}$$

$$42$$

$$33$$

$$9$$

2. In what time will 269 l. - s. 10 $\frac{1}{2}$ d. amount to 309 l.
 16 s. 10d. at 4 $\frac{1}{2}$ per cent. per annum?

$$269.044627, \text{ principal.}$$

$$309.841668, \text{ amount.}$$

$$573.4, \text{ rate reversed.}$$

$$269.044627, \text{ principal.}$$

$$10761785$$

$$40.797039, \text{ interest.}$$

$$807134$$

$$188330$$

$$13482$$

$$\begin{array}{r} 11.77070, \text{ year's interest.} \\ \hline \end{array}$$

$$X \ 4$$

$$As$$

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As 11.7707 : 1 year :: 40.797039 : 3.465982 years.
 11.7707) 40.797039 (3.465982 = $3\frac{1}{2}$ years, 2 months, 18 days.
 253121 — .25 = $\frac{1}{4}$ year.

$$\begin{array}{r} 548403 \quad 215982 \\ 470828 \quad \frac{1}{8} = 166666 = 2 \text{ months.} \end{array}$$

$$\begin{array}{r} 776659 \quad .049316 = 18 \text{ days.} \\ 706242 \quad , \quad 365 \end{array}$$

$$\begin{array}{r} 70417 \quad 246580 \\ 58854 \quad 295896 \\ \hline 147948 \end{array}$$

$$\begin{array}{r} 11563 \\ 10593 \quad 18.000340 \end{array}$$

970

942

28

23

5

QUESTIONS in the three last CASES resolved by the RULE OF FIVE.

1. What money, at $3\frac{1}{2}$ per cent., will clear 38 l. 10s. in a year and quarter's time?

$$\begin{array}{ccc} \text{P.} & \text{T.} & \text{G.} \\ 100 & . \quad 1 & . \quad 3.5 \\ & & 1.25 \quad . \quad 38.5 \end{array}$$

$100 \times 1 \times 38.5 = 3850$, dividend; and $3.5 \times 1.25 = 4.375$, divisor.

Then $4.375 \overline{) 3850}$ (880 l. the answer.

2. Put out 384 l. to interest, and in $8\frac{1}{4}$ years there were 542 l. 8s. found to be due; what rate of interest could then be implied?

$$542 \text{ l. } 8 \text{ s.} - 384 \text{ l.} = 158 \text{ l. } 8 \text{ s. interest.}$$

$$\begin{array}{ccc} \text{P.} & \text{T.} & \text{G.} \\ 384 & . \quad 8.25 & . \quad 15.4 \\ 100 & . \quad 1 & . \end{array}$$

$158.4 \times 100 \times 1 = 15840$, dividend; and $384 \times 8.25 = 3168$, divisor.

$3168 \overline{) 15840}$ (5 per cent. per annum, the answer.

3. Lent

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3. Lent 109 guineas at 4 per cent. which, by the 18th of August 1760, was raised by the interest to 50 many moidores, bating 2 s. 6 d.; pray on what day did the bond bear date?

109 guineas = 114 l. 9 s. = 114.45, principal.

109 moidores = 147 l. 3 s. which — 2 s. 6 d. = 147 l. — s. 6 d. amount.

Then 147 l. — s. 6 d. — 114 l. 9 s. = 32 l. 11 s. 6 d. = 32.575, interest.

P.	T.	G.
100	1	4
114.45	.	32.575

$100 \times 1 \times 32.575 = 3257.5$, dividend; and $114.45 \times 4 = 457.8$.

$457.8 \div 3257.5 (7.11555 = 7 \text{ years and } 42 \text{ days})$.

July hath 31 days.

Till August 18

Sum 49 — 42, gives July the 7th, the answer.

4. If 100 l. in 12 years be allowed to gain 39 l. 19 s. 8 d. in what time will any other sum of money double itself by the same rate of interest?

P.	T.	G.
100	12	39.983
1	.	1

$39.983 \div 1200 = 33.319166$
 $3998 \div 120 = 33.316666$

35.985) 1080.000 (30.0125 = 30 years and $4\frac{1}{2}$ days, answ.

5. In what time will the interest of 49 l. 3 s. equal the proceed of 19 l. 6 s. at use 47 days, at any rate of interest?

Reciprocally, 19.3 l. : 47 days :: 49.15 l. : 18.45 days, the answer.

5. A bond was made on the 7th of August 1713, at 6 per cent. per annum, for 1114 l. 10 s.; on the 11th of May, 1718, 140 l. was paid off, and a fresh bond entered into for the remainder, at $5\frac{1}{2}$ per cent. per annum; at the time the interest for this last was 21 l. 16 s. 8 d. there was paid off 87 l. 11 s. 9 d. The old bond being then taken up, a new one was given for the residue, which being paid off September 11, 1724, the bond-owner took no more than

1409 l.

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 1409 l. 16s. 8d, in full payment. At what rate then did
 he take interest per cent. per annum, upon the last renewal
 of the bond?

To the 11th of May, 1718, are 4 years 277 days =
 4.758904 $1114.5 \times 6 = 6687$
 78.66, year's interest reversed.

28553424
 2855342
 381712
 32312

318.2279, interest for 4 years, 277 days.
 140 paid off.

178.2279, surplus.

$1114.5 l. + 178.2279 = 1292.7279 l.$ new principal.

P.	T.	G.
100	.	5.25
1292	.	21.83

$6787.8215) 2183.33332 (.3217 = 117$ days, which an-
 swers to the 5th of September 1718.

Then $1292.7279 l. + 21.83 l. = 1314.5612 l.$
— 87.5875 paid off.

£ 1226.9737, new princip.

From September 5, 1718, 'till September 11, 1724, are
 six years, six days, = 6.01644.

And $1409.83 - 1226.9737 l. = 182.8596 l.$ interest.

P.	T.	G.
1226.9737	.	6.01644
100	.	182.8596

$\therefore 7382.0136) 18285.96 (2.477096 = 2 l. 9 s. 6 \frac{1}{2} d.$ the
 answer required.

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S E C T. II.

I N S U R A N C E.

INSURANCE is security given in consideration of
 so much per cent. paid in hand to make good ships,
 merchandizes, houses, &c. to the value of that for which
 the

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the premium is received, in case of loss by storm, pirates, fire, or the like.

This, as well as brokerage and commission, is computed in the same manner as simple interest for a year.

What is the insurance of 737 l. 18 s. at $3\frac{1}{2}$ per cent. ?

$$\begin{array}{r}
 \text{l. s.} \\
 737 \text{ } 18 \\
 \quad 3\frac{1}{2} \\
 \hline
 2213 \text{ } 14 \\
 368 \text{ } 19 \\
 92 \text{ } 4 \text{ } 9 \\
 \hline
 \text{£ } 26.74 \text{ } 17 \text{ } 9 \\
 20 \\
 \hline
 14.97 \text{ s.} \\
 12 \\
 \hline
 11.78 \text{ d.} \\
 4 \\
 \hline
 2.92 \text{ qrs.}
 \end{array}$$

Answer, 26 l. 14 s. 11 $\frac{1}{2}$ d.

$$\begin{array}{r}
 \text{Or,} \\
 \text{737 } 18 \text{ -} \\
 \hline
 2 \text{ s.} = \frac{1}{2} \text{ } 73 \text{ } 15 \text{ } 9\frac{1}{2} \\
 \hline
 1 \text{ s.} = \frac{1}{4} \text{ } 14 \text{ } 15 \text{ } 1\frac{1}{2} \\
 \frac{1}{4} = \frac{1}{8} \text{ } 7 \text{ } 7 \text{ } 6\frac{1}{2} \\
 \frac{1}{8} = \frac{1}{16} \text{ } 3 \text{ } 13 \text{ } 9\frac{1}{2} \\
 \hline
 \text{£ } 26 \text{ } 14 \text{ } 11\frac{1}{2}
 \end{array}$$

What is the insurance of 874 l. 13 s. 6 d. at $13\frac{1}{2}$ per cent. ?

$$\begin{array}{r}
 \text{l. s. d.} \\
 874 \text{ } 13 \text{ } 6 \\
 \quad 13\frac{1}{2} \\
 \hline
 11370 \text{ } 15 \text{ } 6 \\
 437 \text{ } 6 \text{ } 9 \\
 \hline
 118.08 \text{ } 2 \text{ } 3 \\
 20 \\
 \hline
 1.62 \\
 12 \\
 \hline
 7.47 \\
 4 \\
 \hline
 1.88
 \end{array}$$

$$\begin{array}{r}
 \text{Or,} \\
 \text{l. s. d.} \\
 10 \text{ per cent} = \frac{1}{10} \text{ } 874 \text{ } 13 \text{ } 6 \\
 \hline
 2 = \frac{1}{5} \text{ } 87 \text{ } 9 \text{ } 4 \\
 1 = \frac{1}{10} \text{ } 17 \text{ } 9 \text{ } 10\frac{1}{2} \\
 \frac{1}{5} = \frac{1}{2} \text{ } 8 \text{ } 14 \text{ } 11 \\
 \hline
 4 \text{ } 7 \text{ } 5\frac{1}{2} \\
 \hline
 118 \text{ } 1 \text{ } 6\frac{1}{2}
 \end{array}$$

Answer, 118 l. 1 s. 7 $\frac{1}{2}$ d.

Primage

Primage is an allowance paid to mariners at their first sailing out of port for their loading the ship.

Stowage is the money paid for stowing the goods in a vessel.

Average is the quota or proportion, which each proprietor of a ship, or the goods therein, is adjudged, on a reasonable estimation, to contribute toward the losses which are sustained by some of the goods being cast overboard for the preservation of the rest, and of the ship.

What is the insurance of an East-India ship and cargo, valued at 35727 l. 17 s. 6 d. at $17\frac{1}{4}$ per cent.?

	l.	s.	d.		l.	s.	d.
	35727	17	6		$\frac{1}{16}$ 35727	17	6
			$17\frac{1}{4}$				
10 times	357278	15	—	10 per cent. =	3572	15	9
7 times	250095	2	6	5 per cent. =	1786	7	$10\frac{1}{2}$
$\frac{1}{2}$ - -	17863	18	9	$2\frac{1}{2}$ per cent. =	893	3	$11\frac{1}{2}$
$\frac{1}{4}$ - -	8931	29	$4\frac{1}{2}$	$\frac{1}{4}$ per cent. =	89	6	$4\frac{1}{2}$
$\frac{1}{8}$ - -	4465	19	$8\frac{1}{4}$	$\frac{1}{8}$ per cent. =	44	13	$2\frac{1}{2}$
	<u>£ 6386.35</u>	15	$3\frac{1}{2}$	11 $\frac{1}{4}$ per cent. =	<u>6386</u>	15	$3\frac{1}{2}$
	20						
	<u>7.14 s.</u>						
	12						
	<u>1.83 d.</u>			Answer, 6386 l. 7 s. $1\frac{1}{2}$ d.			
	4						
	<u>3.35 qrs.</u>						

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S E C T. III.

B R O K A G E.

BROKERAGE, or BROKAGE, is the fee or reward paid unto a person called a broker, for assisting a merchant or factor in buying or selling goods, &c. This business

finest was formerly carried on by broken merchants or traders, from whence their name derived; and in London they are not to act without licence from the lord mayor.

What is the brokerage of 856 l. 6 s. 8 d. at 6 s. per cent.?

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 856 \quad 6 \quad 8 \\ \underline{20} \end{array}$$

$$11.26 \text{ s.}$$

$$\underline{12}$$

$$3.20 \text{ qrs.}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ \frac{1}{4} \quad 8 \quad 11 \quad 3 \end{array}$$

$$\begin{array}{r} \frac{3}{4} \quad 2 \quad 2 \quad 9\frac{1}{2} \\ - \quad 8 \quad 6\frac{1}{2} \end{array}$$

Answer, £ 2 11 4½

What is the brokerage of 737 l. 13 s. at 4 s. 9 d. per cent.?

$$\begin{array}{r} \text{l.} \quad \text{s.} \\ 737 \quad 13 \\ \underline{20} \end{array}$$

$$7.53 \text{ s.}$$

$$\underline{12}$$

$$6.36 \text{ d.}$$

$$\underline{4}$$

$$1.44 \text{ qrs.}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ \frac{1}{4} \quad 7 \quad 7 \quad 6\frac{1}{2} \end{array}$$

$$\begin{array}{r} \frac{1}{4} \quad 1 \quad 9 \quad 6 \\ \frac{1}{4} \quad 3 \quad 8\frac{1}{2} \\ \hline 1 \quad 10 \end{array}$$

Answer, £ 1 15 ¼

What is the brokerage of 2572 l. 15 s. at ½ per cent.?

$$\begin{array}{r} \text{l.} \quad \text{s.} \\ 2572 \quad 15 \\ \underline{20} \end{array}$$

$$14.55 \text{ s.}$$

$$\underline{12}$$

$$6.60 \text{ d.}$$

$$\underline{4}$$

$$2.40 \text{ qrs.}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ \frac{1}{4} \quad 25 \quad 14 \quad 6\frac{1}{2} \end{array}$$

$$\begin{array}{r} \frac{1}{4} \quad 6 \quad 8 \quad 7\frac{1}{2} \\ - \quad 3 \quad 4 \quad 3\frac{1}{2} \end{array}$$

Answer, £ 9 12 11½

S E C T. IV. P U R C H A S I N G S T O C K S.

R U L E.

M U L T I P L Y the sum to be purchased by the ex-
cels above 100, and then proceed as before directed
in computing interest, the product of which added to the
given stock, gives the purchase; or you may find it by
practice, if more convenient.

What is the purchase of 987 l. 15 s. South-Sea stock,
at 113 $\frac{1}{2}$ per cent.?

l.	s.
987	15
	13 $\frac{1}{2}$
<hr/>	
12840	15 -
493	17 6
246	11 9
128	9 4 $\frac{1}{2}$

£ 137.05 - 7 $\frac{1}{2}$

1.00 s.

£ 987 15
+ 137 1

£ 1124 16, the answ.

l.	s.	d.
987	15	-
98	15	6
19	15	12 $\frac{1}{2}$
9	17	6 $\frac{1}{2}$
4	18	9 $\frac{1}{2}$
2	9	4 $\frac{1}{2}$
1	4	8 $\frac{1}{2}$

£ 1124 15 11 $\frac{1}{2}$, answer.

2. 769 l. India stock, at 117 $\frac{1}{2}$ per cent.?

l.	s.	d.	l.	s.	d.
769	-	-	10	-	-
76	18	-	5	-	-
38	9	-	2	5	-
15	7	7 $\frac{1}{2}$	-	5	-
1	18	5 $\frac{1}{2}$	-	2	6
-	19	2 $\frac{1}{2}$	-	-	-

£ 902 12 3, the answer.

3. What

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3. What is the purchase of 1797l. 10s. Bank stock, at 93½ per cent.?

l.	s.	d.	l.
1797	10		
384	7	6	25
76	17	6	5
19	4	4½	1

£ 2017 16 4½, the answer required.

4. 1812l. Bank annuities, at 93½ per cent.?

First, 100	l.
— 93½	1812
6½	90 12
— 3	22 13 d.
1812	—
— 115 10 3½	£ 115 10 3½

£ 1696 9 1½, the answer required.

5. What is the purchase of 1727l. Bank stock, at 119 per cent.?

l.	s.	d.	By PRACTICE.
1727			1727
19			172
— 100			86
£ 328 13			34 10 9½
20			34 10 9½
— 100			
260 s.			£ 2055 2 7, as before.
12			
— 100			
720 d.			

l.	s.	d.
1727	—	—
328	2	7

£ 2055 2 7, the answer.

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6. June the 23d, 1745, bought 900 l. of new South-Sea annuities, at $112\frac{1}{2}$ per cent. viz. the day before the closing the books, the brokerage whereof is alway 2 s. 6 d. per cent. on the capital, whether you buy or sell. The Midsummer dividend, 2 per cent. became due and payable on the 10th of August following; by which time the rebellion growing considerable in the north, the said annuities were down at $92\frac{1}{2}$ per cent. In the general alarm, sold 400 l. capital at that price; but continued the remainder 'till a second, third, fourth, and fifth dividend, as before, came due: and on opening the books on the 10th of August 1747, sold out at $102\frac{1}{2}$ per cent. Now reckoning I might have made five per cent. of my money, had I kept it out of the stocks, how stood this article in point of profit and loss?

	l.	s.	d.
100 : $112\frac{1}{2}$:: 900 :	1002	7	6
Brokerage of 900 l. at 2 s. 6 d. per cent.		1	2
	1003	10	-
1745. Midsummer dividend, at 2 per cent.	18	-	-
	985	10	-
Inter. of 1003 l. for 45 days, at 5 per cent. per ann.	6	14	8
Brokerage of 400 l. at 2 s. 6 d. per cent.	-	-	10
	992	14	8
Sold 400 l. at $92\frac{1}{2}$ per cent.	370	-	-
	622	14	8
Interest for $\frac{1}{4}$ a year, due the 10th of Feb. 1746	15	11	$4\frac{1}{2}$
	638	6	$\frac{1}{4}$
Dividend received at that time	10	-	-
	628	6	$\frac{1}{4}$
Interest due the 10th of August, 1746	15	14	$1\frac{1}{2}$
	644	-	2
Dividend received at that time	10	-	-
	634	-	2
Interest due the 10th of February 1747	15	17	-
	649	17	2
Dividend received then	10	-	-
Carried over	639	17	2

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	l.	s.	d.
Brought over	639	17	2
Interest the 10th of August, 1747	15	19	11
	655	17	1
Midsummer dividend received Aug. 10, 1747	10	-	-
	645	17	1
Sold off 500 l. at 102½ per cent.	512	2	6
	132	14	7
Brokerage	-	12	6
To my damage in the whole	132	2	1



S E C T. V.

REBATE, or DISCOUNT.

REBATE, or DISCOUNT, is an abatement of a sum of money due some time hence, in consideration of the prompt or present payment of the remainder.

The ready money that will satisfy the debt, is called the present worth; because if it was put out to interest at the given rate per cent. per annum for the time the discount is computed, it would amount to the given debt.

The true method of finding the discount of any sum, is by Case II. in simple interest; or,

R U L E,

First find the interest of 100 l. for the time mentioned; then, as 100 l. with the interest is : to the interest :: so is the debt or sum proposed to be discounted : to the discount required; which, subtracted from the debt, leaves the present worth, or money to be paid down.

1. What is the discount of 57 l. 18 s. due 12 months hence, at five per cent. per annum?

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$$\begin{array}{rcl}
 105 : 5 :: 57.9 & \text{l. s. d.} & \\
 & 57 \text{ 18} & \text{debt.} \\
 105) 289.5 (2.75238 & = & 2 \text{ 15} \text{ } \frac{1}{2} \text{ discount.} \\
 & \underline{\text{£ } 55 \text{ 2 11} \frac{1}{2}} & \text{present worth.}
 \end{array}$$

2. What is the discount of 573 l. 15 s. due three years hence, at $4\frac{1}{2}$ per cent. per annum?

$$4.5 \times 3 = 13.5 = \text{interest of 100 l. for three years.}$$

$$113.5 : 13.5 :: 573.75$$

$$\begin{array}{r}
 13.5 \\
 \hline
 286875 \\
 172125 \\
 \hline
 57375
 \end{array}$$

$$\begin{array}{rcl}
 113.5) 7745.625 (68.24339 & = & 573 \text{ 15} \text{ } \frac{1}{2} \text{ discount.} \\
 & & \underline{505 \text{ 10 } \frac{1}{2} \text{ Q. E. F.}}
 \end{array}$$

3. What is the discount of 725 l. 16 s. for five months, at $3\frac{1}{4}$ per cent. per annum?

$$\begin{array}{rcl}
 & \text{M.} & \text{G.} & \text{M.} & \text{G.} \\
 \text{As } 12 : 3.875 & :: & 5 : 1.614583 \\
 101.614583 : 1.614583 & :: & 725.8
 \end{array}$$

$$\begin{array}{r}
 725.8 \\
 \hline
 12918666 \\
 80729186 \\
 322918666 \\
 \hline
 11302083333
 \end{array}$$

$$\begin{array}{rcl}
 101.614583) 1171.864583 & (& \\
 10161458 & & 117186458
 \end{array}$$

$$\begin{array}{rcl}
 91.453185) 1054.678125 & (11.532435 = & 11 \text{ l. } 10 \text{ s. } 7 \frac{1}{4} \text{ d.} \\
 140146275 & & \\
 48693090 & & \\
 2966497 & & \\
 222901 & & \\
 39995 & & \\
 3259 & & \\
 510 & &
 \end{array}$$

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Or to find the present money, observe the following

R U L E.

First find the interest of 100 l. for the time mentioned as before; then say, As 100 l. with the interest added : is to 100 l. :: so is the debt or sum proposed to be discounted : to the present money.

4. What ready money will discharge a debt of 543 l. 7 s. due four months and 18 days hence, at $4\frac{1}{4}$ per cent. per annum?

$$\begin{array}{r} 4 \text{ months} = .233333 \\ 18 \text{ days} - = .049315 \\ \hline \end{array}$$

$$\begin{array}{l} \text{Time} = .382648 \times 4.625 = 1.769748, \text{ int. of } 100 l. \\ \text{As } 101.769748 : 100 :: 543.35 : 533.9012 = 533 l. 18 s. - \frac{1}{4} d. \end{array}$$

$$\begin{array}{r} \text{Debt} - - - 543 \quad 7 \quad - \\ \text{Present worth } 533 \quad 18 \quad - \frac{1}{4}, \text{ the answer.} \end{array}$$

$$\text{Discount} - \underline{\underline{\pounds 9 \quad 8 \quad 11\frac{1}{4}}}$$

5. What ready money will discharge a debt of 1377 l. 13 s. 4 d. due two years, three quarters, 25 days hence, discount $4\frac{1}{2}$ per cent. per annum?

$$\begin{array}{r} 2\frac{1}{2} \text{ years} = 2.75 \\ 25 \text{ days} = .068493 \\ \hline \end{array}$$

$$\begin{array}{l} 2.818493 \times 4.375 = 12.330907 = \text{int. of } 100 l. \\ 112.330907 : 100 :: 1377.8 : 1226.4359858. \\ \text{Answer, } 1226.4359858 = 1226 l. 8 s. 8\frac{1}{2} d. \text{ present money.} \end{array}$$

6. What difference is there between the interest of 500 l. at five per cent. per annum, for 12 years, and the discount of the same sum, at the same rate, for the same time?

$$\begin{array}{ccc} \text{P.} & \text{T.} & \text{G.} \\ 100 & . & 1 \\ 500 & . & 12 \\ & & 5 \end{array}$$

$$5 \times 500 \times 12 = 30000, \text{ dividend.}$$

$$100 \times 1 = 100 \quad 30000 \quad (300, \text{ the interest.})$$

$$\text{Then } 12 \times 5 = 60, \text{ the interest of } 100 l. \text{ for } 12 \text{ years.}$$

$$\text{And } 100 + 60 = 160, \text{ its amount.}$$

$$\text{As } 160 : 60 :: 500 : 187 l. 10 s. \text{ the discount.}$$

$$\therefore 300 l. - 187 l. 10 s. = 112 l. 10 s. \text{ advantage to the interest.}$$

S E C T. VI.

EQUATION OF PAYMENTS.

WHEN several debts are payable at different times, and it is mutually agreed between debtor and creditor, that all those several sums be paid at such a time, that neither debtor nor creditor may be wronged thereby, this is called the equated time of payment. The rule given by Mr. Cocker and others for finding this equated time is,

R U L E,

Multiply each payment by its time, and divide the sum of all these products by the whole debt ; the quotient was by him accounted the equated time.

A person dying, bequeaths to a younger son 1000 l. to be paid as follows ; viz. 300 l. at one year's end ; 300 l. more at a year and a half ; and the remainder at the end of two years and a half. Now the executor agrees with the legatee, to pay the whole at one payment ; how long from the death of the father must this payment be, so that neither party be wronged, or suffer loss ?

$$\begin{array}{rcl} \text{By the foregoing rule } 300 \times 1 & = & 300 \\ 300 \times 1\frac{1}{2} & = & 450 \\ 400 \times 2\frac{1}{2} & = & 1000 \\ \hline & & 1750 \end{array}$$

1000) 1750 (.75, or $1\frac{3}{4}$ year,
the answer.

Mr. Kersey finds fault with the foregoing method, as no interest is thereby implied, and thinks that a discount (statute interest) should be allowed for each payment ; and to find the equated time, gives a rule to the following purport.

R U L E.

Find the present worth of each payment, according to its respective time and rate ; then add all the present worths together, and call their sum the principal ; lastly, having the principal, amount, and rate of interest, find the time by Case IV. of simple interest.

Now allowing a discount of five per cent. the solution of the foregoing question will be as follows :

I

viz.

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$$\text{viz. } 105 : 100 :: 300 : 285.7142$$

$$107.5 : 100 :: 300 : 279.6697$$

$$112.5 : 100 :: 300 : 355.5555$$

$$\underline{920.3394}$$

5 per cent.

$$\underline{\pounds 46.01697, \text{ int. for a year.}}$$

1000, amount.

920.3394, principal.

$$46.01697 : 1 :: 79.6606 : 1.7311 = 1 \text{ year, 8 months, 23 days, the equated time.}$$

But the learned Mr. Alexander Malcolm justly observes, that though the debtor gains the interest of what he keeps after it was due, that he loses only the discount of what he pays before it was due, which is less than the interest; and that therefore the creditor may justly except against Mr. Cocker's method; and I apprehend, that for the same reason the debtor may have as just an exception against Mr. Kersey's.

The before-mentioned Mr. Malcolm, from an algebraic way of reasoning founded on the principles of simple interest, raises and demonstrates a theorem, from whence is deduced the following

R U L E.

Find one year's interest of the debt that is first payable, by which divide the sum of the debts (of the first and second payments) and to the quotient add the sum of the times; call this the first number found.

Then multiply each debt by its time, and divide the sum of the products by one year's interest of the first payable debt; which quotient added to the product of the two times, call the second number found.

Subtract the second number from $\frac{1}{4}$ of the square of the first number, and out of the difference extract the square root; which root being added to, or subtracted from half the first number found, the sum or difference will be the time sought.

N. B. As this rule is ambiguous, if you take the sum, if that happens to be greater than the time to the term of the last payable debt, the difference will be the time sought. Or

Y 3

if

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if you take the difference, and that be less than the time to the term of the first payable debt, the sum will be the time sought. But if both the sum and difference happen between the two given terms, it must be examined which of them will make an equality of interest and discount.

I shall here reassume the foregoing question, allowing the interest and dividend at five per cent. per annum?

$$\text{Debts } \left\{ \begin{array}{l} 300, \text{ its interest for a year } 15 \text{ l. } 300 \times 1 = 300 \\ 300 \quad \quad \quad \quad \quad \quad \quad \quad 300 \times 1.5 = 450 \end{array} \right.$$

$$15) \underline{600}$$

$$\underline{40}$$

$$+ 2.5, \text{ sum of the times.}$$

$$\underline{42.5, \text{ first N}^{\circ} \text{ found.}}$$

$$15) \underline{750}$$

$$\underline{50}$$

$$+ 1.5$$

$$\underline{\text{Second N}^{\circ} 51.5}$$

$$\text{Then } 42.5 \times 42.5 = 1806.25, \text{ which } \div 4 = 451.5625.$$

$$\text{Also } 451.5625 - 51.5 = 400.0625.$$

$$\sqrt{400.0625} = 20.0015624; \text{ and } 2) 42.5 (21.25.$$

$$\therefore 21.25 - 20.0015624 = 1.248437 \text{ years, the equated time for the two first payments.}$$

Then to find the true equated time when the whole 1000 l. must be paid together,

$$\text{Put } 600 \text{ l. for the first payment, interest for a year } 30 \text{ l.} \\ 400 \text{ l. second payment.}$$

$$30) \underline{1000}$$

$$33-333333$$

$$\underline{3-748437}$$

$$2) \underline{37.08177, \text{ first number found.}}$$

$$\underline{18.54088, \text{ its half.}}$$

$$\text{Also } 600 \text{ l. } \times 1.2484376 = 749.06256$$

$$400 \times 2.5 = 1000$$

$$30) \underline{1749.06256}$$

$$\underline{58.302085}$$

$$1.2484376 \times 2.5 = \underline{3.121094}$$

$$\text{Second number found} = \underline{61.423179}$$

And

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$$\text{And } 37.08177 \times 37.08177 = 1375.057666$$

$$4) 1375.057666 \quad (343.764416$$

$$343.764416 - 61.423179 = 282.341237$$

$$\sqrt{282.341237} = 16.80284$$

$\therefore 18.54088 - 16.80284 = 1.73804 = 1 \text{ year, } 8 \text{ mon. } 26 \text{ days, the true equated time required.}$

2. At Michaelmas, seventeen hundred nineteen,
My writings will shew (which are yet to be seen)
That to me were three hundred and twenty pounds due,
And half of that sum, besides forty-two, viz. 202 l.
Just five years after, I then might demand,
But would fain have the whole somewhat sooner (in
I agree to rebate for the latter sum too, hand.)
The same rate (simple) interest our statutes allow,
But then I expect some use will accrue
From my sixteen-score pounds, that last year were due.
Now to know on what day, I should be very fond,
To receive my five hundred and twenty-two pound.

Ladies Diary, 1720.

Debts $\left\{ \begin{array}{l} 320 \text{ l. its interest for one year is } 16 \text{ l.} \\ 202 \end{array} \right.$

$$16) 522 \quad (32.625 + 5 \text{ time} = 37.625, \text{ the 1st number.}$$

The 320 being according to the securities to be paid down.

$$\text{So } 202 \times 5 = 1010$$

$$\text{And } 16) 1010 \quad (63.125, \text{ the second number.}$$

$$\text{Then } 37.625 \times 37.625 = 1415.640625$$

$$\text{Also } 4) 1415.640625$$

$$\underline{353.91015625}$$

$$- 63.125$$

18.8125, half the first number.

$$\sqrt{295.78515625} = 17.0524$$

$$1.7601 = 1 \text{ year, } 277 \text{ days,}$$

which answers to July 4, 1721. Q. E. F.

As there are only a few days difference between this and the other method, and that this method will be operose, particularly when the payments are to be made at many different times, either of the former methods may do without any considerable wrong to either party; yet, in my opinion, truth is worth enquiring after.

CHAPTER III.

TARE AND TRET.

TARE is an allowance in merchandize made by the king to the importer, or to the buyer by the merchant, for the weight of the bag, cask, chest, wrappers, &c. in which any goods are put; several sorts of goods have their tares ascertained in a table annexed to the book of rates.

Gross weight is the whole weight of goods, with the chest, cask, bag, &c. that contains them.

Tret is an allowance, in weighable goods, of 4 lb. in 104 lb. made by merchants in London to their tradesmen and retailers for break, waste or dust, yet himself is only allowed tare in paying custom; so that he payeth as well for the bad as the best commodity.

Cloff, clough, or draught, is a small allowance made by the king to the importer, or by the seller to the buyer, to cause the weight to hold out when goods are weighed again. The king allows 1 lb. draught for goods under 1 cwt.; 2 lb. from 1 to 2 cwt.; 3 lb. from 2 to 3 cwt. 4 lb. from 3 to 10 cwt.; 7 lb. from 10 to 18 cwt.; and 9 lb. from 11 to 30 cwt. and upwards.

Subtile, or futtle, is the weight of the goods when the tare is deducted, but not the tret.

Net weight is the remainder, when both, if both be allowed, are taken away.

ALIIQUOT PARTS.

Cwt.		lb.		lb.
10 = $\frac{7}{8}$	} of a tun.	14 = $\frac{1}{8}$	} of a cwt.	14 = $\frac{1}{2}$
5 = $\frac{1}{2}$		16 = $\frac{1}{7}$		7 = $\frac{1}{2}$
4 = $\frac{1}{3}$		14 = $\frac{1}{4}$	} of $\frac{1}{2}$ cwt.	4 = $\frac{1}{4}$
2 = $\frac{1}{4}$		8 = $\frac{1}{7}$		3 = $\frac{1}{4}$
2 = $\frac{1}{10}$		7 = $\frac{1}{8}$		

CASE I.

When the given tare is the aliquot part of an hundred, as 14 or 16 lb.

RULE,

Divide the given weight by the denominator of the fraction representing the part, the quotient will be the tare.

1. What

1. What is the net weight of four barrels of figs?

	Cwt.	qrs.	lb.	
viz. N ^o 1	- - -	3	1	18
2	- - -	4	3	24
3	- - -	6	-	20
4	- - -	5	2	26
				} tare 14 lb. per cwt.
8) 20 1 4, gros.				
2 2 4, tare.				
Cwt. 17 3 → net.				

2. What is the net weight of five bags of cinnamon?

	Cwt.	qrs.	lb.	
viz. N ^o 1	- - -	1	3	24
2	- - -	3	1	18
3	- - -	2	2	16
4	- - -	2	2	26
5	- - -	1	3	22
				} tare 16 lb. per cwt.
7) 12 3 22, gros.				
1 3 11, tare.				
Cwt. 11 - 11, net.				

CASE II.

When the tare is not an aliquot part of an hundred, but the aliquot part of $\frac{1}{2}$ or $\frac{1}{4}$ of a cwt.

RULE,

Take first the aliquot part of an hundred, and then part of that part, agreeable to the nature of the question, until you have found the true tare.

Another way of finding the tare when it is not an aliquot part of 112 lb.

RULE,

Multiply the hundreds by the tare to be allowed for 1 cwt. and for the quarters and pounds, in the gross weight, take a propor-

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proportional part of the said allowance, and the sum is the tare in pounds; which is either to be reduced into hundreds, and deducted from the gross weight, or the gross weight into pounds, and then deduct the tare.

What is the net weight of 19 cwt. 2 qrs. 12 lb. gross, tare 7 lb. per cwt.?

Cwt. qrs. lb.

$$\begin{array}{r} 4 \overline{) 19 \ 2 \ 12 \text{ gross.}} \\ 4 \overline{) \ 4 \ 3 \ 17} \\ \hline \ 1 \ - \ 25\frac{1}{2} \text{ tare.} \end{array}$$

Cwt. 18 1 14 $\frac{1}{2}$, net.

Otherways.

$$\begin{array}{r} 19 \times 7 = 133 \text{ lb.} \\ \frac{1}{2} \times 7 = 3\frac{1}{2} \\ \text{lb. } 8 = \frac{1}{2} \\ 4 = \frac{1}{2} \end{array}$$

lb. 137 $\frac{1}{2}$ = 1 cwt. 25 $\frac{1}{2}$ lb.
tare, as by the other method.

2. What is the net weight of 15 cwt. 2 qrs. 21 lb. tare 8 lb. per cwt.?

Cwt. qrs. lb.

$$\begin{array}{r} 7 \overline{) 15 \ 2 \ 21} \\ 2 \overline{) \ 2 \ - \ 27} \\ \hline \ 1 \ - \ 13\frac{1}{2} \text{ tare.} \\ \hline 14 \ 2 \ 7\frac{1}{2} \text{ net.} \end{array}$$

Or, 15 \times 8 = 120

$$\begin{array}{r} \frac{1}{2} \times 8 = 4 \\ \text{lb. } 14 = 1 \\ 7 = \frac{1}{2} \end{array}$$

Tare 125 $\frac{1}{2}$ = 1 cwt. 13 $\frac{1}{2}$ lb.
as before.

3. What is the net weight of four bags of hops, tare 4 lb. per cwt.?

Cwt. qrs. lb.

viz. No 1 - - 4 1 18
2 - - 3 3 24
3 - - 4 2 16
4 - - 4 3 4

Otherways thus.

$$\begin{array}{r} 17 \times 4 = 68 \\ \frac{1}{2} \times 4 = 2 \\ \frac{1}{4} \times 4 = 1 \end{array}$$

$$\begin{array}{r} 4 \overline{) 17 \ 3 \ 6 \text{ gross.}} \\ 7 \overline{) \ 4 \ 1 \ 22\frac{1}{2}} \\ \hline \ 2 \ 15 \text{ tare.} \end{array}$$

17 - 19, net.

lb. 71 = 2 qrs. 15 lb.
tare, as before.

CASE III.

When the tare is no aliquot part of an hundred weight, quarter, &c.

R U L E,

Divide the given tare into aliquot parts of an hundred, quarter, &c. the sum of which will be the answer.

1. What is the net weight of 19 cwt. 3 qrs. 14 lb. of antimony, tare 6 lb. per cwt.?

Cwt.	qrs.	lb.
7	19	3 14
2	2	3 10
	1	1 19
4	4	1 1
	1	- 7 $\frac{1}{2}$ tare.
	18	3 6 $\frac{1}{2}$ net.

Or thus.

$$\begin{aligned}
 19 \times 6 &= 114 \\
 \frac{1}{2} \times 6 &= 3 \\
 \frac{1}{4} \times 6 &= 1\frac{1}{2} \\
 \text{lb. } 14 &= \frac{7}{2} \\
 \hline
 119\frac{1}{4} &= 1 \text{ cwt.} \\
 7\frac{1}{2} \text{ lb.} &\text{ tare, as before.}
 \end{aligned}$$

2. What is the net weight of 71 cwt. 3 qrs. 21 lb. of pot-ashes, tare 10 lb. per cwt.?

Cwt.	qrs.	lb.
7	71	3 21
4	10	1 3
	2	2 7 $\frac{1}{2}$
2	12	3 10 $\frac{1}{2}$
	6	1 19 $\frac{1}{2}$ tare.
	65	2 1 $\frac{1}{2}$ net.

Or thus.

$$\begin{aligned}
 71 \times 10 &= 710 \\
 \frac{1}{2} \times 10 &= 5 \\
 \frac{1}{4} \times 10 &= 2\frac{1}{2} \\
 14 &= 1\frac{1}{2} \\
 7 &= \frac{7}{2} \\
 \hline
 \text{Tare } 719\frac{1}{4} &= 6 \text{ cwt.} \\
 1 \text{ qr. } 19\frac{1}{4} \text{ lb.} &\text{ as before.}
 \end{aligned}$$

3. What is the net weight of five casks of alum, tare 12 lb. per cwt.?

Cwt. qr. lb.				Otherways.	
viz. N ^o	1	- - 2	1 27	$13 \times 12 =$	156
	2	- - 1	3 25	$\frac{1}{2} \times 12 =$	6
	3	- - 3	- 18	lb. 14 =	$1\frac{1}{2}$
	4	- - 2	2 21	lb. 8 =	$\frac{3}{4}$
	5	- - 3	1 15		

2	13	2	22
7	6	3	11
2	-	3	$25\frac{1}{2}$
	-	1	$26\frac{3}{4}$

Tare 1 1 $24\frac{1}{4}$

Net 12 - $25\frac{1}{4}$

Tare $164\frac{1}{4}$, as before =
1 cwt. 1 qr. $24\frac{1}{4}$ lb.

4. What is the net weight of five casks of oil, weighing as follows, tare 18 lb. per cwt.?

Cwt. qrs. lb.				Otherways.	
N ^o	1	- - 3	3 19	$21 \times 18 =$	378
	2	- - 4	1 25	$\frac{1}{2}$ of 18 =	9
	3	- - 3	2 21	$\frac{1}{4}$ of 18 =	$4\frac{1}{2}$
	4	- - 5	- 17	$\frac{1}{7}$ of 18 =	$2\frac{1}{2}$
	5	- - 4	2 18		

7	21	3	16
8	3	-	$14\frac{1}{2}$
	-	1	$15\frac{3}{4}$

Tare 3 2 2

Net 18 1 14

Tare 394 = 3cwt. 2qrs.
2 lb. as before.

5. What

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5. What is the net weight of 27 cwt. 1 qr. 21 lb. of prunes in casks, tare 20 lb. per cwt.?

Cwt. qrs. lb.

$$\begin{array}{r} 7 \overline{) 27 \quad 1 \quad 21} \\ 4 \quad 3 \quad 3 \quad 19 \\ - \quad 3 \quad 25\frac{1}{2} \end{array}$$

4 3 16 $\frac{1}{2}$, tare.

Cwt. 22 2 4 $\frac{1}{4}$, net.

Otherways.

$$\begin{array}{l} 27 \times 20 = 540 \\ \frac{1}{4} \text{ of } 20 = 5 \\ \frac{1}{8} \text{ of } 20 = 2\frac{1}{2} \\ \frac{1}{16} \text{ of } 20 = 1\frac{1}{4} \end{array}$$

Tare 548 $\frac{1}{2}$ = 4cwt. 3qrs. 16 $\frac{1}{2}$ lb. as before.

6. What is the net weight of seven fats of hogs bristles, each containing 3 qrs. 19 lb. tare 17 lb. per cwt.?

Cwt. qrs. lb.

$$\begin{array}{r} - \quad 3 \quad 19 \\ \times 7 \\ 8 \overline{) 6 \quad 1 \quad 21} \\ 7 \quad - \quad 3 \quad 6\frac{1}{2} \\ 2 \quad - \quad 12\frac{1}{2} \\ - \quad 6\frac{1}{2} \\ - \quad 3 \quad 25\frac{1}{4}, \text{ tare.} \end{array}$$

Cwt. 5 1 23 $\frac{1}{4}$

Otherways.

$$\begin{array}{l} 6 \times 17 = 102 \\ \frac{1}{4} \times 17 = 4\frac{1}{4} \\ \frac{1}{8} \times 17 = 2 \\ \text{lb. } 7 = 1 \end{array}$$

Tare 109 $\frac{1}{4}$ = 3 qrs. lb. 25 $\frac{1}{4}$

In many commodities the allowance for tare is not reckoned by the hundred weight, but so much of the gross; this is called invoice tare.

CASE IV.

When the tare of raw silk from Smyrna or Cyprus is to be deducted,

RULE,

For 3cwt. and upwards allow 16 lb. tare; from 3cwt. down to 2cwt. 14 lb. tare; and from 2cwt. downwards, 12 lb. tare.

Likewise in Virginia tobacco :

For all hogheads under 3 cwt. allow 70 lb. tare; from 3 to 4 cwt. 80 lb.; from 4 to 5 cwt. 90 lb.; and from 5 cwt. upward, 100 lb. tare.

I. What

1. What is the tare of eight hogsheds of tobacco ?

		Cwt. qrs. lb.				qrs. lb.	
viz. N ^o	1	-	2	2	18	2	14
	2	-	3	1	21	-	2 24
	3	-	4	2	8	-	3 6
	4	-	3	3	12	-	2 24
	5	-	5	2	12	-	3 16
	6	-	4	3	19	-	3 6
	7	-	5	2	27	-	3 16
	8	-	5	3	-	-	3 16
Grofs		36	2	5		Tare	6 1 10
Net cwt. 30 - 23							

1. What is the tare of six bales of raw silk ?

		lb.			
viz. N ^o	1	-	325	16	
	2	-	185	12	
	3	-	274	14	Grofs 1505
	4	-	377	16	Tare 84
	5	-	129	12	
	6	-	215	14	Net 1421

CASE V.

When allowance is required for tare and tret,

RULE,

Find what is to be allowed for tare, according to the foregoing rules; which having deducted, the remainder is futtle, which divide by $\frac{104}{4} = 26$, and the quotient is what is to be allowed for tret, which deduct from the futtle, and that remainder is net.

1. What

1. What is the net weight of a puncheon of prunes, gross 13 cwt. 1 qr. 21 lb. tare 14 lb. per cent. tret 4 lb. in 104?

Cwt. qrs. lb.	
8	13 1 21
<hr/>	
	1 2 20, tare.
<hr/>	
104 ÷ 4	26 11 3 1, futtle = 1317 lb.
<hr/>	
	1 22, tret = 50 lb.
<hr/>	
	11 1 7, net = 1267 lb.

2. A merchant buys six hogheads of tobacco, each containing 9 cwt. 1 qr. 14 lb. gross; tare 1 cwt. - qr. 18 lb. per hoghead; tret 4 lb. in 104; and cloff 3 lb. in every 336 gross; what will the net weight come to, at 6½ d. per pound?

Cwt. qr. lb.	Cwt. qrs. lb.	lb.
9 1 14 × 6	= 56 1	gross = 6300
1 - 18 × 6	= 6 3 24,	tare = 780
<hr/>		
	49 1 4,	ftuttle = 5520
<hr/>		
	1 3 16,	tret = 212
<hr/>		
	47 1 16	= 5308
<hr/>		
$\frac{336}{3} = 112$	6300 (= 56 lb. - 2 - ,	cloff = 56
<hr/>		
	46 3 16,	net = 5252

40	5252	s. d.
<hr/>		
12	131 6 -	
	10 18 10	
<hr/>		
	142 4 10,	the answer.

London, March 10, 1758.

Mr. JAMES DENTON,

Bought of JOHN SANDS, six casks of Barbadoes sugar.

N ^o		Cwt. qrs. lb.		Tare	
		8	16	3	7 each.
1	-	7	3 20		× 6
2	-	8	1 16		
3	-	8	- 12	Cwt. 4	3 14
4	-	8	2 21		
5	-	8	3 23		
6	-				

Gross 50 - 24

Tare 4 3 14

Net 45 1 10, at 2l. 7s. 6d. per cwt. - 107l.
13s. 7d.

Computation.

Cwt. qr. lb. l. s. d.

45 1 10, at 2 7 6

l. s. d.

2 7 6

5

11 17 6

9

106 17 6

 $\frac{1}{7}$ 11 10 $\frac{1}{2}$ 1 8 $\frac{1}{2}$ $\frac{2}{7}$ 1 8 $\frac{1}{4}$

- 10

price of { 49 cwt.
1 cwt.
4 lb.
4 lb.
2 lb.

£ 107 13 7

N. B. Below is the computation of the bill of parcels on the next page.

lb. s. d.
20 | 25 19, at 1 2

6 | 125 19
20 19 10

£ 146 18 10

lb. d.
3 | 16 21, at 4

20 | 540 4
27 - 4

Mr.

Chap. III. TARE AND TRET.

§ 37

MR. PETER MASON,

London, April 3, 1758.

Bought of HENRY EUSTACE JOHNSON, Esq; for ready
money, cotton 13 bags.

Cwt. qrs. lb.				Cwt. qrs. lb.			
viz. N ^o 1	-	3	1 7	N ^o 8	-	2	3 16
2	-	2	3 -	9	-	3	- 27
3	-	2	3 5	10	-	2	3 4
4	-	3	- 15	11	-	3	1 10
<hr/>				<hr/>			
11 3 27				Cwt. 12 1 1			
12 1 1				<hr/>			

24 1 —, total gross.
— 3 13, tare allowed.

23 I 15, futtle = 2619 lb.
 tret = 100

net - 2519 lb. l. s. d.
at 1 s. 2 d. per lb. - - - - 146 18 10

More, viz.	Cwt.	qrs.	lb.	
N ^o 17	-	2	2	7
18	-	3	2	8
19	-	3	1	26
20	-	3	-	10
21	-	2	3	12

} damaged.

Gross	15	2	7
Tare	—	2	2

15 - 5 fittle = 1685
tret = 64

net = 1621 lb.

at 4 d. per lb. - - - - - 27 - 4

£ 173 19 - 2

March 24, 1758.

Sir ANDREW BUCKWORTH, and Company,

Bought of the United East-India company, at 4 months,

Pepper, 2 lots, viz.

	Cwt.	qrs.	lb.	lb.
N ^o 15 - 10 bags - wt.	27	3	18	Tare 150
19 - 10 ditto - - -	24	1	24	- - 138
	<hr/>			cwt. qrs. lb.
Gross	52	1	14	Tare 288 = 2 2 8
Tare	2	2	8	<hr/>
Net	49	3	6	= 5578 lb.

at 10½ d. per lb. - - - - 241 2 7½

Redwood, 2 lots, viz.

	Ton.	cwt.
N ^o 42 - 120 sticks - wt.	10	13½
43 - 100 ditto - - -	11	12
	<hr/>	
220 sticks - wt.	22	5½

at 3l. 7s. per ton. - - - - 74 13 3

Wormseed, 3 bales, viz.

	Cwt.	qr.	lb.
N ^o 16 - - - - wt.	3	1	19
19 - - - - -	4	2	-
27 - - - - -	2	3	10

Gross	10	3	1
Tare	1	-	15

Net 9 2 14 = 1078 lb.

at 1s. 1½ d. per lb. - - - - 60 12 9

£ 376 8 7½

5578 lb.

lb.	d.	cwt. qrs. lb.	
$\frac{1}{2}$ 5578,	at $10\frac{1}{2}$.	Or, 49 3 6,	at $10\frac{1}{2}$ d. per lb.
$\frac{1}{2}$ 2789			
$\frac{1}{2}$ 1394	6		
$\frac{1}{2}$ 464	10		
$\frac{1}{2}$ 116	$2\frac{1}{2}$		
$\frac{1}{2}$ 58	$1\frac{1}{2}$		
$\frac{1}{2}$ 4822	$7\frac{1}{2}$		
$\frac{1}{2}$ £ 241	2 $7\frac{1}{2}$		

4
3 $5\frac{1}{2}$
7
1 4 $2\frac{1}{2}$ per qr.
4
4 16 10 per cwt.
7
33 17 10
7
237 4 10
3 12 $7\frac{1}{2}$
5 $2\frac{1}{2}$
£ 241 2 $7\frac{1}{2}$

As before, £ 241 2 $7\frac{1}{2}$

l. s.	
22 tun $5\frac{1}{2}$ cwt. at 3 7	
7	
23 9 -	
3	
70 7 -	
3 7 -	
- 16 9	
- 1 8	
- - 10	
£ 74 13 3	

price of

tun. cwt.	
21 -	
1 -	
- 5	
- $2\frac{1}{2}$	
- $1\frac{1}{2}$	
22 $5\frac{1}{2}$	

Cwt. qrs. lb.	s. d.
9 2 14,	at 1 $1\frac{1}{2}$ per pound.
7	
7 $10\frac{1}{2}$,	price of 7 lb.
8	
3 3 -	
2	
6 6 -	per cwt.
9	
56 14 -	
3 3 -	
15 9	
£ 60 12 9	

The net proceeds of a hoghead of Barbadoes sugar were 4l. 14s. 6d.; the custom and fees 2l. 8s. 6d.; freight 22s. 8d.; factorage 4s. 6d.; the gross weight was 9 cwt. 94 lb.; tare $\frac{1}{15}$: pray how was the sugar rated in the bill of parcels?

	l.	s.	d.
Net proceeds	4	14	6
Custom, &c.	2	8	6
Freight - -	1	2	8
Factorage - -	4	6	
	<hr/>		
	8	10	2 = 8.5087l.

Cwt.	qrs.	lb.	cwt.
9	3	10	= 9.83928577
			Tare .08392857
			<hr/>
			Net 8.8553572

8.8553572) 8.5083333 (.960812 = 19s. 2½d. the answer.

$$\begin{array}{r} 53851185 \\ 719042 \\ 10613 \\ 1758 \\ \hline \end{array}$$

I have imported 80 jars of Lucca oil, each containing 1180 solid inches; what came the freight to, at 4s. 6d. per cwt. tare 1 in 10, counting 7½ lb. of oil to the wine gallon of 231 cubic inches?

$$\begin{aligned} 1180 \times 80 &= 94400 \text{ inches.} \\ 231) 94400 &(408.658 \text{ gallons.} \\ 408.658 \times 7.5 &= 3064.935 \text{ pounds.} \\ 10 - 1 &= 9) 3064.935 \text{ grofs.} \end{aligned}$$

$$\begin{array}{r} 340.548 \\ \hline \end{array}$$

$$112) 3405.483 (30.406 \text{ cwt.}$$

$$4 \text{ s. } 6 \text{ d. } = .225 \text{ l.}$$

$$30.406 \times .225 = 6.84135 = 6 \text{ l. } 18 \text{ s. } 9 \frac{1}{4} \text{ d. the answer.}$$

CHAPTER IV. FELLOWSHIP.

THE rule of fellowship is that by which the accompts of several partners trading in company are adjusted, made up, or divided; so that every partner may have his just part of the gain (or loss) in proportion to the money he hath in the joint stock, and to the time of its continuance therein.



SECT. I. SINGLE FELLOWSHIP.

By single fellowship is adjusted the accompts of such partners that put all their several, and, perhaps, different sums of money into one common stock at the same time; and therefore it is usually called the rule of fellowship without time.

RULE.

As the whole stock : is to the whole gain or loss :: so is every man's particular part of that stock : to his particular share of the gain or loss.

1. Three merchants, A, B, C, enter upon a joint adventure; A puts into the common stock 175l. 13s. 4d.; B 117l. 16s. 8d.; and C 98l. 17s. 7d.: with this stock they trade and gain 264l.; I demand each merchant's share of the gain?

$$\text{A's stock } 175\text{l. } 13\text{s. } 4\text{d.} = 175.6666\bar{6}$$

$$\text{B's } - - 117\text{l. } 16\text{s. } 8\text{d.} = 117.8333\bar{3}$$

$$\text{C's } - - 98\text{l. } 17\text{s. } 7\text{d.} = 98.8791\bar{6}$$

$$392.3791\bar{6}$$

$$392.37916\text{l.} : 264\text{l.} ::$$

$$\left\{ \begin{array}{l} 175.6666\bar{6} : 118.191802 = 118 \ 3 \ 10 = \text{A's} \\ 117.8333\bar{3} : 79.280459 = 79 \ 5 \ 7\frac{1}{4} = \text{B's} \\ 98.8791\bar{6} : 66.527743 = 66 \ 10 \ 6\frac{3}{4} = \text{C's} \end{array} \right\} \text{gain.}$$

$$\text{£ } 264. - - - = \text{whole gain.}$$

Z 3

But

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But questions of the same kind with the foregoing, and those relating to bankruptcies, the readiest way of solution will be, by dividing the whole gain by the whole stock, or the bankrupt's whole estate by the sum of his debts; the quotient will be a common multiplier, or so much a pound as the bankrupt's estate will pay.

2. A merchant breaking owes his creditors as follows :

		l.	s.	d.		l.
viz. To	Mr. Trust	-	3725	17	3	= 3725.8625
	Mr. Credit	-	7967	14	9	= 7967.7375
	Mr. Gripe	-	5674	12	6	= 5674.625
	Mr. Covet	-	967	10	4 $\frac{1}{2}$	= 967.51875
	Mr. Squeeze	-	734	6	2 $\frac{1}{4}$	= 734.309375
	Mr. Hard	-	873	18	6	= 873.925
	Mr. Near	-	382	14	3	= 382.7125
	Mr. Dunn	-	125	16	7 $\frac{1}{2}$	= 125.83125
	Mr. Diffident	-	637	18	6 $\frac{1}{2}$	= 637.928125

In all, £ 21090 9 - = 21090.45

His whole estate is 17500*l.* what is each creditor's part of that in proportion to his debt ?

whole debt. whole estate. s. d.
21090.45 : 17500 :: 1 : .829759441 = 16 7 $\frac{1}{2}$ per lb.

	i.	s.	d.	
3725.8625 } x {	3091.569586 =	3091 II	4 $\frac{1}{2}$ T	
7967.7375 }	6611.305415 =	6611 6	1 $\frac{1}{2}$ C.	
5674.625 }	4708.573669 =	4708 II	5 $\frac{1}{2}$ G.	
967.51875 }	802.807817 =	802 16	1 $\frac{1}{2}$ C.	
734.309375 }	609.300137 =	609 6	- Sq.	
873.925 }	725.147520 =	725 2	II $\frac{1}{2}$ H.	
382.7125 }	317.559311 =	317 I	2 $\frac{1}{2}$ N.	
125.83125 }	104.409669 =	104 8	2 $\frac{1}{2}$ D.	
637.928125 }	529.326885 =	529 6	6 $\frac{1}{2}$ D.	

$$17500.001802 = 17500 \quad - \quad -$$

In cases of bankruptcy, when there are many creditors, first find what the bankrupt's estate will pay in the pound; and then each particular part may be found by the rule of practice, very near the truth: and here note, that the small redundancy in the larger sums, in this question, is owing to 16s. 7½d. being taken a small matter more than the bankrupt's

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rupt's estate would allow; and the deficiency in the smaller ones to the fraction of a farthing omitted.

Trust.			Credit.			Gripe.		
l.	s.	d.	l.	s.	d.	l.	s.	d.
$\frac{1}{2}$ 3725	17	3	$\frac{1}{2}$ 7967	14	9	$\frac{1}{2}$ 5674	12	6
$\frac{1}{2}$ 1862	18	$7\frac{1}{2}$	$\frac{1}{2}$ 3983	17	$4\frac{1}{2}$	$\frac{1}{2}$ 2837	6	3
$\frac{1}{2}$ 931	9	$3\frac{1}{2}$	$\frac{1}{2}$ 1991	18	$8\frac{1}{4}$	$\frac{1}{2}$ 1418	13	$1\frac{1}{2}$
$\frac{1}{2}$ 180	5	$10\frac{1}{4}$	$\frac{1}{2}$ 398	7	8	$\frac{1}{2}$ 283	14	$7\frac{1}{2}$
$\frac{1}{2}$ 93	2	11	$\frac{1}{2}$ 199	3	$10\frac{1}{4}$	$\frac{1}{2}$ 141	17	$3\frac{1}{2}$
$\frac{1}{2}$ 15	10	$5\frac{1}{2}$	$\frac{1}{2}$ 33	3	$11\frac{1}{2}$	$\frac{1}{2}$ 23	12	$10\frac{1}{2}$
$\frac{1}{2}$ 2	4	$4\frac{1}{4}$	$\frac{1}{2}$ 4	14	10	$\frac{1}{2}$ 3	7	$6\frac{1}{2}$
£ 3091	11	$6\frac{1}{2}$	£ 6611	6	$5\frac{1}{4}$	£ 4708	11	$8\frac{1}{2}$
Covet.			Squeeze.			Hard.		
l.	s.	d.	l.	s.	d.	l.	s.	d.
$\frac{1}{2}$ 967	10	$4\frac{1}{2}$	$\frac{1}{2}$ 734	6	$2\frac{1}{4}$	$\frac{1}{2}$ 873	18	6
$\frac{1}{2}$ 483	15	$2\frac{1}{4}$	$\frac{1}{2}$ 367	3	1	$\frac{1}{2}$ 436	19	3
$\frac{1}{2}$ 241	17	7	$\frac{1}{2}$ 183	11	$6\frac{1}{2}$	$\frac{1}{2}$ 218	9	$7\frac{1}{2}$
$\frac{1}{2}$ 48	7	6	$\frac{1}{2}$ 36	14	$3\frac{1}{2}$	$\frac{1}{2}$ 43	13	11
$\frac{1}{2}$ 24	3	9	$\frac{1}{2}$ 18	7	$1\frac{1}{2}$	$\frac{1}{2}$ 21	16	$11\frac{1}{2}$
$\frac{1}{2}$ 4	-	$7\frac{1}{2}$	$\frac{1}{2}$ 3	1	$2\frac{1}{4}$	$\frac{1}{2}$ 3	12	$9\frac{1}{2}$
$\frac{1}{2}$ -	11	6	$\frac{1}{2}$ -	8	$8\frac{1}{2}$	$\frac{1}{2}$ -	10	$4\frac{1}{2}$
£ 802	16	$1\frac{1}{2}$	£ 609	5	$11\frac{1}{2}$	£ 725	2	$11\frac{1}{2}$
Near.			Dunn.			Diffident.		
l.	s.	d.	l.	s.	d.	l.	s.	d.
$\frac{1}{2}$ 382	14	3	$\frac{1}{2}$ 125	16	$7\frac{1}{2}$	$\frac{1}{2}$ 637	18	$6\frac{1}{2}$
$\frac{1}{2}$ 191	7	$1\frac{1}{2}$	$\frac{1}{2}$ 62	18	$3\frac{1}{4}$	$\frac{1}{2}$ 318	19	$3\frac{1}{4}$
$\frac{1}{2}$ 95	13	$6\frac{1}{2}$	$\frac{1}{2}$ 31	9	$1\frac{1}{4}$	$\frac{1}{2}$ 159	9	$7\frac{1}{2}$
$\frac{1}{2}$ 19	2	$8\frac{1}{2}$	$\frac{1}{2}$ 6	5	$9\frac{1}{4}$	$\frac{1}{2}$ 31	17	11
$\frac{1}{2}$ 9	11	$4\frac{1}{4}$	$\frac{1}{2}$ 3	2	$10\frac{1}{4}$	$\frac{1}{2}$ 15	18	$11\frac{1}{2}$
$\frac{1}{2}$ 1	11	$10\frac{1}{2}$	$\frac{1}{2}$ -	10	$5\frac{1}{2}$	$\frac{1}{2}$ 2	13	$1\frac{1}{4}$
$\frac{1}{2}$ -	4	$6\frac{1}{2}$	$\frac{1}{2}$ -	1	$5\frac{3}{4}$	$\frac{1}{2}$ -	7	7
£ 317	11	2	£ 104	8	$1\frac{1}{2}$	£ 529	6	6

As there is no general rule for solving all questions that may occur or be proposed in partnership, or other branches of

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of trade, the answer must be found by the ingenuity of the arithmetician; who, by this time, may be supposed to be pretty well grounded in figures.

3. A hath in stock 35 l. B 20 l.; they trade and gain 40 l. and agree that it shall be divided so, that A is to have 10 per cent. and B only 8; what must each have of the gain?

35 l. at 10 per cent. is 3.5 l. }
20 l. at 8 per cent. is 1.6 l. } their sum 5.1 l.

l. s. d.

∴ 5.1 : 40 :: 3.5 : 27.45098 = 27 9 $\frac{1}{4}$ = A's } share.
5.1 : 40 :: 1.6 : 12.54902 = 12 10 $\frac{3}{4}$ = B's }

4. A, B and C put in money together; A puts in 20 l.; B and C put in 85 l.; they gained 63 l. of which B took up 21 l.; what did A and C gain, and B and C put in?

A 20 }
B + C 85 } sum 105.

105 : 63 :: 20 : 12, A's } gain. } Which
12 + 21 = 33, and 63 — 33 = 30, C's } were to
And 63 : 105 :: 21 : 35, B's } stock, } be found
63 : 105 :: 30 : 50, C's }

5. Some others advance in trade as follow; viz. W, X and Y raised 350 l. 10 s.; W, X and Z 344 l. 10 s.; X, Y and Z made up together 400 l.; and W, Y and Z contributed 378 l. 4 s. in the conclusion they parted with their joint property for 450 guineas; what did they gain or lose by their adventure?

l. s.
W, X, Y 350 10 }
W, X, Z 344 10 } each partner being mentioned three
X, Y, Z 400 - } times.
W, Y, Z 378 4 }

3) 1473 4

d.

491 1 4, the whole stock.

20) 450 guineas.

22 10

472 10 made of their joint property.

Then 491 l. 1 s. 4 d. — 472 l. 10 s. = 18 l. 11 s. 4 d. loss,
the answer. 6. A,

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6. A, B and C put in trade 360l. and gained 270l.; of which as often as A took up 3l. B took up 5l.; and as often as B took up 5l. C took up 7l.; what did each gain and put in?

$$\begin{array}{r} 3 \\ + 5 \\ + 7 \\ \hline \end{array}$$

15) 360 (24, the common multiplier for the stock.

$$\begin{array}{l} 3 \times 24 = 72 = A's \\ 5 \times 24 = 120 = B's \\ 7 \times 24 = 168 = C's \end{array} \left. \vphantom{\begin{array}{l} 3 \times 24 \\ 5 \times 24 \\ 7 \times 24 \end{array}} \right\} \text{stock.}$$

15) 270 (18, the common multiplier for the gain.

$$\begin{array}{l} \text{and } 3 \times 18 = 54 = A's \\ 5 \times 18 = 90 = B's \\ 7 \times 18 = 126 = C's \end{array} \left. \vphantom{\begin{array}{l} 3 \times 18 \\ 5 \times 18 \\ 7 \times 18 \end{array}} \right\} \text{gain.}$$

7. A, B and C put in money together; A puts in 20l.; B 30l.; C a sum unknown: they gained 36l. whereof C took 16l.; what did A and B gain, and C put in?

$$\begin{array}{r} 20 \qquad 36 \\ + 30 \quad - 16 \\ \hline \end{array}$$

50 : 20 :: 20 : 8, A's gain.

20 — 8 = 12, B's gain.

8 : 20 :: 16 : 40, C's stock.

8. A, B, C and D put in money together, and gained a sum of money, of which A, B and C took 60l.; B, C and D took 90l.; A, C and D took 80l.; and A, B and D took up 70l.; what distinct gain did each take up?

$$\begin{array}{l} 1. \\ A, B, C \quad 60 \\ B, C, D \quad 90 \\ A, C, D \quad 80 \\ A, B, D \quad 70 \\ \hline \end{array}$$

$$3) 300$$

$$\begin{array}{l} 1. \\ 100 - 90 = 10 = A's \\ 100 - 80 = 20 = B's \\ 100 - 70 = 30 = C's \\ 100 - 60 = 40 = D's \end{array} \left. \vphantom{\begin{array}{l} 100 - 90 \\ 100 - 80 \\ 100 - 70 \\ 100 - 60 \end{array}} \right\} \text{gain, the answer.}$$

9. A

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9. A and B clear by an adventure at sea 50 guineas, with which they agreed to buy a horse and chaise; whereof they were to have the use, in proportion to the sums adventured, which was found to be, A 10 : to B 7; they cleared 45 l. per cent. what money did they each send abroad?

$$\begin{aligned} 50 \text{ guineas} &= 52 \text{ l. } 10 \text{ s.}; \text{ and } 10 + 7 = 17. \\ 45 : 100 :: 52.5 : 116.6 &= 116 \text{ l. } 13 \text{ s. } 4 \text{ d. whole sum.} \\ 17 : 10 :: 116.6 : 68.6274 &= 68 \text{ l. } 12 \text{ s. } 6\frac{1}{2} \text{ d. A's} \\ 17 : 7 :: 116.6 : 48.0392 &= 48 \text{ l. } - \text{ s. } 9\frac{1}{2} \text{ d. B's} \end{aligned} \left. \vphantom{\begin{aligned} 45 : 100 :: 52.5 : 116.6 \\ 17 : 10 :: 116.6 : 68.6274 \\ 17 : 7 :: 116.6 : 48.0392 \end{aligned}} \right\} \text{stock.}$$

10. A father divided his fortune amongst his sons, giving A 7, as often as B 4; to C he gave as often 2, as to B 5; and yet the dividend of C came to 2166 $\frac{1}{2}$ l.: what was the value of the whole legacy?

$$\begin{array}{rcl} \text{C.} & \text{l.} & \text{s.} & \text{d.} & \text{B.} & \text{l.} & \text{s.} & \text{d.} \\ 2 : 2166 & 7 & 6 & :: & 5 : 5415 & 18 & 9, & \text{B's} \\ \text{B.} & \text{l.} & \text{s.} & \text{d.} & \text{A.} & & & \\ 4 : 5415 & 18 & 9 & : & 7 : 9477 & 17 & 9\frac{1}{2}, & \text{A's} \end{array} \left. \vphantom{\begin{array}{rcl} 2 : 2166 & 7 & 6 \\ 4 : 5415 & 18 & 9 \end{array}} \right\} \text{share.}$$

and, as above, 2166 7 6, C's

17060 4 $-\frac{3}{4}$, the answer.

11. Part 1500 acres of land; give B 72 more than A, and C 112 more than B.

$$\begin{array}{rcl} \text{B} & - & - & - & - & - & 72 \\ \text{C} & 72 & + & 112 & = & 184 \end{array} \left. \vphantom{\begin{array}{rcl} \text{B} & - & - & - & - & - & 72 \\ \text{C} & 72 & + & 112 & = & 184 \end{array}} \right\} \text{more than A.}$$

sum 256

Then $1500 - 256 = 1244$

3) 1244 (414 $\frac{2}{3}$ = A's

Also $414\frac{2}{3} + 72 = 486\frac{2}{3}$ = B's

And $486\frac{2}{3} + 112 = 598\frac{2}{3}$ = C's

12. Divide 1000 crowns; give A 129 more than B, and B 178 fewer than C.

$$\begin{array}{rcl} & 1000 & \\ 129 & + & 178 & = & 307 \end{array}$$

3) 693 (231 = B's.

231 + 129 = 360 = A's.

231 + 178 = 409 = C's.

13. Part

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13. Part 250 l. give A 37 more than B, and let C have 28 fewer than B.

First, $37 - 28 = 9$ - - - Also $250 - 9 = 241$.

$$\begin{array}{l} 3) 241 \quad (80\frac{1}{3} = B's \\ 80\frac{1}{3} + 37 = 117\frac{1}{3} = A's \\ 80\frac{1}{3} - 28 = 52\frac{1}{3} = C's \end{array} \left. \vphantom{\begin{array}{l} 3) 241 \\ 80\frac{1}{3} + 37 \\ 80\frac{1}{3} - 28 \end{array}} \right\} \text{part.}$$

14. In an article of trade, A gains 14 s. 6 d. and his adventure was 35 s. more than B's, whose share of profit is but 8 s. 6 d.; what are the particulars of their stock?

First, 14 s. 6 d. — 8 s. 6 d. = 6 s. difference of their gain.

$$\begin{array}{ccccccc} \text{s.} & \text{s.} & & \text{s.} & & \text{l.} & \text{s.} & \text{d.} \\ \text{Then as } 6 : 35 :: 14.5 : 84.583 = 447 = A's \\ & 6 : 35 :: 8.5 : 49.583 = 297 = B's \end{array} \left. \vphantom{\begin{array}{l} 6 : 35 :: 14.5 \\ 6 : 35 :: 8.5 \end{array}} \right\} \text{stock.}$$

15. Three persons entered joint trade, to which A contributed 210 l. B 312 l.; they clear 140 l. whereof 37 l. 10 s. belongs of right to C; that person's stock, and the several gains of the other two are required?

$$210 \text{ l.} + 312 = 522 \text{ l.} = A's \text{ stock} + B's.$$

$$140 \text{ l.} - 37.5 = 102.5 \text{ l.} = A's \text{ gain} + B's.$$

$$\text{As } 102.5 : 522 :: 37.5 : 190.8756 \text{ l.} = 190 \text{ l. } 19 \text{ s. } \frac{6}{41},$$

C's share.

$$210 \text{ l.} + 312 + 190.8756 = 712.8756 \text{ l.} = \text{whole stock.}$$

$$\begin{array}{l} 712.8756 \text{ l.} : \left\{ \begin{array}{l} 210 : 41.2357 = 41 \text{ l. } 4 \text{ s. } 8\frac{1}{4} \text{ d.} + A's \\ 140 : \left\{ \begin{array}{l} 312 : 61.2643 = 61 \text{ l. } 5 \text{ s. } 3\frac{1}{2} \text{ d.} - B's \end{array} \right. \end{array} \right. \text{gain.} \end{array}$$

16. A and B venturing equal sums of money, clear by joint trade 154 l.; by agreement A was to have 8 per cent. because he spent time in execution of the project; and B was only to have 5: the question is, what was allotted A for his trouble?

$$\begin{array}{ccccccc} & & & & \text{l.} & \text{s.} & \text{d.} \\ \text{As } 8 + 5 = 13 : 8 :: 154 : 94 \text{ l. } 15 \text{ s. } 4\frac{2}{3}, & A's \\ & 13 : 5 :: 154 : 59 \text{ l. } 4 \text{ s. } 7\frac{1}{3}, & B's \end{array} \left. \vphantom{\begin{array}{l} 154 : 94 \\ 154 : 59 \end{array}} \right\} \text{gain.}$$

$$\text{Answer, } \underline{\underline{\text{£ } 35 \text{ } 10 \text{ } 9\frac{1}{3}}}$$

17. A, B and C play a concert at hazard, and making up accmpts, it appears that A and B together brought off 13 l.

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13 l. 10 s.; B and C together 12 l. 12 s.; and A and C together won 11 l. 16 s. 6 d.; what did they severally get?

$$13 \text{ l. } 10 \text{ s.} + 12 \text{ l. } 12 \text{ s.} + 11 \text{ l. } 16 \text{ s. } 6 \text{ d.} = 37 \text{ l. } 18 \text{ s. } 6 \text{ d.}$$

$$\begin{array}{r} 2) \quad 37 \quad 18 \quad 6 \\ \hline \end{array} \quad \begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \\ 18 \quad 19 \quad 3 - 12 \quad 12 \quad - = 6 \quad 7 \quad 3, \text{ A's} \\ 18 \quad 19 \quad 3 - 11 \quad 16 \quad 6 = 7 \quad 2 \quad 9, \text{ B's} \\ 18 \quad 19 \quad 3 - 13 \quad 10 \quad - = 5 \quad 9 \quad 3, \text{ C's} \end{array} \left. \vphantom{\begin{array}{r} 18 \quad 19 \quad 3 \\ 18 \quad 19 \quad 3 \\ 18 \quad 19 \quad 3 \end{array}} \right\} \text{share.}$$

18. A, B and C are three horses belonging to different men, and are employed as a team to draw a load of wheat from Hertford for 30 s.; A and B are deemed to do $\frac{2}{7}$ of the work; A and C $\frac{3}{8}$; and B and C $\frac{3}{10}$ of it: they are to be paid proportionably, and you know how to divide it as it should be.

$$\frac{2}{7} = \frac{16}{56} = \frac{160}{560}, \text{ and } \frac{3}{8} = \frac{21}{56}.$$

$$\frac{21}{56} - \frac{16}{56} = \frac{5}{56} = \text{C's share more than B's,}$$

$$\frac{3}{8} = \frac{30}{80}, \text{ and } \frac{3}{10} = \frac{24}{80}.$$

$$\frac{30}{80} - \frac{24}{80} = \frac{6}{80} = \frac{42}{560} = \text{A's share more than B's.}$$

$$\frac{160}{560} - \frac{42}{560} = \frac{118}{560}, \text{ } \frac{2}{1} \frac{118}{560} \left(\frac{59}{56} = \text{B's.} \right)$$

$$\frac{59}{560} + \frac{42}{560} = \frac{101}{560} = \text{A's; and } \frac{59}{560} + \frac{50}{560} = \frac{109}{560} = \text{C's,}$$

Then rejecting the common denominator,

$$\begin{array}{r} 101 + 59 + 109 = 269, \text{ sum of the numerators,} \\ \text{s.} \qquad \qquad \text{s.} \quad \text{d.} \end{array}$$

$$\begin{array}{r} 269 : 30 :: 101 : 11 \frac{3 \frac{45}{56}}{56} = \text{A's} \\ 269 : 30 :: 59 : 6 \frac{2 \frac{59}{56}}{56} = \text{B's} \\ 269 : 30 :: 109 : 12 \frac{1 \frac{109}{56}}{56} = \text{C's} \end{array} \left. \vphantom{\begin{array}{r} 269 : 30 \\ 269 : 30 \\ 269 : 30 \end{array}} \right\} \text{share of the money.}$$

$$\underline{\underline{\pounds 110 -}}, \text{ proof.}$$

19. Three persons purchase together a West-India sloop, towards the payment whereof A advanced $\frac{1}{2}$, B $\frac{1}{3}$, and C $\frac{1}{6}$.

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22l. 1s.: 19l. 19s. 8d.: 127l. 1s. sum: 115l. 1s. 11d. cost.

$$127 \times 1 = 2541$$

$$115 \times 11 = 2301 \text{ 11 d.}$$

$$55) 4842. \quad (88 \text{ ankers, and 2s. 11 d. over.}$$

$$88) 127 \quad 1 (1 \text{ l. 8s. } 7\frac{1}{4} \text{ d. gain per anker.}$$

22. In raising a joint stock of 400 l. A advances $\frac{4}{11}$; B $\frac{12}{11}$ of $\frac{3}{4}$; C $\frac{1}{6}$ more; the difference between A's adventure and B's, and D the rest of the money; what did every one subscribe?

$$\left. \begin{aligned} \frac{4}{11} &= \frac{264}{858} = 123 \text{ l. } 1 \text{ s. } 6 \text{ d. } \frac{26}{111} = \text{A's} \\ \frac{12}{11} \text{ of } \frac{3}{4} &= \frac{9}{22} = \frac{351}{858} = 163 \text{ l. } 12 \text{ s. } 8 \text{ d. } \frac{624}{111} = \text{B's} \\ \frac{351}{858} - \frac{264}{858} &= \frac{87}{858} \\ \frac{1}{6} \text{ or } \frac{143}{858} + \frac{87}{858} &= \frac{230}{858} = 107 \text{ l. } 4 \text{ s. } 6 \text{ d. } \frac{228}{111} = \text{C's} \\ \frac{264}{858} + \frac{351}{858} + \frac{230}{858} &= \frac{845}{858} \\ \frac{858}{858} - \frac{845}{858} &= \frac{13}{858} = 6 \text{ l. } 1 \text{ s. } 2 \text{ d. } \frac{468}{111} = \text{D's} \end{aligned} \right\} \text{Ans.}$$

23. A father devised $\frac{3}{4}$ of his estate to one of his sons, and $\frac{1}{4}$ of the residue to another, and the surplus to his relict for life; the children's legacies were found to be 257 l. 3 s. 4 d. different $= \frac{1543}{6}$; pray what money did he leave the widow the use of?

$$\begin{aligned} \text{Whole estate } \frac{83}{83} - \frac{34}{83} &= \frac{49}{83}. \text{ Then } \frac{34}{83} = \frac{2822}{6889}, \text{ and } \frac{14}{83} \\ \text{of } \frac{49}{83} &= \frac{1666}{6889}. \\ \text{And } \frac{2822}{6889} - \frac{1666}{6889} &= \frac{1156}{6889} = 257 \text{ l. } 3 \text{ s. } 4 \text{ d. Also } \frac{2822}{6889} \\ + \frac{1666}{6889} &= \frac{4488}{6889}. \\ \text{Then } \frac{6889}{6889} - \frac{4488}{6889} &= \frac{2401}{6889} = \text{widow's part of the estate.} \end{aligned}$$

As

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As $\frac{1156}{6889} : \frac{1543}{6} :: \frac{2822}{6889} : \frac{2177170}{3468} = 627 \text{ l. } 15 \text{ s. } 9 \text{ d. } 1\frac{1}{11}$
 eldest son.

As $\frac{1156}{6889} : \frac{1543}{6} :: \frac{1666}{6889} : \frac{1285319}{3468} = 370 \text{ l. } 12 \text{ s. } 5 \text{ d. } 1\frac{1}{11}$
 youngest.

As $\frac{1156}{6889} : \frac{1543}{6} :: \frac{2401}{6889} : \frac{3704743}{6936} = 534 \text{ l. } 2 \text{ s. } 8 \text{ d. (nearly) widow.}$

24. A father, ignorant in numbers, ordered 500l. to be divided among his five sons, thus: Give A, says he, $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, D $\frac{1}{6}$, and E $\frac{1}{7}$; part this equitably among them according to the father's intention.

First $\frac{1}{3} = \frac{140}{420}$, $\frac{1}{4} = \frac{105}{420}$, $\frac{1}{5} = \frac{84}{420}$, $\frac{1}{6} = \frac{70}{420}$, and $\frac{1}{7} = \frac{60}{420}$.

Then $\frac{140}{420} + \frac{105}{420} + \frac{84}{420} + \frac{70}{420} + \frac{60}{420} = \frac{459}{420}$, their sum.

As $\frac{459}{420} : \frac{500}{1} :: \frac{140}{420} : 152 \text{ } 10 \text{ } 1 \text{ } 1\frac{10}{11} = \text{A's.}$

$\frac{459}{420} : \frac{500}{1} :: \frac{105}{420} : 114 \text{ } 7 \text{ } 6 \text{ } 3\frac{1}{11} = \text{B's.}$

$\frac{459}{420} : \frac{500}{1} :: \frac{84}{420} : 91 \text{ } 10 \text{ } - \text{ } 3\frac{6}{11} = \text{C's.}$

$\frac{459}{420} : \frac{500}{1} :: \frac{70}{420} : 76 \text{ } 5 \text{ } - \text{ } 2\frac{1}{11} = \text{D's.}$

$\frac{459}{420} : \frac{500}{1} :: \frac{60}{420} : 65 \text{ } 7 \text{ } 2 \text{ } 1\frac{4}{11} = \text{E's.}$

£ 500 - -

25. A in a scuffle seized on $\frac{2}{3}$ of a parcel of sugar-plums; B caught $\frac{1}{4}$ of them in his hands; and C laid hold on $\frac{1}{8}$ more; D ran off with all A had left, except $\frac{1}{7}$, which E afterwards secured sily for himself: then A and C jointly set upon B, who in the conflict shed half he had, which were equally picked up by D and E, who lay perdue. B then kicked down C's hat, and to work they all went a-new for what it contained; of which

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which A got $\frac{1}{4}$, B $\frac{1}{5}$, D $\frac{2}{7}$, and C and E equal shares of what was left of that stock. D then struck $\frac{1}{3}$ of what A and B last acquired out of their hands; they with difficulty recovered $\frac{1}{5}$ of it in equal shares again, but the other three carried off $\frac{1}{4}$ a-piece of the same. Upon this they called a truce, and agree that the $\frac{1}{3}$ of the whole left by A, at first should be equally divided amongst them; how much of the prize, after this distribution, remained with each of the competitors?

Though A at the first seized $\frac{1}{3}$, he lost all again this heat.

$$\left. \begin{array}{l} \frac{3}{8} \text{ of } \frac{2}{3} = \frac{1}{4} = \text{B's} \\ \frac{3}{10} \text{ of } \frac{2}{3} = \frac{1}{5} = \text{C's} \end{array} \right\} \text{first acquisition. Their sum} = \frac{2}{15}$$

$$\frac{2}{3} - \frac{9}{20} = \frac{13}{60} = \frac{91}{420}$$

$$\left. \begin{array}{l} \frac{1}{7} \text{ of } \frac{13}{60} = \frac{13}{420} = \text{E's} \\ \text{and } \frac{91}{420} - \frac{13}{420} = \frac{78}{420} = \text{D's} \end{array} \right\} \text{first acquisition.}$$

Thus ended the first heat.

$$\left. \begin{array}{l} \text{Again, } \frac{1}{2} \text{ of } \frac{1}{4} = \frac{1}{8} = \text{B's} \\ \text{Retained} \dots \frac{1}{5} = \text{C's} \\ \frac{13}{70} + \frac{1}{16} = \frac{139}{560} = \text{D's} \\ \frac{13}{420} + \frac{1}{16} = \frac{157}{1680} = \text{E's} \end{array} \right\} \text{part at the end of the second scuffle.}$$

$$\left. \begin{array}{l} \text{Proceeding, } \frac{1}{4} \text{ of } \frac{1}{5} = \frac{1}{20} = \text{A's} \\ \frac{1}{3} \text{ of } \frac{1}{5} + \frac{1}{8} = \frac{23}{120} = \text{B's} \\ \frac{2}{7} \text{ of } \frac{1}{5} + \frac{139}{560} = \frac{171}{560} = \text{D's} \end{array} \right\} \text{part after the third smush.}$$

$$\text{Then } \frac{1}{20} + \frac{1}{15} + \frac{2}{35} = \frac{73}{420}$$

$$\left. \begin{array}{l} \frac{1}{5} - \frac{73}{420} = \frac{11}{420}, \text{ and } \frac{1}{2} \text{ of } \frac{11}{420} = \frac{11}{840} = \text{C's} \\ \frac{157}{1680} + \frac{11}{840} = \frac{179}{1680} = \text{E's} \end{array} \right\} \text{part of the third smush.}$$

Further,

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Further, $\frac{1}{20} + \frac{1}{15} = \frac{7}{60}$, and $\frac{3}{4}$ of $\frac{7}{60} = \frac{7}{80}$, left by A and B.

$$\left. \begin{aligned} \frac{5}{16} \text{ of } \frac{7}{80} + \frac{1}{4} \text{ of } \frac{1}{20} &= \frac{51}{1280} = \text{A's} \\ \frac{5}{16} \text{ of } \frac{7}{80} + \frac{1}{4} \text{ of } \frac{1}{15} \text{ of } \frac{1}{8} &= \frac{649}{3840} = \text{B's} \\ \frac{1}{8} \text{ of } \frac{7}{80} + \frac{11}{840} &= \frac{323}{13440} = \text{C's} \\ \frac{1}{8} \text{ of } \frac{7}{80} + \frac{171}{840} &= \frac{1417}{4480} = \text{D's} \\ \frac{1}{8} \text{ of } \frac{7}{80} + \frac{179}{1680} &= \frac{1579}{13440} = \text{E's} \end{aligned} \right\} \text{part after the last scuffle.}$$

$$\left. \begin{aligned} \text{Then } \frac{51}{1280} + \frac{1}{15} &= \frac{2863}{26880} = \text{A's} \\ \frac{649}{3840} + \frac{1}{15} &= \frac{6335}{26880} = \text{B's} \\ \frac{323}{13440} + \frac{1}{15} &= \frac{2438}{26880} = \text{C's} \\ \frac{1417}{4480} + \frac{1}{15} &= \frac{10294}{26880} = \text{D's} \\ \frac{1579}{13440} + \frac{1}{15} &= \frac{4950}{26880} = \text{E's} \end{aligned} \right\} \text{share carried off at the last.}$$

So that if the number of fugar-plumbs were 26880,

$$\left. \begin{array}{lll} \text{A got} & - & - & 2863 \\ \text{B} & - & - & 6335 \\ \text{C} & - & - & 2438 \\ \text{D} & - & - & 10294 \\ \text{E} & - & - & 4950 \end{array} \right\} \text{sum 26880.}$$

26. If A, having $\frac{1}{2}$ of $\frac{1}{2}$ of the half of a trading sloop and cargo worth 16131 l. 14 s. sells his brother B $\frac{1}{2}$ of $\frac{1}{2}$ of his interest therein at prime cost; what did it cost the brother, and what did his cousin P pay at the same time for $\frac{2}{11}$ of the remainder?

$$\frac{1}{2} \text{ of } \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } 16131.7 = 4234.57125 \text{ l.} = 4234 \text{ l. } 11 \text{ s. } 5 \text{ d. A at first.}$$

$$\frac{1}{2} \text{ of } \frac{1}{2} \text{ of } 4234.57125 = 2032.5942 = 2032 \text{ l. } 11 \text{ s. } 20 \frac{1}{2} \text{ d. sold B.}$$

$$16131.7 - 4234.57125 = 11897.12875, \text{ remainder.}$$

$$\frac{2}{11} \text{ of } 11897.12875 = 9734.01443 = 9734 \text{ l. } - \text{ s. } 3 \frac{1}{2} \text{ d. cousin P.}$$

A a

27. Two

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27. Two merchants company, A put in 20 l. and B put in 135 ducats; they gain 67 l. 10 s. of which A took 30 l. what is the value of a ducat?

$$67 \text{ l. } 10 \text{ s.} = 30 \text{ l.} = 37 \text{ l. } 10 \text{ s.} = \text{B's gain.}$$

$$30 : 20 :: 37.5 : 25 \text{ l. B's stock} = 135 \text{ ducats.}$$

$$135 \text{ ducats} = 500 \text{ s. (3 s. } 8 \frac{1}{2} \text{ d. value of a ducat, the answer.}$$

28. Three merchants, A, B and C, freight ships to Lisbon with sugar, to the value of 15778 l. 2 s. 6 d. sterling. A bought 250 cwt. 1 qr. 22 lb. at 2 l. 16 s. per cwt. B paid 2 l. 6 s. 8 d. per cwt. for his; but meeting with a storm at sea, the mariners were constrained, for the safety of their lives, to cast out part of the ship's lading. A's proportion ejected part was $\frac{1}{100}$ th part of the ship's lading; and $3\frac{1}{2}$ times the quantity cast over board, was $3\frac{1}{2}$ times the whole freight of A and B. When they came to land, A sold his remaining part for 4 l. 4 s. per cwt. and found himself a loser 10 per cent. besides charges. B advanced the remaining part of his commodity 20 per cent. and C gained 4 s. 8 d. per cwt. by the quantity he saved. Quere, What did each merchant lose by this voyage, the charge of the same amounting to 500 guineas? By Mr. Flower.

$$1 \text{ cwt.} : 2 \text{ l. } 16 \text{ s.} = \frac{14}{5} :: 250 \text{ cwt. } 1 \text{ qr. } 22 \text{ lb.} = \frac{14025}{56}$$

$$: 701 \text{ l. } 5 \text{ s. A's cost besides charges.}$$

$$\frac{100}{1} : \frac{10}{1} :: 701 \text{ l. } 5 \text{ s.} = \frac{2805}{4} : \frac{561}{8} = 701 \text{ l. } 2 \text{ s. } 6 \text{ d. A's}$$

$$\text{loss besides charges.}$$

$$701 \text{ l. } 5 \text{ s.} - 701 \text{ l. } 2 \text{ s. } 6 \text{ d.} = 631 \text{ l. } 2 \text{ s. } 6 \text{ d.} = \frac{5049}{8},$$

value of A's remaining part.

$$4 \text{ l. } 4 \text{ s.} = \frac{21}{5} : 1 :: \frac{5049}{8} : \frac{8415}{56} = 150 \text{ cwt. } 1 \text{ qr. } 21 \text{ lb.}$$

A's remaining part.

$$250 \text{ cwt. } 1 \text{ qr. } 22 \text{ lb.} - 150 \text{ cwt. } 1 \text{ qr. } 2 \text{ lb.} = 100 \text{ cwt.}$$

$$20 \text{ lb.} = \frac{2805}{28}, \text{ A's ejected part.}$$

$$\frac{2809}{28} \times 100 = \frac{70125}{7} = 10017 \text{ cwt. } 3 \text{ qrs. } 12 \text{ lb. whole}$$

cargo.

$$\frac{14025}{56} : \frac{2805}{28} :: \frac{70125}{7} : 4007 \text{ cwt. } 16 \text{ lb. whole ejected}$$

$$\text{part} = \frac{28050}{7}.$$

$$\frac{10}{3}$$

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$$\frac{10}{3} : \frac{25}{4} :: \frac{28090}{7} : \frac{126325}{28} = 4508 \text{ cwt. 4 lb. A's cargo} \\ + \text{B's.}$$

$$4508 \text{ cwt. 4 lb.} - 250 \text{ cwt. 1 qr. 22 lb.} = 4257 \text{ cwt.} \\ 2 \text{ qrs. 10 lb.} = \frac{238425}{56}, \text{ B's cargo.}$$

$$10017 \text{ cwt. 3 qrs. 12 lb.} - 4508 \text{ cwt. 4 lb.} = 5509 \text{ cwt.} \\ 3 \text{ qrs. 8 lb.} = \frac{154275}{28}, \text{ C's cargo.}$$

$$\frac{70125}{7} : \frac{28050}{7} :: \frac{238425}{56} : \frac{47685}{28} = 1703 \text{ cwt. 4 lb. B's} \\ \text{ejected part.}$$

$$100 \text{ cwt. 20 lb.} + 1703 \text{ cwt. 4 lb.} = 1803 \text{ cwt. 24 lb.} \\ \text{A's + B's.}$$

$$4007 \text{ cwt. 16 lb.} - 1803 \text{ cwt. 24 lb.} = 2203 \text{ cwt. 20 lb.} \\ \text{C's ejected part.}$$

$$1 : \frac{7}{3} :: \frac{238425}{56} : \frac{556325}{56} = 9934 \text{ l. 7 s. 6 d. B's cost, be-} \\ \text{sidcs charges.}$$

$$701 \text{ l. 5 s.} + 9934 \text{ l. 7 s. 6 d.} = 10635 \text{ l. 12 s. 6 d. A's} \\ + \text{B's cost.}$$

$$15778 \text{ l. 2 s. 6 d.} - 10635 \text{ l. 12 s. 6 d.} = 5142 \text{ l. 10 s.} \\ \text{C's cost, besides charges.}$$

$$4257 \text{ cwt. 2 qrs. 10 lb.} - 1703 \text{ cwt. 4 lb.} = 2554 \text{ cwt.} \\ 2 \text{ qrs. 6 lb.} = \frac{141055}{56}, \text{ B's remainder.}$$

$$5509 \text{ cwt. 3 qrs. 8 lb.} - 2203 \text{ cwt. 2 lb.} = 3305 \text{ cwt.} \\ 3 \text{ qrs. 16 lb.} = \frac{92565}{28}, \text{ C's remainder.}$$

$$1 : \frac{7}{3} :: \frac{141055}{56} : 5960 \text{ l. 12 s. 6 d. value of B's remain-} \\ \text{ing part at prime cost.}$$

$$\frac{100}{1} : \frac{120}{1} :: \frac{47685}{8} : \frac{57222}{8} = 7152 \text{ l. 15 s. advanced,} \\ \text{value of B's.}$$

$$5934 \text{ l. 7 s. 6 d.} - 7152 \text{ l. 15 s.} = 2781 \text{ l. 12 s. 6 d. B's} \\ \text{loss, charge excepted.}$$

$$1 : \frac{30}{7} :: \frac{92565}{28} : \frac{6171}{8} = 771 \text{ l. 7 s. 6 d. C gained by} \\ \text{what he saved.}$$

$$\frac{154275}{28} : 5142 \text{ l. 10 s.} = \frac{10285}{2} :: \frac{92565}{28} : 3085 \text{ l. 10 s.} \\ \text{value of C's remainder.}$$

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3085l. 10s. + 771l. 7s. 6d. = 3856l. 17s. 6d. advanced, value of C's.

5142l. 10s. - 3856l. 17s. 6d. = 1285l. 12s. 6d. C's loss, besides charges.

$$\frac{70125}{7} : \frac{525}{1} :: \frac{14025}{56} : \frac{105}{8} = 13l. 2s. 6d. A's$$

$$\frac{14025}{56} : \frac{105}{8} :: \frac{238425}{56} : \frac{1785}{8} = 223l. 2s. 6d. B's$$

$$13l. 2s. 6d. + 223l. 2s. 6d. = 236l. 5s. -d.$$

$$525l. - 236l. 5s. -d. = 288l. 15s. -d. C's$$

$$701l. 2s. 6d. + 13l. 2s. 6d. = 83l. 5s. -d. A's$$

$$2781l. 12s. 6d. + 223l. 2s. 6d. = 3004l. 15s. = B's$$

$$1285l. 12s. 6d. + 288l. 15s. = 1574l. 7s. 6d. = C's$$

part of the charges.
whole loss.

29. There were at a feast 20 men and 30 women, and 15 servants, who spent 24l. and for every 10s. that a man paid, a woman paid 6s. and a servant 2s.; what did each person pay?

20 × 10 = 200, 30 × 6 = 180, and 15 × 2 = 30, their sum 410.

$$\begin{array}{l} 410 : 24 :: 200 : 11.7073 = 11\ 14\ 1\frac{1}{2} \text{ men} \\ 410 : 24 :: 180 : 10.7305 = 10\ 10\ 8\frac{1}{2} \text{ women} \\ 410 : 24 :: 30 : 1.5609 = 1\ 15\ 1\frac{1}{2} \text{ servants} \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{pay.}$$

20) 11l. 14s. 1½d. (11s. 8½d. nearly each man.
30) 10l. 10s. 8½d. (7s. ¼d. ⅓ each woman.
15) 1l. 15s. 1½d. (2s. 4d. ⅓ each servant.

30. It is proposed to divide 300l. amongst three persons, so that A gets 6l. more than ⅔ B, 12l. more than ⅓, and C 8l. less than ⅓; what is the share of each?

According to the most obvious meaning of this question, the solution is as follows:

$$\frac{1}{3} \text{ of } 300l. = 100l. \quad \frac{2}{3} \text{ of } 300l. = 200l. \quad \text{and } \frac{1}{3} \text{ of } 300l. = 100l.$$

$$\text{Also } 100l. + 6 = 106; 100 + 12 = 112; \text{ and } 200 - 8 = 192.$$

And 106 + 112 + 192 = 410, their sum.

$$410 : 300 :: 106 : 78l. 14s. 9d. \frac{2}{3} = A's$$

$$\text{Also } 410 : 300 :: 112 : 81l. 10s. 10d. \frac{1}{3} = B's$$

$$\text{And } 410 : 300 :: 192 : 125l. 4s. 4d. \frac{1}{3} = C's$$

share.

Others

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Others taking the question in a different sense, solve it thus :

$$6 + 12 - 8 = 10; \text{ and } 300 - 10 = 290.$$

Then $\frac{1}{2} = \frac{2}{4}$; $\frac{1}{3} = \frac{2}{6}$; and $\frac{2}{3} = \frac{4}{6}$; their sum $\frac{8}{6}$.

$$\frac{2}{6} : 290 :: \frac{1}{6} : 96\frac{2}{3} + 6 = 102\text{l. } 13\text{s. } 4\text{d. for A.}$$

$$\frac{2}{6} : 290 :: \frac{2}{6} : 64\frac{2}{3} + 12 = 76\text{l. } 8\text{s. } 10\text{d. } \frac{2}{3} - \text{B.}$$

$$\frac{2}{6} : 290 :: \frac{4}{6} : 120\frac{2}{3} - 8 = 120\text{l. } 17\text{s. } 9\text{d. } \frac{1}{3} - \text{C.}$$

31. It being agreed that the French King, Pope, and Pretender, are to share 100000 acres in the infernal regions, in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ respectively; but the Pretender relinquishing his right, how is the territory to be divided betwixt the other two, without the help of a lawyer?

Palladium.

$\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ reduced, so as to have one common denominator, will be $\frac{20}{60}$, $\frac{15}{60}$, and $\frac{12}{60}$; rejecting the denominator 20, 15, and 12.

$$\text{Then } 20 + 15 + 12 = 47.$$

$$\text{As } 47 : 100000 :: \begin{cases} 20 : 42553\frac{1}{47}, \text{ French King.} \\ 15 : 31914\frac{2}{47}, \text{ Pope.} \\ 12 : 25531\frac{3}{47}, \text{ Pretender.} \end{cases}$$

But 25531 $\frac{3}{47}$ acres being relinquished by the Pretender, must be divided between the French King and the Pope, as 4 to 3;

$$\text{viz. } 4 + 3 = 7 : 25531\frac{3}{47} :: 4 : 14589\frac{12}{47}.$$

$$7 : 25531\frac{3}{47} :: 3 : 10942\frac{9}{47}.$$

$\therefore 42553\frac{1}{47} + 14589\frac{12}{47} = 57142\frac{13}{47}$, for the French King.

And $31914\frac{2}{47} + 10942\frac{9}{47} = 42857\frac{11}{47}$, for the Pope.
Q. E. F.

32. Bought 100 quarters of malt, meal and oatmeal, together for 142 l. for every five bushels of malt, I had three of meal; and for every eight of meal, I had seven of oatmeal: pray what did they cost me severally a bushel, the malt being half as dear again as the meal, and the meal double the price of the oatmeal?

$$3 \text{ meal} : 5 \text{ malt} :: 8 \text{ meal} : 13\frac{1}{2} = \frac{40}{3}, \text{ malt.}$$

$$\text{Then } 8 + 13\frac{1}{2} + 7 = 28\frac{1}{2} = \frac{85}{2}, \text{ their sum.}$$

$$\frac{85}{2} : \frac{40}{3} :: \frac{100}{1} : \frac{800}{17} = 47\frac{1}{17} = 376\frac{8}{17} \text{ malt} = \frac{6400}{17}.$$

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$$\frac{85}{3} : \frac{8}{1} :: \frac{100}{1} : \frac{480}{17} = 28\frac{4}{7} = 225\frac{1}{4} \text{ meal} = \frac{3840}{17}$$

$$\frac{85}{3} : \frac{7}{1} :: \frac{100}{1} : \frac{420}{17} = 24\frac{1}{7} = 197\frac{1}{4} \text{ oatmeal} = \frac{3360}{17}$$

$$\text{And } \frac{800}{17} \times \frac{2}{3} = \frac{1200}{17}$$

$$\left. \begin{array}{l} \frac{480}{17} \\ \frac{420}{17} \end{array} \right\} \text{for the price of the } \left\{ \begin{array}{l} \text{malt.} \\ \text{meal.} \\ \text{oatmeal.} \end{array} \right.$$

$$\frac{420}{17} \times \frac{1}{2} = \frac{210}{17}$$

The denominators may be omitted, and each numerator divided by 30, the quotients will still retain the same proportion ;

$$\text{viz. } 40 + 16 + 7 = 63.$$

$$\text{As } 63 : 142 :: 40 : \frac{5680}{63} = 90 \text{ } 3 \text{ } 2\frac{2}{7}$$

$$63 : 142 :: 16 : \frac{2272}{63} = 36 \text{ } 1 \text{ } 3\frac{1}{7}$$

$$63 : 142 :: 7 : \frac{994}{63} = 15 \text{ } 15 \text{ } 6\frac{1}{7}$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} \text{price of the } \left\{ \begin{array}{l} \text{malt.} \\ \text{meal.} \\ \text{oatmeal.} \end{array} \right.$$

$$\frac{80}{17} \left(\frac{71}{63} \right) \left(\frac{1207}{5040} \right) = 4 \text{ s. } 9\frac{10}{17} \text{ d. malt}$$

$$\frac{120}{17} \left(\frac{71}{63} \right) \left(\frac{1207}{7560} \right) = 3 \text{ s. } 2\frac{10}{17} \text{ d. meal}$$

$$\frac{240}{16} \left(\frac{71}{63} \right) \left(\frac{1207}{15120} \right) = 1 \text{ s. } 7\frac{10}{17} \text{ d. oatmeal}$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} \text{per bushel. Q. E. F.}$$

33. Three men, A, B, C, buy a ship for 310 l. 15 s. of which A paid an unknown sum; B paid $2\frac{1}{2}$ as much; and C $3\frac{1}{3}$ as much: how much did each man pay?

$$A = \frac{6}{6}, B = 2\frac{1}{2} = \frac{5}{2} = \frac{15}{6}, C = 3\frac{1}{3} = \frac{10}{3} = \frac{20}{6}$$

$$\text{Then } \frac{6}{6} + \frac{15}{6} + \frac{20}{6} = \frac{41}{6}$$

$$\text{And } 310 \text{ l. } 15 \text{ s.} = 310\frac{1}{4} = \frac{1241}{4}$$

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$$\begin{array}{r} \frac{41}{8} \frac{1243}{81} \frac{13729}{181} = 45 \text{ l. } 9 \text{ s. } 6 \frac{6}{11} \text{ d.} = \text{A's} \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{part.}$$

Then $45 \text{ l. } 9 \text{ s. } 6 \text{ d.} \times 2 \frac{1}{2} = 113 \text{ l. } 13 \text{ s. } 9 \frac{9}{11} \text{ d.} = \text{B's}$
 And $45 \text{ l. } 9 \text{ s. } 6 \text{ d.} \times 3 \frac{1}{2} = 151 \text{ l. } 11 \text{ s. } 8 \frac{9}{11} \text{ d.} = \text{C's}$

£ 310 15 -

34. There were 25 cobblers, 20 taylors, 18 weavers, and 12 combers, spent 133 shillings at a meeting; to which reckoning five cobblers paid as much as four taylors, 12 taylors as much as nine weavers, and six weavers as much as eight combers; how much did each company pay, and what each man?

Per question, $\left. \begin{array}{l} 5 \text{ cobblers} \\ 4 \text{ taylors} \\ 3 \text{ weavers} \\ 4 \text{ combers} \end{array} \right\} \text{paid like fums.}$

Then $\left. \begin{array}{l} 1 \text{ cobbler} \\ 1 \text{ taylor} \\ 1 \text{ weaver} \\ 1 \text{ comber} \end{array} \right\} \times \left. \begin{array}{l} 25 \\ 20 \\ 18 \\ 12 \end{array} \right\} = \left. \begin{array}{l} 5 \\ 5 \\ 6 \\ 3 \end{array} \right\} \text{parts of the reckoning.}$

19

$1 : 133 :: \left\{ \begin{array}{l} 5 : 35 \\ 5 : 35 \\ 6 : 42 \\ 3 : 21 \end{array} \right\} \text{for the } \left\{ \begin{array}{l} \text{cobblers.} \\ \text{taylors.} \\ \text{weavers.} \\ \text{combers.} \end{array} \right.$

s. d.

$\left. \begin{array}{l} 25 | 35 | 1 | 4 \frac{1}{2} \\ 20 | 35 | 1 | 9 \\ 18 | 42 | 2 | 4 \\ 12 | 21 | 1 | 9 \end{array} \right\} \text{for each } \left\{ \begin{array}{l} \text{cobbler.} \\ \text{taylor.} \\ \text{weaver.} \\ \text{comber.} \end{array} \right.$

35. Once as I walked upon the banks of Rye,
 To see the purling streams glide gently by,
 And hear the pretty birds to chirp and sing,
 Making the groves with melody to ring;
 I in the meads three beauteous nymphs did spy,
 That for their pleasure came as well as I;
 And unto me their steps they did direct,
 Saluting me with most benign respect;

A a 4

Saying,

Saying, Well met, we've business to impart,
Which we cannot decide without your art;
Our granum's dead, and left a legacy,
Which is to be divided amongst us three:
In pounds it is two hundred twenty-nine;
Also a good mark, being sterling coin.
Then spake the eldest of the lovely three,
I'll tell you how it must divided be;
Likewise our names I unto you will tell,
Mine is Moll, the other Anne and Nell:
As oft as I five and five-ninths do take,
Anne takes four and three-sevenths her part to make;
As oft as Anne four, and one-ninth does tell,
Three and two-thirds must be took up by Nell.

L. D. 1717.

First $229\text{ l. } 13\text{ s. } 4\text{ d.} = 129.6\text{ };$ $5\frac{5}{9} = 5.5\text{ };$ $4\frac{1}{7} = 4.2857\text{ }.$

And $4\frac{1}{7} = 4.7\text{ };$ and $3\frac{1}{3} = 3.6\text{ }.$

As $4.7 : 3.6 :: 4.2857\text{ } : 3.949809$

Hence as often as Moll takes 5.55555

Anne takes 4.2857

And Nell 3.949809

13.933935

$\therefore 13.933935 : \begin{cases} 5.55555 : 91.569 = 91\text{ } 11\text{ } 4\frac{1}{2}, \text{ Moll.} \\ 4.2857 : 72.995 = 72\text{ } 19\text{ } 10\frac{1}{2}, \text{ Anne.} \\ 229.6 : 3.933935 : 65.102 = 65\text{ } 2\text{ } -\frac{1}{2}, \text{ Nell.} \end{cases}$

Q. E. F.

£ 229.66



SECT. II.

DOUBLE FELLOWSHIP;

OR,

FELLOWSHIP WITH TIME.

DDOUBLE FELLOWSHIP is a rule whereby we compute the gain or loss of several merchants who employ different sums of money different times in partnership.

R U L E.

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R U L E.

As the sum of the products of each man's stock and time : is to the whole gain or loss :: so is the particular product of each man's stock and time : to each man's particular gain or loss.

1. Three persons, A, B and C, enter into partnership thus : A puts in 65 l. for eight months ; B puts in 78 l. for 12 months ; and C puts in 84 l. for six months. With this they traffic, and gain 166 l. 12 s. I demand each man's share of the gain in proportion to his stock and time of employing it ?

$$\begin{array}{l} \text{A's} \\ \text{B's} \\ \text{C's} \end{array} \left. \begin{array}{l} \text{stock} \\ \text{time} \end{array} \right\} \begin{array}{l} \left\{ \begin{array}{l} 65 \times 8 \\ 78 \times 12 \\ 84 \times 6 \end{array} \right\} \begin{array}{l} \text{time} \\ \text{stock} \end{array} \end{array} \left\{ \begin{array}{l} = 520 \\ = 936 \\ = 504 \end{array} \right.$$

1960

$$1960 : 166 :: \left\{ \begin{array}{l} 520 : 44 \text{ l. } 4 \text{ s. } - \text{d.} = \text{A's} \\ 936 : 79 \text{ l. } 14 \text{ s. } 2 \frac{1}{2} \text{ d.} = \text{B's} \\ 504 : 42 \text{ l. } 16 \text{ s. } 9 \frac{1}{2} \text{ d.} = \text{C's} \end{array} \right\} \text{Ans.}$$

Or by finding a common multiplier ; viz. 1960)166.6(.085.

$$\begin{array}{l} \text{Then } 520 \times .085 = 44.2, \text{ for A} \\ \text{Also } 936 \times .085 = 79.56, \text{ for B} \\ \text{And } 504 \times .085 = 42.84, \text{ for C} \end{array} \left. \vphantom{\begin{array}{l} 520 \\ 936 \\ 504 \end{array}} \right\} \text{as before.}$$

2. Three persons, A, B, C, hired a certain pasture for 24 l. in which A keeps 40 cows for four months ; B keeps 30 cows for two months ; and C keeps 36 cows for five months ; how much of the rent ought each of them to pay ?

$$\begin{array}{l} \text{A } 40 \times 4 = 160 \\ \text{B } 30 \times 2 = 60 \\ \text{C } 36 \times 5 = 180 \end{array}$$

$$\begin{array}{l} 1. \\ 400 : 24 \left\{ \begin{array}{l} :: 160 : 9 \text{ } 12 = \text{A's} \\ :: 60 : 3 \text{ } 12 = \text{B's} \\ :: 180 : 10 \text{ } 16 = \text{C's} \end{array} \right. \left. \begin{array}{l} \text{part of} \\ \text{the} \\ \text{rent.} \end{array} \right\} \end{array}$$

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3. Six merchants, viz. A, B, C, D, E and F, enter into partnership, and compose a joint-stock in this manner:

	l.	s.			
A puts in	64	10	=	64.5	
B - - -	78	15	=	78.75	
C - - -	100	-	=	100.	
D - - -	80	10	=	80.5	
E - - -	74	12	=	74.6	
F - - -	125	15	=	125.75	

viz. $\left. \begin{array}{l} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \\ \text{F} \end{array} \right\} \text{for } \left\{ \begin{array}{l} 4\frac{1}{2} \\ 6 \\ 8\frac{1}{2} \\ 12 \\ 9\frac{1}{2} \\ 7 \end{array} \right\} \text{months:}$

they traffic and gain 258 l. 18 s. 4½ d. It is required to find every man's share of the gain, according to his stock, and the time it was employed?

	l.	s.	months.	
A's stock	64.5	×	4.5	= 290.25
B's - -	78.75	×	6.	= 472.5
C's - -	100.	×	8.25	= 825.
D's - -	80.5	×	12.	= 966.
E's - -	74.6	×	9.5	= 708.7
F's - -	125.75	×	7.	= 880.25
Sum	= 4142.7			

The whole gain is 258 l. 18 s. 4½ d. = 258.91875.
Then 4142.7) 258.91875 (.0625, common multiplier.

	l.	s.	d.	
290.25 × .0625	= 18.140625	= 18	2	9¼ = A's
472.5 × .0625	= 29.53125	= 29	10	7½ = B's
825. × .0625	= 51.5625	= 51	11	3 = C's
966. × .0625	= 60.375	= 60	7	6 = D's
708.7 × .0625	= 44.29375	= 44	5	10½ = E's
880.25 × .0625	= 55.015625	= 55	-	3¼ = F's

Here,

Their whole gain £ 258 18 4½

4. A and B in partnership equally divide the gain; A's money, which was 84 l. 12 s. 6 d. lay for 19 months; and B's for no more than seven: the adventure of the latter is sought.

Reciprocally, 19 mon. : 84.625 :: 7 mon. : 229.696 l. = 229 l. 13 s. 11 d. answer.

5. A,

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5. A, B, C have a common stock of 1000 l. A gains 100 l. in nine months; B 80 l. in 12 months, and C 120 l. in eight months: what was each of their particular stocks?

9) 100 (11.1
12) 80 (6.7
80) 120 (15.

				l.	s.	d.	
32. # : 1000 :: 11. # :	338.98305 =	338	19	8	A's	} Rock.	
32. # : 1000 :: 6. # :	203.38983 =	203	7	9	B's		
32. # : 1000 :: 15. # :	457.62712 =	457	12	6	C's		

6. A hath 200l. more stock than B; but A continued his only five months, and B nine, and drew equal gains: what are their stocks?

$$\begin{array}{rcccl} & \text{m. l.} & \text{m. l.} & & \\ \hline 9 - 5 = 4 : 200 :: 5 : 250 = & \text{A's stock.} & & & \\ & 4 : 200 :: 9 : 450 = & \text{B's stock.} & & \\ & & 700 = & \text{whole stock.} & \end{array}$$

7. A and B paid equally for a horse, February 7, 1756; A on the 10th took him a journey into the west, and returned on the 10th of June following; B on the 2d of August took him into Scotland, and stayed till November 13, and then concluded his service this year. From January 17 following, A used him 10 days; and in six weeks after his return, employed him till April 30; B then rode him from May-day to Midsummer; A had him from July 14, till 14 days after St. James's tide; B, on September 30, took him into Norfolk, and came back October 19. He then was sold for 7l. 10s. and they would have the money equitably parted between them; viz. in proportion to the use each made of their steed.

	Days.	
From Feb. 10 till June 10, are - - -	122	} A in all 208 days.
Between Jan. 17 and April 30 - - -	61	
From July 14 till 14 after St. James -	24	
From Aug. 2 till Nov. 13 - - - - -	104	} B in all 179 days.
May 1 till June 24 - - - - -	55	
Sept. 30 till October 19 - - - - -	20	

Then $208 + '179 = 387$ days, the horse was in life.

As $387:7.5::179$, B's time: $39\frac{4}{5}$, A's } share of the
 $317:7.5::208$, A's time: $4-7\frac{7}{10}$, B's } money.
8. A

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8. A for a nine months adventure received 20 l. B for one of seven months received 25 guineas ; and C for lying out of his contribution five months has a title to 32 l. The total of their adventures multiplied into their respective times was 640 l. what then was the particulars ?

$$25 \text{ guineas} = 26.25 \text{ l.}$$

Then $20 + 26.25 + 32 = 78.25$, the whole gain.

$$78.25 : 640 :: \begin{cases} 20 : 163.57837 \\ 26.25 : 214.69648 \\ 32 : 261.72524 \end{cases}$$

$$\begin{array}{r|l} 9 & 163.57837 \\ 7 & 214.69648 \\ 5 & 261.72524 \end{array} \quad \begin{array}{l} 18.175 \\ 30.6709 \\ 52.345 \end{array} = \begin{array}{l} 18 \\ 30 \\ 52 \end{array} \quad \begin{array}{l} 3 \\ 13 \\ 6 \end{array} \quad \begin{array}{l} 6 \\ 5 \\ 10\frac{1}{2} \end{array} = \begin{array}{l} \text{A's} \\ \text{B's} \\ \text{C's} \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{adventure.}$$

9. Ten pounds a quarter is allowed the five auditors of a fire-office ; they attend about seven times in a quarter, and the absentee's money is always divided equally among such as do attend. A and B on these occasions never miss ; C and D are generally twice in a quarter absent, and E only once ; at the payment what had each man to receive ?

5) 10 (2 l. each man's equal share.

7) 2 (5 s. 8 d. each man for his day.

5 s. 8 d. $\times 2 = 11$ s. 5 d. C and D to share each for two days absence.

2 l. — 11 s. 5 d. = 1 l. 8 s. 6 d. to C and D each for attendance.

2 l. — 5 s. 8 d. = 1 l. 14 s. 3 d. to E for his attendance.

11 s. 5 d. $\times 2 = 1$ l. 2 s. 10 d. C's more, D's defaults.

3) 1 l. 2 s. 10 d. (7 s. 7 d. A, B and E's share of these defaults.

4) 5 s. 8 d. (1 s. 5 d. A, B, C and D's share of E's default.

Then 2 l. + 7 s. 7 d. + 1 s. 5 d. = 2 l. 9 s. 4 d. A's and B's each.

1 l. 8 s. 6 d. + 1 s. 5 d. = 1 l. 10 s. C and D each.

1 l. 14 s. 3 d. + 7 s. 7 d. = 2 l. 1 s. 10 d. E's share.

10. A, B and C enter into partnership ; A puts in on the 1st of March 60 l. B puts in the 1st of May 160 yards of broad-cloth ; and C puts in on the 1st of June 405 dyets. On the 1st of January following they accounted their gain, of which A and B took up 456 l. B and C took up 431 l. and C and A took up 375 l. I demand what was

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 was gained as well in the whole as a part; what B valued a yard of his cloth at, and what was C's ducats apiece?

$$\begin{array}{rcl} & 1. & \\ A's \text{ gain} + B's & = & 456 \\ B's - - + C's & = & 431 \\ A's - - + C's & = & 375 \end{array}$$

2) 1262 sum, each being named twice.

631, whole gain.

$$\begin{array}{lcl} \text{Then } 631 - 431 & = & 200 \text{ l.} = A's \\ \text{Also } 631 - 375 & = & 256 \text{ l.} = B's \\ \text{And } 631 - 456 & = & 175 \text{ l.} = C's \end{array} \left. \vphantom{\begin{array}{lcl} \text{Then } 631 - 431 \\ \text{Also } 631 - 375 \\ \text{And } 631 - 456 \end{array}} \right\} \text{gain.}$$

P.	T.	G.	
60	10	200	$60 \times 10 \times 256 = 153600$
.	8	256	$8 \times 200 = 1600$
1600	153600	(96 l. value of B's cloth.)	
160	96 l.	= 1920 s. (12 s. B's cloth per yard.)	

P.	T.	G.	
60	10	200	$60 \times 10 \times 175 = 105000$
.	7	175	$200 \times 7 = 1400$
1400	105003	(75 l. value of C's ducats.)	
1405	75 l.	= 1500 s. (3 s. 8½ d. value of one ducat.)	
	285		
	12		
	<u>3420</u>		
	180		

9) $\frac{180}{\frac{10}{11} \times \frac{10}{11}} = \frac{2}{1}$

11. A clears 12 l. in six months; B 18 l. in five months; and C 21 l. in nine months, with a stock of 72 l. 10 s. what then did the general stock amount to?

P.	T.	G.
72.5	9	23
	6	13
72.5	9	13

$72.5 \times 9 \times 13 = 8482.5$, dividend.

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$$\begin{array}{rcl}
 23 \times 6 = 138 & 8482.5 & (61.46739 = 61 \ 9 \ 4 = A's) \\
 \text{P.} & \text{T.} & \text{G.} \\
 72.5 & . \ 9 & . \ 23 \\
 & 5 & . \ 18 \\
 72.5 \times 9 \times 18 = 11745, & \text{dividend.} & \\
 23 \times 5 = 115 & 11745 & (102.130435 = 102 \ 4 \ 7\frac{1}{2} = B's) \\
 & & 72 \ 10 \ - = C's
 \end{array}
 \left. \vphantom{\begin{array}{rcl} 23 \times 6 = 138 & 8482.5 & (61.46739 = 61 \ 9 \ 4 = A's) \\ \text{P.} & \text{T.} & \text{G.} \\ 72.5 & . \ 9 & . \ 23 \\ & 5 & . \ 18 \\ 72.5 \times 9 \times 18 = 11745, & \text{dividend.} & \\ 23 \times 5 = 115 & 11745 & (102.130435 = 102 \ 4 \ 7\frac{1}{2} = B's) \\ & & 72 \ 10 \ - = C's \end{array}} \right\} \text{Stock.}$$

Answer, the whole stock £ 236 1 11 $\frac{1}{2}$

12. A, B and C enter partnership; A puts in the 1st of January 100 l. and the 1st of May puts in 150 l. more; and on the 1st of September takes out 30 l. the remainder stays in till the year's end.

B puts in the 1st of January 250 l. and on the 1st of June 60 l. more; and on the 1st of November 100 l. more; which continues till the year's end.

C puts in the 1st of January 300 l. and the 1st of April takes out 200 l. and on the 1st of August takes out 50 l. more; the remainder stays in till the year's end; what must each have of the gain, which was 133 l.?

	l.		l.
A from 1 January	- - - - 100	$\times 12 =$	1200
1 May	- - - - 150	$\times 4 =$	600
1 September	- - - 120	$\times 4 =$	480
			<u>2280</u>

B from 1 January	- - - - 250	$\times 12 =$	3000
1 June	- - - - 60	$\times 7 =$	420
1 November	- - - 100	$\times 2 =$	200
			<u>3620</u>

C from 1 January	- - - - 300	$\times 3 =$	900
1 April	- - - - 100	$\times 4 =$	400
1 August	- - - 50	$\times 5 =$	250
			<u>1550</u>

Then $2280 + 3620 + 1550 = 7450.$

	l.	s.	d.	qrs.	
7450 : 133 ::	{ 2280 : 40	14	$\frac{1}{2}$	$\frac{1}{4}$	A's
	{ 3620 : 64	12	6	$\frac{1}{4}$	B's
	{ 1550 : 27	13	5	$\frac{1}{4}$	C's

gain.

13. A,

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18. A, B and C in company, and put in together 3822 l. A's money was in three months, B's money was in five months, and C's money was in seven months; they gained 234 l. which was so divided, as $\frac{1}{4}$ of A's gain was equal to $\frac{1}{2}$ of B's gain; and $\frac{1}{4}$ of B's gain was equal to $\frac{1}{4}$ of C's gain: what did each merchant gain and put in?

Suppose A's gain to be 2
Then will B's be - - 3 } by the question.
And C's - - - - - 4 }

Their sum = 9

1. $\frac{2}{3} : \frac{52}{78} = \frac{A's}{B's}$ gain.
Also $9 : 234 :: \frac{3}{4} : \frac{104}{104} = \frac{C's}{C's}$

And $52 \times 3 = 156$

$78 \times 5 = 390$

$104 \times 7 = 728$

1274, their sum.

1274) 3822 (3, common multiplier.

156 $\times 3 = 468 = A's$
390 $\times 3 = 1170 = B's$
728 $\times 3 = 2184 = C's$ } stock.

3822, whole stock.

SECT. III.

FACTORAGE.

WHEN a person does not transact business himself, but commissions another to act for him, the person so commissioned is called a factor, and the business he transacts is called factorship or factorage.

1. What

1. What is the commission of 793 l. 17 s. 6 d. at 2½ per cent.?

$$\begin{array}{r} \text{l. s. d.} \\ \frac{1}{4} 793 \ 17 \ 6 \\ \hline 2\frac{1}{2} \end{array}$$

$$\begin{array}{r} 1587 \ 15 \ 0 \\ \frac{1}{4} 396 \ 18 \ 9 \\ 99 \ 4 \ 8\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£ } 20.83 \ 18 \ 5\frac{1}{2} \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 16.78 \text{ s.} \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 9.41 \text{ d. anf. } 20 \text{ l. } 16 \text{ s. } 9\frac{1}{2} \text{ d.} \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 1.64 \text{ qr.} \\ \hline \end{array}$$

$$\begin{array}{r} \text{Or,} \\ 10 \text{ per cent. or } \frac{1}{10} \end{array} \begin{array}{r} \text{l. s. d.} \\ 793 \ 17 \ 6 \\ \hline \end{array}$$

$$\begin{array}{r} \frac{1}{10} 79 \ 7 \ 9 \\ \hline \end{array}$$

$$\begin{array}{r} \frac{1}{10} 15 \ 17 \ 6\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{1}{10} 3 \ 19 \ 4\frac{1}{2} \\ - 19 \ 10 \\ \hline \end{array}$$

$$\text{Answer, } \text{£ } 20.16 \ 9$$

2. What is the commission of 967 l. 13 s. 4 d. at 3½ per cent.?

$$\begin{array}{r} \text{l. s. d.} \\ \frac{3}{8} 967 \ 13 \ 4 \\ \hline 3\frac{1}{2} \end{array}$$

$$\begin{array}{r} 2903 \ - \ - \\ \frac{3}{8} 483 \ 16 \ 8 \\ \frac{3}{8} 241 \ 18 \ 4 \\ 120 \ 19 \ 2 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£ } 37.49 \ 14 \ 2 \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 9.94 \text{ s.} \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 11.30 \text{ d.} \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 1.20 \text{ qr.} \\ \hline \end{array}$$

Or,

$$\begin{array}{r} \text{l. s. d.} \\ \frac{3}{8} 967 \ 13 \ 4 \\ \hline \end{array}$$

$$\begin{array}{r} \frac{1}{8} 96 \ 15 \ 4 \\ \hline \end{array}$$

$$\begin{array}{r} \frac{1}{8} 24 \ 3 \ 19 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \ 1 \ 11 \\ 1 \ 4 \ 2\frac{1}{2} \\ \hline \end{array}$$

$$\text{£ } 37 \ 9 \ 11\frac{1}{2} \text{ answer.}$$

3. A merchant's real stock being 100*l.* and the factor's 30*l.* who received $\frac{1}{5}$ of the gain; what was his services valued at?

$\frac{2}{3} : 100 :: \frac{1}{5} : 50$; therefore $50 - 30 = 20$, the answer.

Otherways.

$100 + 30 = 130$ *l.* whole stock. $\frac{3}{5}$ 130 (43*l.* 6*s.* 8*d.*
 $\therefore 43$ *l.* 6*s.* 8*d.* $- 30$ *l.* = 13*l.* 6*s.* 8*d.* value of the factor's service.

4. A merchant delivers to his factor 100*l.* allowing him to join to it 30*l.* and values his service at 40*l.* what share of the gain ought the factor to have, the whole gain being 75*l.*?

There are two ways of solving questions of this kind; but if the merchant and factor previously agree (as to prevent disputes they always should) the method is determined.

The most common method:

$30 + 40 = 70$, factor's stock.
 100, merchant's stock.

	l.	s.	d.	
170 : 75 :: 70 :	30	17	$7\frac{1}{2}$	factor's } share of
170 : 75 :: 100 :	44	2	$4\frac{1}{4}$	merch. } the gain
	75	-	-	

But if the gain be made upon the real stock 130*l.* and not upon the imaginary one 170*l.* the factor ought to be gratified for his service, by being allowed the profit of 40*l.* of the real stock more than what he actually put in.

In consideration whereof the above question must be solved as follows:

$100 - 40 = 60$, merchant's stock.
 $30 + 40 = 70$, factor's stock.

	l.	s.	d.	
130 : 75 :: 60 :	34	12	$3\frac{1}{2}$	merch. } share.
150 : 75 :: 70 :	40	7	$8\frac{1}{2}$	factor's }
	£ 75	-	-	

B b

g. A

5. A merchant delivers unto his factor 500 l. and allows his person at 200 l. when they made up their accounts they find they have gained 20 per cent. what is the factor's part?

By the first method.

First, $500 \text{ l.} + 200 = 700$; and $100 : 20 :: 700 : 140$.

$\therefore 700 : 140 :: 200 : 40$; factor's part.

$140 - 40 = 100 \text{ l.}$ merchant's part.

By the other method, the whole stock being but 500 l.

As $100 : 20 :: 500 : 100 \text{ l.}$ the whole gain.

Also $100 : 20 :: 300 : 60 \text{ l.}$ for the merchant.

$\therefore 100 : 20 :: 100 : 40 \text{ l.}$ for the factor.

6. A merchant's real stock being 100 l. and the factor's service valued at 20 l. who received $\frac{1}{4}$, what was the factor's real stock?

First $100 \text{ l.} - 20 = 80$, which the factor must put in.

But by the other method, $100 \text{ l.} - 20 = 80 = \frac{1}{4}$, the real stock.

$\therefore 160 - 100 = 60$, which the factor in this case puts in.

7. A merchant's real stock 100 l. and the factor being allowed $\frac{1}{4}$ of the gain for his service; what real stock must he join to have $\frac{1}{4}$ of the gain?

By the first method.

4) $100 (25$; and $100 + 25 = 125 \text{ l.}$ imaginary stock.

3) $125 (41 \text{ l. } 13 \text{ s. } 4 \text{ d.}$ the factor being to have $\frac{1}{4}$.

$41 \text{ l. } 13 \text{ s. } 4 \text{ d.} - 25 = 16 \text{ l. } 13 \text{ s. } 4 \text{ d.}$ the factor must put in.

By the other method.

3) $100 (33 \text{ l. } 6 \text{ s. } 8 \text{ d.} = \frac{1}{4}$, imaginary stock.

Also $100 \text{ l.} + 33 \text{ l. } 6 \text{ s. } 8 \text{ d.} = 133 \text{ l. } 6 \text{ s. } 8 \text{ d.}$ whole imaginary stock.

3) $133 \text{ l. } 6 \text{ s. } 8 \text{ d.} (44 \text{ l. } 8 \text{ s. } 10 \frac{2}{3} \text{ d.}$ factor's $\frac{1}{4}$.

$\therefore 44 \text{ l. } 8 \text{ s. } 10 \frac{2}{3} \text{ d.} - 33 \text{ l. } 6 \text{ s. } 8 \text{ d.} = 11 \text{ l. } 2 \text{ s. } 2 \frac{2}{3} \text{ d.}$ the answer.

8. A merchant's real stock being 120 l. and the factor's 60 l. they agreed, that at the year's end the factor should have

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have half of both stock and gain; but they broke up at eight months end, having gained 150*l.* how much ought the factor to have?

First $120 + 60 = 180$, whole stock; and 2) $180 (90$ *l.* the share of each of the stock at the year's end: so that the factor was to have 30*l.* of the merchant's stock, had it continued in trade for 12 months.

But $12 : 30 :: 8 : 20$ *l.* the factor's due of the merchant.
Also 120 *l.* — $20 = 100$ *l.* merchant's } stock at eight
And $60 + 20 = 80$ *l.* factor's - - - } months.

l. s. d.

$180 : 150 :: \left\{ \begin{array}{l} 100 : 83 \text{ } 6 \text{ } 8, \text{ merchant's } \\ 80 : 66 \text{ } 13 \text{ } 4, \text{ factor's } \end{array} \right\}$ gain.

l. s. d. l. s. d.

$\therefore 100$ *l.* + $83 \text{ } 0 \text{ } 8 = 183 \text{ } 6 \text{ } 8$, merchant's }
And 80 *l.* + $66 \text{ } 13 \text{ } 4 = 146 \text{ } 13 \text{ } 4$, factor's - } part.

9. It is proposed by an elderly person in trade, desirous of a little respect, to admit a sober and industrious young fellow to a share in the business, and to encourage him offer, that if his circumstances will allow him to advance 100*l.* his pay should be 40*l.* a year; if he shall be able to put 200*l.* into the stock, he shall have 55*l.* a year; and if 300*l.* he shall receive 70*l.* annually: in this proposal what was allowed for his attendance simply?

First 70 *l.* — 55 *l.* = 15 *l.* } hence it is plain he proposed
And 55 *l.* — 40 *l.* = 15 *l.* } to allow him 15 per cent.
for his money.

$\therefore 40$ *l.* — $15 = 25$ *l.* the answer.

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SECT. IV.

LOSS AND GAIN.

BY this rule we discover what is got or lost by any parcel of goods, or how much per cent. is got or lost according to the price bought and sold at; by which we are instructed to raise or fall the price of commodities in such proportion, that neither our gain may be so exorbitant as to injure our customers, nor our loss so great as to impoverish ourselves.

1. At what price must I sell 1 cwt. of sugar, which cost 2 l. 6 s. 8 d. to gain 10 per cent.?

$$\begin{array}{r} \text{10 per cent. is } \frac{1}{10} \end{array} \begin{array}{r} \text{l.} \\ 2 \\ 6 \\ 8 \end{array} \begin{array}{r} \text{s.} \\ 6 \\ 8 \end{array} \begin{array}{r} \text{d.} \\ 8 \end{array}$$

$$\text{Answer, } \underline{\underline{2 \text{ l. } 11 \text{ s. } 4 \text{ d.}}}$$

2. A Manchester man buyeth yarn for 6 s. for a bundle, which not proving so good as was expected, would put it off again, so as but to lose 6 per cent. by it; what is the selling price?

$$\text{10 per cent. is } \frac{1}{10} \begin{array}{r} \text{s.} \\ 6 \end{array} \begin{array}{r} \text{d.} \\ - \end{array}$$

$$\text{5 per cent. is } \frac{1}{2} - 7.2$$

$$\text{1 per cent. is } \frac{1}{10} - 3.6$$

$$\text{Loss - } 4.32$$

$$\therefore 6 \text{ s.} - 4.32 = 5 \text{ s. } 7.68 \text{ d. the selling price.}$$

3. If a tun of wine cost 45 l. 19 s. 10 d. how must I sell it a tun to gain $26\frac{1}{2}$ per cent.?

$$\begin{array}{r} \text{20 per cent. is } \frac{1}{5} \begin{array}{r} \text{l.} \\ 45 \\ 19 \\ 10 \end{array} \begin{array}{r} \text{s.} \\ 19 \\ 10 \end{array} \begin{array}{r} \text{d.} \\ 10 \end{array} \\ \text{5 per cent. is } \frac{1}{20} \begin{array}{r} 9 \\ 3 \\ 11\frac{1}{2} \end{array} \\ \text{1 per cent. is } \frac{1}{200} \begin{array}{r} 2 \\ 5 \\ 11\frac{1}{2} \end{array} \\ \frac{1}{2} \text{ per cent. is } \frac{1}{400} \begin{array}{r} - \\ 9 \\ 2\frac{1}{2} \end{array} \\ \begin{array}{r} - \\ 4 \\ 7 \end{array} \end{array}$$

$$\underline{\underline{\text{£ } 58 \text{ } 3 \text{ } 6\frac{1}{2}, \text{ the answer.}}}$$

4. If I buy broad-cloth for 11 s. 6 d. a yard; how must I sell it to gain 20 per cent.?

$$\text{20 per cent. is } \frac{1}{5} \begin{array}{r} \text{s.} \\ 11 \\ 6 \end{array} \begin{array}{r} \text{d.} \\ 6 \end{array}$$

$$\text{Answer, } \underline{\underline{13 \text{ s. } 9\frac{1}{2} \text{ d.}}}$$

5. If a pack of yarn, weighing 240 lb. cost 13 l. what must it be sold at a pound to gain $15\frac{1}{2}$ per cent.?

$$\begin{array}{r}
 10 \text{ per cent. is } \frac{10}{100} \\
 5 \text{ per cent. is } \frac{5}{100} \\
 \frac{1}{2} \text{ per cent. is } \frac{1}{200}
 \end{array}
 \left|
 \begin{array}{r}
 13 \\
 1 \cdot 6 \\
 - 13 \\
 - 1
 \end{array}
 \right.
 \begin{array}{r}
 \\
 \\
 \\
 3\frac{1}{2}
 \end{array}$$

equal to the pound's weight in a pack, it will be 1 s. 3 d. a pound, and $3\frac{1}{2}$ d. over in the whole.

6. If I gain 2 d. in the shilling, what is my gain per cent.?

$$2 \text{ d. is } \frac{2}{100}$$

Answer, $\pounds 16 \cdot 13 \cdot 4$ per cent.

7. By a quantity of damaged lump-sugar lost $5\frac{1}{2}$ d. in the shilling, what did I lose per cent.?

$$\begin{array}{r}
 \frac{1}{2}, \frac{1}{4} \pounds 100 \\
 25 \\
 16 \quad 13 \quad 4 \\
 6 \quad 5 \quad -
 \end{array}$$

Answer, $\pounds 47 \cdot 18 \cdot 4$ per cent.

8. If by the sale of a chest of lemons I gained 4 s. in the pound, what is my gain per cent.?

$$4 \text{ s. is } \frac{4}{100}$$

Answer, $\pounds 20$ per cent.

9. A grocer bought 3 cwt. 1 qr. 14 lb. weight of cloves, at the rate of 2 s. 4 d. per pound, and sold them for 52 l. 14 s. whether did he gain or lose by the bargain, and how much?

s. d.
2 4 per lb.
7

16 4
4

3 5 4
4

$\frac{1}{2}$ 13 1 4, price of 1 cwt.
3

	cwt.	qrs.	lb.
39 4 - - - -	3	-	-
$\frac{1}{2}$ 3 5 4 - - - -	-	1	-
1 12 8 - - - -	-	-	14

£ 44 2 - - - - 3 1 14

10. A merchant bought 436 yards of broad-cloth for 8s. 6d. a yard, and sold it again for 10s. 4d. a yard; what did he gain by the whole?

s. d.
10 4, sold at a yard.
8 6, cost.
1 10, gained per yard.

$\frac{1}{2}$ 436	872
$\frac{1}{2}$ 145 4	727
$\frac{1}{2}$ 799 4	39 10 4, the anfw.

11. Sold goods for 50 l. 12 s. 6 d. and gained $3\frac{1}{2}$ d. in the shilling, what did I gain per cent. and what the prime cost?

3 d. is $\frac{1}{4}$ 100 20 per cent. ; 50 12 6

$\frac{1}{2}$ d. is $\frac{1}{8}$ 25
4 3 4

Per cent. £ 29 3 4

$\frac{1}{2}$ 100	2 6
$\frac{1}{2}$ 2 -	6
$\frac{1}{2}$ 2 -	6
$\frac{1}{2}$ - 10	12
$\frac{1}{2}$ - 1	8

Gain £ 14 15 3, subtract

Prime cost £ 35 17 3

12. If I buy 1 cwt. of tobacco for 4l. 13s. 4d. and sell it again for 5l. 2s. 8d. a pound; what do I gain or lose, and what per cent.?

$$\frac{1}{8} \times 7 = 6 \frac{1}{2}, \text{ price of 7 lb.}$$

$$\begin{array}{r} 4 \\ 5 \\ 8 \\ \hline 4 \end{array}$$

$$\pounds 5 \ 2 \ 8 - 4 \text{ l. } 13 \text{ s. } 4 \text{ d.} = 9 \text{ s. } 4 \text{ d. gain.}$$

As 4s. : 5s. 2 : 100 : 125 his gain was 25 per cent.

13. A Manchester man buys 20 ton of cheese, with which he went into Ireland; it cost him 400 l. the freight and custom came to 50 l. his own expenses and charges were 16 l. 13 s. 4 d. How must he sell it a pound to gain 20 l. per cent.?

$$\begin{array}{r} \text{Cost} \quad \quad \quad 400 \\ \text{Freight and custom} \quad 50 \\ \text{Charges} \quad \quad \quad 16 \ 13 \ 4 \\ \hline 20 \text{ per cent. } \left(\frac{1}{5} \right) \left. \begin{array}{r} 466 \ 13 \ 4 \\ 93 \ 6 \ 8 \end{array} \right\} \\ \hline \pounds 560, \text{ selling} \\ \text{price.} \end{array}$$

$$\begin{array}{r} \text{20 } \left\{ \begin{array}{l} 5 \ 28 \text{ per ton,} \\ 4 \ 5 \ 12 \text{ s.} \\ 7 \ 1 \ 8 \text{ per cwt.} \\ 4 \text{ } - \ 4 \text{ s.} \\ 4 \text{ } - \ 1 \text{ s.} \end{array} \right. \end{array}$$

Answer, - - 3 d. p. lb.

14. A stationer sold quills at 11 s. a thousand, by which he cleared $\frac{1}{4}$ of the money; but growing scarce, raised them 13 s. 6 d. a thousand; what might he clear per cent. by the latter price?

$$\frac{3}{8} \times \frac{11}{1} = \frac{33}{8} = 4 \text{ s. } 1 \frac{1}{2} \text{ d. gained per thousand by the first sale.}$$

$$11 \text{ s. } - 4 \text{ s. } 1 \frac{1}{2} \text{ d.} = 6 \text{ s. } 10 \frac{1}{2} \text{ d.} = \frac{11}{12}, \text{ cost.}$$

$$13 \text{ s. } 6 \text{ d.} - 6 \text{ s. } 10 \frac{1}{2} \text{ d.} = 6 \text{ s. } 7 \frac{1}{2} \text{ d.} = \frac{53}{100}, \text{ gain per 1000 by the second sale.}$$

$$\frac{11}{12} : \frac{53}{100} :: \frac{100}{1} : \frac{1060}{11} = 96 \text{ l. } 7 \text{ s. } 3 \frac{1}{2} \text{ d. the answer.}$$

(P. 1.)

B b 4

15. Bought

15. Bought hosiery in London at 4s. 3d. the pair, and sold them afterwards in Dublin at 6s. the pair; now taking the charges at an average to be 2d. the pair; and considering I must lose 12 per cent. by remitting my money home again; what must I gain per cent. by the article of trade?

$$4s. 3d. = \frac{51}{240} + 2d. = \frac{2}{240} = 4s. 5d. = \frac{53}{240}; \text{ and}$$

$$6s. = \frac{3}{10}.$$

$$\text{As } \frac{53}{240} : \frac{3}{10} :: \frac{100}{1} : \frac{7200}{53}. \text{ Then } 100 - 12 = 88.$$

$$\text{Also } \frac{100}{1} : \frac{88}{1} :: \frac{7200}{53} : \frac{6336}{53} = 119l. 10s. 11\frac{1}{2}d.$$

$$\therefore 119l. 10s. 11\frac{1}{2}d. - 100l. = 19l. 10s. 11\frac{1}{2}d. \text{ the answer,}$$

16. If my factor at Leghorn returns me 800 barrels of anchovies, each weighing 14 l. net, worth 12½ d. per pound, in lieu of 7400 pounds of Virginia tobacco; and if I find that I have gained after the rate of 17 per cent. by the said consignment, pray how was my said tobacco invoiced per pound to the factor, that is, what was the prime cost?

$$\text{Barrels } 800 \times 14 \text{ lb.} = 11200, \text{ at } 12\frac{1}{2}d. = \frac{1750}{3}.$$

$$\frac{100}{1} : \frac{17}{1} :: \frac{1750}{3} : \frac{595}{6}.$$

$$\frac{1750}{3} - \frac{595}{6} = \frac{2905}{6} \dots \frac{7400}{1} \times \frac{2905}{6} \left(\frac{2905}{44940} \right) = 1s. 3\frac{1}{2}d. \text{ the answer.}$$

17. If by selling hops at 3 l. 10s. per cwt. the planter clears 30 per cent. what was his gain per cent. when the same goods sold at 4 l. and a crown?

$$100 + 30 = 130 : 100 :: 3.5l. : 2.6923 = \text{cost per cwt.}$$

$$4.25 - 2.6923 = 1.5577l. \text{ gain per cwt.}$$

$$2.6923 : 1.5577 :: 100 : 52.58414 = 52\frac{1}{2}l. 11s. 8.19d. \text{ the answer.}$$

18. Sold a repeating watch for 50 guineas, and by so doing lost 17 per cent. whereas I ought in dealing to have cleared 20 per cent. then how much was it sold under the just value?

$100 - 17 = 83$ l. : 100 l. :: 52.5 : 63.253012, cost.
 Then 100 : 100 l. 63.253012 : 75.903614 l.
 $75.903614 - 52.5 = 23.403614 = 23$ l. 8s. -d. 3f.
 answer.

19. If by remitting to Holland, at 31s. 9d. Flemish per pound sterling, five per cent. is gained; how goes the exchange, when by remittance I clear 10 per cent.?

105 l. : 31.75 :: 110 l. : 33.2619 s. = 1 l. 13 s. 3.143 d. the answer.

20. If by sending pewter for Turkey, and parting with it at 25 $\frac{1}{2}$ d. per pound, the merchant clears cent. per cent. what does he clear in Holland, where he disposes of the cwt. for 8 l.?

3	2.0
12	1.6
20	2.138

$1060 \times 118 = 11.9$ l. sold at per cwt. in Turkey.
 And as his gain was cent. per cent. it cost him 5.94 l. per cwt.

Therefore 8 l. - 5.94 = 2.07 = 2 l. -s, 2 $\frac{1}{2}$ d. the answer.

21. Bought comfits to the value of 41 l. 3s. 4d. for 3s. 1d. per pound: it happened that so many of them were damaged in carriages, that by selling what remained good at 4s. 6d. the pound, my returns were no more than 34 l. 2s. 6d. pray how much of these goods were spoiled, and what did this part stand me in?

34 l. 2s. 6d. = 34.125 - - 3s. 1d. = .15416 - -
 4s. 6d. = .225.

As .225 l. : 1 lb. :: 34.125 : 151.6, remained good.
 $1 - .15416 = .84584$:: 151.6 : 23.38194 l. = 23 l. 7s. 7 $\frac{1}{2}$ d.
 the goods cost.

Then 41 l. 13s. 4d. - 23 l. 7s. 7 $\frac{1}{2}$ d. = 18 l. 5s. 8 $\frac{1}{2}$ d.
 the damaged cost.

22. A had 15 pipes of Malaga wine, which he parted with to B at 4 $\frac{1}{3}$ per cent. profit, who sold them to C for 38 l.

38 l. 11 s. 6 d. advantage; C made them over to D for 500 l. 11 s. 6 d. and cleared thereby $6\frac{1}{2}$ per cent. what did the wine cost A a gallon?

l. s. d.

$$500 \text{ } 16 \text{ } 8 = 500.83$$

$$\text{As } 100 : 6.5 : 500.83 : 32.5416 = \text{C's } \left. \begin{array}{l} \text{profit.} \\ + 38 \text{ } 11 \text{ } 6 = 38.575 = \text{B's} \end{array} \right\}$$

$$500.83 - 71.12916 = 429.70416, \text{ B's cost.}$$

$$\text{As } 100 : 4.3 : 429.70416 : 18.6205134, \text{ A's profit.}$$

$$429.70416 - 18.6205134 = 411.083657, \text{ A's cost.}$$

$$126 \times 15 = 1890 \quad 411.083657 \div (.2175045) = 4 \text{ s. } 4\frac{1}{2} \text{ d. per gallon, the answer.}$$

23. Having bought a parcel of goods for 180 l. and sold them again immediately for 188 l. 10 s. with four months credit; what is gained per cent. per annum?

l. mon. gain.

$$180 \quad . \quad 4 \quad . \quad 8.5$$

$$100 \quad . \quad 12 \quad .$$

$$180 \times 4 = 720 \quad 10200 (14 \text{ l. } 3 \text{ s. } 4 \text{ d. the answer.}$$

24. Having bought 160 gallons of French brandy, at 6 s. 6 d. a gallon, there chanced to leak out 18 gallons; at what rate per gallon may I sell the remainder, with eight months credit, so as to gain upon the whole prime cost, at the rate of 12 per cent. per annum?

$$\begin{array}{r} \text{gal.} \\ 160 \\ \hline 32 \\ 20 \end{array}$$

l. mon. gain.

$$100 \quad . \quad 12 \quad . \quad 12$$

$$152 \quad . \quad 8$$

$$2 \times 52 \times 8 = 4992, \text{ dividend.}$$

£ 52, prime cost.

$$100 \times 12 = 1200 \quad 4992 \div 4.16 \text{ l. gain by the whole.}$$

$$\text{Also } 52 \text{ l. } + 4.16 = 56.16 \text{ l. and } 160 - 18 = 142 \text{ gallons.}$$

$$142 \times 56.16 \div .3955 = 7 \text{ s. } 11 \text{ d. nearly, the answer.}$$

25. Having paid 14 s. a yard for 100 yards of cloth, I propose to gain 25 per cent. ready money; and if I sell it upon time, to have moreover 10 per cent. per annum for the

the forbearance, what must be the price of one yard, with six months credit, to make both these gains?

$$\frac{1}{2}, \frac{1}{2} 100$$

$$25 \text{ per cent. is } \frac{1}{4} 70$$

$$\begin{array}{r} 100 \\ 50 \\ 25 \\ \hline 75 \end{array}$$

$$\begin{array}{r} 70 \\ 17.5 \\ 87.5 \end{array}$$

70, prime cost.

$$\begin{array}{r} \text{£.} \quad \text{T.} \quad \text{Gain.} \\ 100 \quad 12 \quad 10 \\ 87.5 \quad 6 \end{array}$$

$$1200) 9450 (4.375 = 4 \text{ l. } 7 \text{ s. } 6 \text{ d.}$$

$$87.5 + 4.375 = 91.875 = 91 \text{ l. } 17 \text{ s. } 6 \text{ d.}$$

$$100) 91.875 (91875 = 184 \frac{1}{2} \text{ d. the answer.}$$

26. Laid out in a lot of muslin 480 l. 12 s. upon examination of which two parts in seven proved damaged; so that I could make but 5 s. 6 d. a yard of the same; and by so doing, find I lost 48 l. 18 s. by it; at what rate per ell am I to part with the undamaged muslin, to make up my said loss?

$$480 \text{ l. } 12 \text{ s.} = \frac{2403}{5} \times \frac{2}{7} = \frac{4806}{35} = \frac{9612}{70} = 137 \text{ } 6 \text{ } 3 \frac{1}{2} \text{ s. d.}$$

cost of the damaged goods.

$$\frac{3423}{70} = 48 \text{ } 18 \text{ } -$$

lost by the damaged goods.

$$\frac{6189}{70} = 88 \text{ } 8 \text{ } 3 \frac{1}{2}$$

made of the damaged goods.

$$5 \text{ s. } 6 \text{ d.} = \frac{11}{40} : 1 :: \frac{6189}{70} : \frac{24756}{77} = 321 \frac{1}{2} \text{ yards damaged.}$$

$$\frac{2}{7} : \frac{24756}{77} : 1 : 1 : \frac{86646}{77} : 1125 \frac{3}{4} \text{ yards in all.}$$

$$\frac{86646}{77} - \frac{24756}{77} = \frac{61890}{77} = 803 \frac{3}{4} \text{ yards undamaged.}$$

$$\frac{2403}{5} = \frac{35642}{70} - \frac{6189}{70} = \frac{27453}{70} = 392 \text{ l. } 3 \text{ s. } 8 \frac{1}{2} \text{ d. made}$$

of the good.

$$\frac{61890}{77} : \frac{27453}{70} :: \frac{5}{4} : \frac{100661}{165030} = 12 \text{ s. } 2 \text{ d. } 1 \frac{5057}{165030} \text{ qr. per ell.}$$

Q. E. F.

CHAPTER V.

BARTER.

WHEN merchants or tradesmen exchange one commodity for another, it is called *bartering*; and by the rule of proportion, the price and quantity of the goods so exchanged are determined, so that neither party shall sustain a loss by such traffic.

In solving all questions in truck, the intrinsic value of the thing received ought to tally with a like value of the thing delivered, where they deal upon a par; for if there be any difference, some one of the parties has the advantage of the other by the value of that difference.

1. How many pounds of sugar, at $4\frac{1}{2}$ d. per pound must be given in barter for 60 gross of inkle, at 8 s. 8 d. per gross?

$$\begin{array}{r} 8 \text{ s. } 8 \text{ d.} \\ 12 \\ \hline 104 \\ 2 \\ \hline 208 \\ 60 \\ \hline \end{array}$$

$4\frac{1}{2} = 9$) 12480 halfpence.

1386 $\frac{1}{2}$ pounds, the answer.

Or, 1386 lb. and 3 d. in money.

2. Two merchants, A and B, barter; A would exchange 5 cwt. 3 qrs. 14 lb. of pepper, at 3 l. 10 s. per cwt. with B for cotton, worth 10 d. per pound; how much cotton must B give A for his pepper?

1. s. 1. s. d.
 $\frac{1}{2}$ 3 10 20 11 3, value of A's pepper.

$$\begin{array}{r} 5 \\ 17 \text{ } 10 \\ 1 \text{ } 15 \\ 17 \text{ } 6 \\ \hline 8 \text{ } 9 \text{ } 10 \end{array} \quad \begin{array}{r} 20 \\ 411 \\ \times 12 \\ \hline 4935 \end{array}$$

112) cwt. qr. lb.

4935 (493 $\frac{1}{2}$ = 4 1 17 cotton, the answer)

45 $\frac{1}{2}$

$$\begin{array}{r} 20 \text{ } 11 \text{ } 3 \\ \hline \end{array}$$

3. A and B barter; A gives 120 yards of kersey, 3 $\frac{1}{2}$ yards whereof cost 15 s. 9 d. for stockings, at 7 s. a pair, and hats at 6 s. 6 d. each, an equal number of hats, and a pair of stockings; how many of each must B give A for his kersey?

$$\begin{array}{rccccccc} \text{yds.} & \text{s.} & \text{d.} & \text{l.} & \text{yds.} & & \\ 3.5 : 15 & 9 & = & .7875 & :: & 120 & \\ & & & 120 & & & \end{array}$$

94.5000 (27 l. value of the kersey.

Then 7 s. + 6 s. 6 d. = 13 s. 6 d. = .675) 27.000 (40
pair of stockings and hats, the answer.

4. Two merchants, A and B, barter; A would exchange 20 cwt. of cheese, at 1 l. 1 s. 6 d. with B for eight pieces of Irish cloth, at 3 l. 14 s. apiece; I demand which must receive money, and how much?

l. s.

B's 8 pieces of cloth, at 3 l. 14 s. per piece, come to 29 12
A's 20 cwt. of cheese, at 1 l. 1 s. 6 d. per cwt. - - 21 10

So that A is debtor to B - - - - - £ 8 2

5. Two merchants, A and B, barter; A hath 86 yards of broad-cloth, worth 9 s. 2 d. per yard, ready money; but in barter he will have 11 s. per yard; B hath shaloon, worth 2 s. 1 d. per yard, ready money; it is required to find how many yards of shaloon B must give A for his cloth, making his gain in barter equal to that of A?

The most common method in authors of solving this question is as follows:

As 9 s. 2 d. = 110 d. : 11 s. = 132 d. :: 2 s. 1 d. = 25 d.
: 30 d. = 2 s. 6 d. the advanced price of a yard of B's shaloon.

Also $86 \times 11 = 946 \text{ s.} = 47 \text{ l. } 6 \text{ s.}$ advanced value of the cloth.

2.5) 946.0 (378 $\frac{2}{3}$ yards of shaloon, the answer.

But when the price of each quantity are raised proportionally, the quantity sought may be found by the ready money values, without having any regard to the advanced prices.

So

So that the foregoing question may be solved as follows :

$$\begin{array}{r}
 \text{s. d.} \quad \text{s. d.} \\
 5 \text{ } 10 \text{ } - \\
 \underline{7} \\
 39 \text{ } 8 \text{ } 4 = 39.41\text{ } \frac{1}{2} \text{, value of the cloth.}
 \end{array}$$

6. A has currants worth 4 d. a pound; but in truck charges 6 d. and also requires $\frac{1}{4}$ of that in ready money; B has candles at 6 s. 8 d. the dozen, and he in barter, honest man, charges but 7 s. should these persons deal together for the value of 20 l. how much will A have got of B?

$$\begin{array}{l}
 6 \text{ d.} : 4 \text{ d.} :: 20 \text{ l.} : 13 \text{ l. } 6 \text{ s. } 8 \text{ d. worth of A's currants.} \\
 13 \text{ l. } 6 \text{ s. } 8 \text{ d.} - 10 \text{ l.} = 3 \text{ l. } 6 \text{ s. } 8 \text{ d.} \\
 7 \text{ s.} = .35, \text{ and } 6 \text{ s. } 6 \text{ d.} = .325 \text{ l.} \\
 33 \text{ l.} : .325 :: 10 : 9.785714 = 9 \text{ } 5 \text{ } 8 \text{ } 2\frac{1}{2}
 \end{array}$$

$$\text{A's advantage} = \text{£ } 5 \text{ } 19 \text{ } - \text{ } 2\frac{1}{2} \text{, answer.}$$

7. Two merchants have various kinds of goods to barter, A hath 735 yards of Indian silk, at 8 s. 6 d. per yard ready money, and in barter 10 s. also 532 canes, at 3 s. apiece ready money, and in barter 3 s. 4 d. and 16 pieces of muslin, at 4 l. apiece ready money, in barter 4 l. 10 s. B hath scarlet cloth, at 1 l. per yard ready money; glass manufacture, at 1 s. 8 d. per pound ready money, and a finer kind at 2 s. 4 d. per pound; how many yards of cloth, and pounds of each kind of glass, of all a like number, must B give A, advancing his goods proportionally also in barter?

$$\begin{array}{r}
 \text{L. s. d.} \\
 \text{A's Indian silk 735 yards, at 8 s. 6 d. is } 312 \text{ } 7 \text{ } 6 \\
 \text{Canes 532, at 3 s. - - - } 79 \text{ } 16 \text{ } - \\
 \text{Muslin 16 pieces, at 4 l. - } 64 \text{ } - \text{ } - \\
 \hline
 \text{£ } 456 \text{ } 3 \text{ } 6 = 456.175
 \end{array}$$

B's

	l.	s.	d.
B's broad cloth, 1 yard	1	—	—
Glass manufacture per lb.	—	—	8
Ditto sugar fort	—	—	4
	<u>£</u>	<u>1</u>	<u>4</u> — = 22

∴ 12) 456.175 (380 $\frac{2}{3}$ of each sort of B's things, the answer.

8. A and B barter; A hath 100 yards of broad-cloth, worth 12 s. a yard ready money; but in barter he will have 13 s. 6 d. and will also have $\frac{1}{2}$ of the barter value in ready money; B hath sugar at 8 d. a pound; how much sugar ought B to deliver, and how is it to be raised to equal the barter?

100 yards of cloth, at 13 s. 6 d. per yard, is 67 l. 5 s.
4) 67 5

16 16 3 ready money, = 16.8125 l. and 12 s. = .6 l.
.6) 16.8125 (28 $\frac{1}{4}$ yards. Also 8 d. = .03
Then 100 — 28 $\frac{1}{4}$ = 71 $\frac{3}{4}$ yds. which at 12 s. is 43 l. 3 s.
.03) 43.1875 9 d. = 43.1875
03 43 1875

.03) 38.86875 (1262 $\frac{1}{2}$ fere, pounds of sugar.
Also 12 s. : 8 d. :: 13.5 s. : 9 d. advanced price of the sugar.

9. A has kerseys at 4 l. 5 s. apiece ready money; in barter they are changed by him at 5 l. 6 s. each, and half of that required down; B has wax at 3 d. a pound; how ought he to rate it in truck, not to be hurt by the extortion of A?

First, 5 l. 6 s. — 4 l. 5 s. = 1 l. 1 s. gain. 2) 5 l. 6 s. (2 l. 13 s. paid.

Then 4 l. 5 s. — 2 l. 13 s. = 1 l. 12 s. = 384 pence, value of the half remaining.

And 1 l. 12 s. + 1 l. 1 s. = 2 l. 13 s. = 636 pence, made of the half remaining.

∴ 384 : 636 :: 3 : 4 $\frac{1}{2}$, the answer required.

10. A hath 40 pair of stockings, at 3 s. a pair ready money, or 3 s. 8 d. in barter; but he is willing to discount three

three per cent. of his barter price, to have $\frac{1}{4}$ of it paid in ready money; how many yards must he deliver with the money A requires, and what is the rate of his cloth to equal the barter?

First, $100 - 3 = 97$, and $3s. 8d. = .18\frac{1}{2}l.$ also $10s. = .5l.$

$100 : 97 :: .18\frac{1}{2} : .17783 =$ barter price after the discount.

And $40 \times .17783 = 7.113l.$ value of the stockings at the said price.

4) $7.113 (1.7783 = 1l. 15s. 6\frac{1}{2}d.$ ready money.

.15) $1.7783 (11.8\frac{1}{2}$ pair in value.

$40 - 11.8\frac{1}{2} = 28.1\frac{1}{2}$ pair, at $3s.$ is $4.221\frac{1}{2}l.$

.5) $4.221\frac{1}{2} (8.443 = 8$ yards, 1 quarter, 3 nails, nearly.

.15 : $.1783 :: .5 : .59\frac{1}{2} = 11s. 10\frac{1}{2}d.$ the advanced price of the broad-cloth.

11. A let B have a hoghead of sugar, of 18 hundred weight, worth $31s.$ for $42s.$ the hoghead, $\frac{1}{4}$ of which he is to pay in cash; B hath paper worth $14s.$ the ream, which it is agreed shall bear no more than $15s. 6d.$ and at that rate truck for the rest; how stood the account?

18 cwt. sugar, at $\begin{matrix} l. & s. \\ 2 & 2 \\ 1 & 11 \end{matrix}$ is $\begin{matrix} l. & s. \\ 37 & 16 \\ 27 & 18 \end{matrix}$, advanced value.
real value.

A $\pounds 9$ 18 advanced his sugar.
3) 37 16 ($12l. 12s.$ ready money, and $25l. 4s.$ in paper.

Then $15s. 6d. = \frac{31}{40}$, $14s. = \frac{28}{40}$, and $25l. 4s. = \frac{126}{5}l.$

And $\frac{31}{40} - \frac{28}{40} = \frac{3}{40} = 1s. 6d.$

As $\frac{31}{40} : \frac{3}{40} :: \frac{126}{5} : \frac{378}{155} = 2l. 8s. 9\frac{2}{11}d.$ B advanced his paper.

$\therefore 9l. 18s. - 2l. 8s. 9\frac{2}{11}d. = 7l. 9s. 2\frac{1}{11}d.$ the answer in A's favour.

12. A barter with B 40 lb. of cloves, at $6s.$ a pound ready money, and $7s. 6d.$ in barter, but is willing to lose 10 per cent. to have $\frac{1}{4}$ ready money; what is the ready money price of a yard of velvet, delivered by B, at $1l. 1s.$ to equal the barter, and how much was delivered?

First, $100 - 10 = 90$, and $7s. 6d. = .375$.

As $100 : 90 :: .375 : .3375 = 6s. 9d. =$ A's barter price, when 10 per cent. is deducted.

$40 \times 5375 = 215 = 131. 10 s.$ value of the cloves at 5 s. 9 d.

$215 = 205 \cdot 195$ (205 yards of velvet B must deliver.

$6.75 : 6 :: 21 : 18.9 = 18 s. 8 d.$ the ready money, price of the velvet. Q. E. F.

I am beholden to the sagacious Alexander Malcolm, teacher of the mathematics at Aberdeen, for the foregoing as well as the 7th, 8th, and 10th questions, and some others in this work; who, after exploding the mistakes of their first proposers, points out, and fully demonstrates the above method to be true.

13. A has 50 broad-cloths, at 11 l. 10 s. apiece; but in change requires 13 l. taking wool at 2 s. 6 d. per stone of B in return, that was really worth but 4 s. 2 d. a tod; the question is, how many sacks of wool will pay for the cloth, and which of the dealers has the better in the bargain?

l. s. l.

13 - $\times 50 = 650$, advanced value of the broad-cloth.

11 10 $\times 50 = 575$, real value.

£ 75, gain by the broad-cloth.

s. d. s. d.

2 6 $\times 2 = 5$ -, advanced value of the wool per tod.

- 4 2, real value.

£ - 10 per tod gain, or 5 d. per stone.

4 s. 6 d. = .125 : 1 :: 650 : 5200 stone.

1 sack = 267 5200 (260 sacks of wool for five pieces of cloth.

11 5200 stone, at 5 d. per stone profit.

s. d.

4 33 1 8

£ 108 6 8, gain by the wool.

Therefore 108 l. 6 s. 8 d. - 75 l. = 33 l. 6 s. 8 d. B's clear gain by this affair.

14. A has 100 ream of paper, at 8 s. ready money, which in barter he sets down at 10 s. B, sensible of this, has pamphlets at 6 d. apiece; ready money, which he adequately

quately charges, and insists to have over and above $\frac{1}{2}$ of the price of those he parts with in specie. What number of books is he to deliver in lieu of A's paper, what cash will make good the difference, and how much is B the gainer by this affair?

As 8s. : 10s. :: 6d. : $7\frac{1}{2}$ d. advanced price of B's pamphlets.

$\frac{1}{2}, \frac{1}{3}$ | 100 reams of paper.

20
+ 20

£ 40, real
£ 50, advanced } value of the paper.

£ 12 10s. B to have in cash.

40, value also of B's pamphlets.
× 40, sixpences in 11.

1600, pamphlets to be delivered.

∴ 40l. — 12l. 10s. = 27l. 10s. what they then stood him in; so that the advantage to B was 12l. 10s.

15. A and B barter; A hath 140lb. 11 oz. of plate, at 6s. 4d. per ounce, which in truck he rates at 7s. 2d. an ounce, and allows a discount on his part to have $\frac{1}{7}$ of that in ready money; B has tea worth 9s. 6d. the pound, which he rates at 11s. 2d. When they come to strike the balance, A received but 7 cwt. 2 qrs. 18lb. of tea: pray what discount did A allow B, which of them had the advantage, and how much, in an article of trade thus circumstanced?

oz. s. d.
 $\frac{1}{7}, \frac{1}{8}$ | 1691 at 6 4
281 16 8
 $\frac{1}{2}$ | 169 2 -
84 11 -

£ 535 9 8, real value
of A's plate.

oz. s. d.
 $\frac{1}{7}, \frac{1}{8}$ | 1691 at 7 2
563 13 4
42 5 6

£ 605 18 10, its advanced value.

605l. 18s. 10d. — 535l. 9s. 8d. = 70l. 9s. 2d.

$\frac{1}{7}$ 605

1840-1841

151-8611 37, prompt payment.

519	7	6 ^s ₇		7	2	18,	at 9	6	per lb.
								8	

cwt.	qr.	lb.	s.	d.
7	2	18,	at 11	2
				8

3:16 -
2

4 9 4
12.

26 12 2

31 5.4
2

53 4 2
7

$\frac{1}{7}$ 62 10, 8
7

372 8 -
26 12 -

437 14 8

19

$$3\frac{1}{8} \cdot 5\frac{1}{8} \cdot 4\frac{1}{8}$$

407 11 -, real value of B's
tea.

479. I -, advanced value of B's tea.

£ 519 7 6⁶ — 479l. 1s. = 40l. 6s. 6⁶d. discount
— 407 18 4 allowed by A.

III 16 '68

—70 9. 2, A's advantage by the use of his plate.

£ 41. 7 4⁵/₇ B's whole advantage. Q. E. F.

16. A, with intention to clear 30 guineas on a bargain with B, rates hops at 16 d. per lb. that stood him in 10 d. B, apprised of that, sets down malt which cost 20s. a quarter at an adequate price; how much malt did they contract for?

As 10 d. : 16 d. :: 20 s. : 32 s. advanced value of the malt.

$$32 - 20 = 12, \text{ B's gain per quarter.}$$

Guineas $30 \times 21 = 630$ shillings.

12) $630 (52\frac{1}{2} \text{ quarters} = 420 \text{ bushels, the answer.}$

Cc 2

17. A

17. A and B truck; A has 14 cwt. 8 lb. of Farnham hops, at 2l. 19s. per cwt. but in barter insists on three guineas; B has wine worth 6s. per gallon, which he raises in proportion to A's demand. On the balance A received but a hoghead and a half of wine, pray what had he in ready money?

cwt. lb. l. s.
14 81 at 3 3
 7

22 1
2

		cwt. qrs. lb.	
44	2 -	14	- -
1	11 6	-	2 -
-	9 -	-	- 16
-	4 6	-	- 8
-	- 6½	-	- 1

£ 46 7 6½

As 2.95 : 3.15 :: .3 : .220339, advanced value of one gallon of wine.

1½ hoghead of wine = 94.5 gal. × .220339 = 30.272.

30.272 l. = 30l. 5s. 5½d. advanced value of the wine.

∴ 46l. 7s. 6½d. — 30l. 5s. 5½d. = 16l. 2s. 1½d.
O. E. F.

18. A, in order to put off 720 ells of damaged Holland, worth 5s. an ell, at 6s. 8d. proposes, in case he has half the value in money, to give B thereon a discount of 10 per cent. the rest A is to take out in saffron; which B, apprised of the whole management, rates in justice at 30s. the pound: pray what was it really worth in ready money; and what quantity of saffron was he to deliver on the change?

720 ells.

144, real	} value of the Holland
240, advanced	

24, discount.

216, remains.

108, paid in ready money.

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And if 116 : 144 :: 30 s : 20 s, per pound, real value of the stuff.

And if 131 : 1 lb. :: 108 : 72 lb. the quantity delivered.

Q. V. F.

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## CHAPTER VI.

### SECT. I.

#### EXCHANGE.

**EXCHANGING** the coins of one country into those of another, is like the business of bartering commodities (that is) it consists of finding what sum of one country coin will be equal in value to any proposed sum of another country's coin; and in order to perform that, it will be necessary to have a true account at all times of the ~~just~~ value of those foreign coins which are to be exchanged, as they are compared in value of our English coin; for the par of exchange (as the merchants call it) differs almost every day from London to other countries; that is, it rises and falls, according as money is plenty or scarce, or according to the time allowed for payment of money in exchange.

If our purchases and payments in foreign countries exactly balance their purchases and payments in ours, there will be just enough of bills on the one to clear accounts with the other; so that in this case the exchange on both sides will be at par; that is, one who gives money in one country, will receive as much in the other in weight and standard.

If a nation supplies us with more than it takes from us, or if we pay that nation more money than it pays to us, there will be a balance against us, which we must necessarily pay; in order to which, the demand for the money of that nation, or its bills of exchange, becomes greater among us than the quantity to supply that demand, which raises the value of their money or bills, and lowers ours; or, in other words, puts the price of their money above par, and ours below it, which constitutes what we call the course of exchange. From hence we may naturally infer,

I. That the course of exchange betwixt two nations is a herald, which proclaims publicly the state of commerce and money-

money-negotiations betwixt them, and which of the two is indebted to the other.

II. That the nation which is indebted hath the disadvantage in commerce and money-transactions; and that the one which hath the balance in its favour, hath in every respect the advantage.

III. That the balance of trade naturally imports specie, and renders money at home more valuable abroad; whereas, on the other hand, when the balance is against a nation, their specie is exported, and becomes thereby less valued.

The English standard for gold coin is 22 carats of fine gold, and two carats or  $\frac{2}{24}$  of alloy.

In the royal mint a pound of standard gold is divided into  $44\frac{1}{2}$  parts, each a guinea, at which rate a guinea will weigh 5 pennyweights, 9.4382 grains.

The English standard for silver is 11 ozs. 2 pwts. of fine silver, and 18 pwts. of alloy, in the pound =  $\frac{1}{16}$ .

The pound weight standard silver is divided into 62 parts, each a shilling; so that a shilling will weigh 3 pwts. 20.9 grains.

*A TABLE of the proportion of the value in several nations of the world between gold and silver, taken from Poffleithway's Universal Dictionary of Trade and Commerce.*

|                                                                                           | gold. | silver,         |
|-------------------------------------------------------------------------------------------|-------|-----------------|
| In Japan one ounce of gold is - - - - -                                                   | 1     | 8               |
| China - - - - -                                                                           | 1     | 10              |
| Mogul empire - - - - -                                                                    | 1     | 12              |
| France - - - - -                                                                          | 1     | 15              |
| Spain and Portugal - - - - -                                                              | 1     | 16              |
| But as they required a premium of six per cent. on payments in silver, it reduces it to } | 1     | $15\frac{1}{3}$ |
| England - - - - -                                                                         |       | $15\frac{1}{3}$ |

*Explanation and use of the following TABLE of coins (viz.)*

|                 |          |                  |
|-----------------|----------|------------------|
| One pound troy  | contains | 12 ounces.       |
| one ounce - - - |          | 20 pennyweights. |
| one pennyweight |          | 24 grains.       |
| one grain - - - |          | 20 mites.        |

The first column expresses the fineness of the assayed piece; the B. signifying better, and the W. worse than the English standard.

The second column is the absolute weight of the piece.

The third column its standard weight, or its quantity of standard metal.

The fourth column its value in English money.

*Ex. gr.* in the second article of silver coin, the new Seville piece of eight is  $1\frac{1}{2}$  pwt. in the pound worse than English standard weight; 13 pennyweights, 21 grains, 15 mites of sterling silver, is in value 43 pence, and 11 decimal parts of a penny.

And in the first article of gold coin, the old Louis d'ors is half a grain worse than English standard; its weight is four pennyweights, 7 grains, 8 mites of English standard gold, and its value 16 shillings,  $9\frac{1}{8}$  pence.

The par of exchange between English and Dutch money is easily found, thus: as by Sir Isaac's table, the ducat of Holland is worth intrinsically 65.59 d. English; which is received at the Bank at 60 stivers, or three guilders, and consequently is equal to 10 shillings Flemish; therefore by the rule of three, as 65.59 d. English is to 10 s. Flemish; so is 240 d. in a pound English, to a fourth number, which will be found to be 36.59 s. Flemish; and so much Bank money at Amsterdam should be received for one pound, or 240 pence sterling.

Sir Isaac Newton's TABLE of the assays, weights and values of most silver and gold coins, actually made at the Mint, by order of the privy-council, with a calculation of the real or intrinsic par of exchange.

# FOREIGN SILVER COINS.

|                                                                                 | Assay. | Weight. |      | Stand. wt. | Value.    |
|---------------------------------------------------------------------------------|--------|---------|------|------------|-----------|
|                                                                                 |        | dwt.    | grs. | dit. grs.  | dit. grs. |
| The piafter of Spain, or Seville piece of eight reals, now ten                  | W. 1   | 17      | 12   | 17         | 10        |
| The new Seville piece of eight                                                  | W. 1   | 17      | 12   | 17         | 10        |
| The Mexico piece of eight                                                       | W. 1   | 17      | 10   | 17         | 8         |
| The pillar piece of eight                                                       | Stand. | 17      | 9    | 17         | 9         |
| The Peru piece of eight, coarser, but of uncertain alloy.                       | W. 1   | 17      | 12   | 17         | 10        |
| The old ecu of France, or piece of 60 sols Tournois                             | W. 1   | 17      | 12   | 17         | 10        |
| The new ecu, or piece of five livres, or 100 sols                               | W. 1   | 17      | 12   | 17         | 10        |
| N. B. The ecu of France should be two pennyweights worse by law.                |        |         |      |            |           |
| The cruzado of Portugal, or ducat, worth 400 reas, raised to 480                | W. 2   | 17      | 12   | 17         | 10        |
| The patacks, or patagons, of Portugal, worth 500 reas, marked and raised to 600 | W. 2   | 17      | 12   | 17         | 10        |
| The ducatoon of Flanders, or piece of 60 sols or patars                         | B. 4   | 20      | 22   | 20         | 8         |
| The patagon of Flanders, or cross dollar, or piece of 48 patars                 | W. 12  | 18      | 4    | 17         | 1         |
| The ducatoon of Holland, or piece of 63 stivers                                 | B. 3   | 20      | 21   | 21         | 3         |
| The patagon leg-dollar, or six-dollar of Holland, or piece of 50 stivers        | W. 14  | 18      | 8    | 16         | 20        |
| The three guilder pieces of Holland, or piece of 60 stivers                     | W. 2   | 20      | 8    | 20         | 3         |
| The guilder, florin, or piece of 20 stivers                                     | W. 2   | 6       | 18   | 6          | 17        |
| The ten shilling piece of Zealand, or piece of 60 stivers                       | W. 12  | 20      | 6    | 20         | 1         |
| The lion-dollar of Holland, or $\frac{1}{2}$ of the ducatoon                    | W. 4   | 17      | 14   | 14         | 2         |

ENGLAND

1791

|                                                                                      |                     |    |    |    |    |       |
|--------------------------------------------------------------------------------------|---------------------|----|----|----|----|-------|
| The ducatoon of Cologne                                                              | B. 3                | 20 | 28 | 24 | 13 | 65.02 |
| The rix-dollar, or patagon of Cologne                                                | W. 13               | 18 | 16 | 16 | 22 | 52.53 |
| The rix-dollar, or patagon of the bishop of Liege                                    | W. 12               | 17 | 16 | 16 | 22 | 55.48 |
| The rix-dollar of Mentz                                                              | W. 6                | 18 | 17 | 17 | 10 | 55.27 |
| The rix-dollar of Francfort                                                          | W. 9                | 18 | 18 | 17 | 14 | 54.53 |
| The rix-dollar of the elector Palatins of the Rhine and Bavaria before 1620          | —                   | 18 | 5  | —  | —  | —     |
| The rix-dollar of Nuremberg                                                          | W. 6                | 18 | 10 | 17 | 22 | 55.55 |
| The old rix-dollar of Lunenburg                                                      | W. 10               | 18 | 11 | 17 | 25 | 54.65 |
| The old rix-dollar of Hanover                                                        | W. 8                | 18 | 12 | 17 | 20 | 55.03 |
| The double gulden of the elector of Hanover                                          | W. 7                | 18 | 18 | 18 | 3  | 56.29 |
| The gulden of the elector of Hanover, or piece of $\frac{1}{2}$                      | B. 17 $\frac{1}{2}$ | 8  | 10 | 9  | 1  | 28.14 |
| The half-gulden of the elector of Hanover, or piece of $\frac{1}{4}$                 | B. 17 $\frac{1}{2}$ | 4  | 5  | 4  | 12 | 14.07 |
| The gulden of the duke of Zell, or piece of 16 gutz grosh                            | W. 43               | 11 | 2  | 8  | 22 | 27.07 |
| The gulden of the bishop of Hildesheim, or piece of 24 manen grosh, now raised to 26 | W. 40 $\frac{1}{2}$ | 11 | 22 | 9  | 17 | 30.21 |
| The rix-dollar of Magdeburg                                                          | W. 10               | 18 | 12 | 17 | 16 | 54.27 |
| The gulden, or guilder of Magdeburg                                                  | W. 44               | 11 | 14 | 9  | 6  | 28.67 |
| The old rix-dollar of the elector of Brandenburg                                     | W. 9                | 18 | 13 | 17 | 19 | 35.17 |
| The old gulden of Brandenburg, now raised from 24 to 26 manen grosh                  | W. 43               | 12 | 4  | 9  | 19 | 30.41 |
| The gulden of Brandenburg, or piece of $\frac{1}{2}$                                 | W. 43               | 11 | 3  | 8  | 23 | 27.81 |
| The half-gulden of Brandenburg, or piece of $\frac{1}{4}$                            | W. 43               | 5  | 13 | 4  | 11 | 13.09 |
| The gulden of the elector of Saxony, or piece of $\frac{1}{2}$                       | W. 41               | 11 | 3  | 9  | 1  | 28.12 |
| The old blank dollar of Hamburg                                                      | W. 8                | 18 | 9  | 17 | 17 | 54.92 |

|                                                                                                                                                               |                     |    |                  |    |    |    |       |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|----|------------------|----|----|----|-------|
| The old rix-dollar of Lubec                                                                                                                                   | W. 8 $\frac{1}{2}$  | 18 | 16               | 17 | 22 | 17 | 55.54 |
| The four-mark piece of Denmark, of coarser alloy                                                                                                              | W. 6 $\frac{1}{2}$  | 14 | 8                | 10 | 9  | 10 | 32.23 |
| The four-mark piece of Denmark, of finer alloy                                                                                                                | W. 21               | 11 | 13 $\frac{1}{2}$ | 10 | 11 | 5  | 32.45 |
| The eight-mark piece of Sweden                                                                                                                                | Stand.              | 20 | —                | 20 | —  | —  | 62.   |
| The four-mark piece of Sweden                                                                                                                                 | W. 58               | 13 | 12               | 9  | 23 | 7  | 30.92 |
| The two-mark piece of Sweden                                                                                                                                  | W. —                | 6  | 19               | —  | —  | —  | —     |
| The old dollar of Dantzick                                                                                                                                    | W. 10 $\frac{1}{2}$ | 18 | 9                | 17 | 12 | 4  | 54.27 |
| The old rix-dollar of Thorn, near Dantzick                                                                                                                    | W. 12               | 18 | 8 $\frac{1}{2}$  | 17 | 8  | 15 | 53.85 |
| The rix-dollars of Sigismund III. and Vladislaus IV. kings of Poland                                                                                          | W. 10               | 18 | 9                | 17 | 13 | 14 | 54.04 |
| The rix-dollar of the late emperor Leopold                                                                                                                    | W. 10 $\frac{1}{2}$ | 18 | 9                | 17 | 12 | 4  | 54.27 |
| The rix-dollar of his predecessor Ferdinand III.                                                                                                              | W. 10 $\frac{1}{2}$ | 18 | 9                | 17 | 12 | 4  | 54.27 |
| The rix-dollar of Ferdinand archduke of Austria                                                                                                               | W. 10 $\frac{1}{2}$ | 18 | 5                | 17 | 8  | 7  | 53.78 |
| The rix-dollar of Basil                                                                                                                                       | W. 7 $\frac{1}{2}$  | 18 | 18 $\frac{1}{2}$ | 18 | 3  | 6  | 56.24 |
| The rix-dollar of Zune                                                                                                                                        | W. 13               | 18 | 1                | 16 | 23 | 13 | 52.65 |
| The old ducat of Venice, with the words <i>Ducatus Venetus</i> upon it; a }<br>piece of six old livres, afterwards raised to six livres four sols de Picoli } | W. 23 $\frac{1}{2}$ | 14 | 15               | 13 | 1  | 17 | 40.50 |
| The half ducat                                                                                                                                                | W. 23 $\frac{1}{2}$ | 7  | 7 $\frac{1}{2}$  | 6  | 12 | 18 | 20.25 |
| The new ducat, with the number 124 upon it, signifying 124 fols, }<br>or six livres four sols de Picoli }                                                     | —                   | 18 | 2                | —  | —  | —  | —     |
| The half thereof                                                                                                                                              | —                   | 9  | 1                | —  | —  | —  | —     |
| The crusado croifat, or St. Mark of Venice, with number 140 upon }<br>it, signifying 140 fols, or seven livres de Picoli }                                    | —                   | 20 | 6                | —  | —  | —  | —     |
| The half crusado of the same form                                                                                                                             | —                   | 10 | 3                | —  | —  | —  | —     |
| The quarter crusado of the same form                                                                                                                          | —                   | 5  | 1 $\frac{1}{2}$  | —  | —  | —  | —     |

|                                                                                                                                                                             |        |    |     |    |    |    |       |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------|----|-----|----|----|----|-------|
| Another coin of Venice                                                                                                                                                      | W. 46  | 17 | 10  | 13 | 19 | 8  | 42.08 |
| The piece of two Jules                                                                                                                                                      | B. 6   | 3  | 15  | 3  | 17 | 7  | 11.05 |
| The ducat de banco of Naples, or piece of 5 tarians, or 10 carlins, or 100 gr.                                                                                              | W. 3   | 14 | 1   | 13 | 1  | -  | 40.43 |
| The half ducat                                                                                                                                                              | W. 3   | 7  | 8   | 6  | 12 | 20 | 20.21 |
| The tarin, or fifth part of the ducat                                                                                                                                       | W. 3   | 2  | 19½ | 2  | 14 | 12 | 8.09  |
| The carlin, or tenth part of the ducat                                                                                                                                      | W. 3   | 1  | 9½  | 1  | 7  | 6  | 4.04  |
| The escudi ecu, or crown of Rome, or piece of 10 Julios, or 100 bayoches                                                                                                    | W. 1   | 20 | 14½ | -  | -  | -  | -     |
| The teston of Rome, or piece of three Julios                                                                                                                                | W. 1   | 5  | 21½ | 5  | 20 | 17 | 18.32 |
| The ducat of Florence and Leghorn, or piece of 7 livres, or 10½ Julios                                                                                                      | B. 8   | 20 | 3   | 20 | 20 | 6  | 64.62 |
| The Julio of Rome                                                                                                                                                           | W. 1   | 2  | 5   | -  | -  | -  | -     |
| The piafter ecu, or crown of Ferdinand II. duke of Tuscany                                                                                                                  | W. 1   | 17 | 12  | 17 | 10 | 2  | 54    |
| The piafter ecu, or crown of Cosmus III. present duke of Tuscany, }<br>whose monies are about four per cent. lighter than those of his }<br>father; this piece is 8½ Julios | W. 1   | 16 | 18  | 16 | 16 | 4  | 51.69 |
| The croisat of Genoa, or piece of 7½ livres                                                                                                                                 | B. 7   | 24 | 15  | 25 | 9  | 11 | 78.74 |
| The ecu d'argent of Genoa, or piece of 7 livres 12 fols                                                                                                                     | -      | -  | -   | -  | -  | -  | -     |
| The piafter ecu, or crown of Milan                                                                                                                                          | -      | 17 | 21  | -  | -  | -  | -     |
| The Philip of Milan, or piece of 7 livres                                                                                                                                   | -      | 20 | 20  | -  | -  | -  | -     |
| The livre, or 20 fols piece of Savoy                                                                                                                                        | -      | 3  | 22  | -  | -  | -  | -     |
| The 10 fols piece of Savoy                                                                                                                                                  | -      | 1  | 23  | -  | -  | -  | -     |
| A roupee                                                                                                                                                                    | B. 16½ | 7  | 10  | 7  | 23 | 4  | 24.07 |
| A gout gulden, or florin d'or, a Dutch coin of 28 stivers                                                                                                                   | W. 75  | 12 | 19  | 8  | 11 | 5  | 26.26 |
| Another gout gulden                                                                                                                                                         | W. 48  | 11 | -   | 8  | 14 | 18 | 26.72 |
| Another                                                                                                                                                                     | W. 48  | 12 | -   | 9  | 9  | 15 | 29.15 |

|                                                             | Assay. | weight.  |          | stand. wt. |     | value. |            |
|-------------------------------------------------------------|--------|----------|----------|------------|-----|--------|------------|
|                                                             |        | car. gr. | dt. grs. | dt. gr.    | mi. | s.     | d.         |
| The old Louis d'or                                          | W.     | —        | 4        | 4          | 7   | 8      | 9.3        |
| The half and quarter in proportion                          | W.     | —        | 2        | 4          | 3   | 14     | 8 5.       |
| The new Louis-d'or                                          | W.     | —        | 5        | 5          | 3   | 18     | 20 0.6     |
| The half and quarter in proportion                          | W.     | —        | 2        | 14         | 2   | 13     | 19 0.3     |
| The old Spanish double doubloon                             | W.     | —        | 17       | 8          | 17  | 5      | 12 6.7     |
| The old Spanish double pistole                              | W.     | —        | 8        | 16         | 8   | 14     | 16 3.3     |
| The old Spanish pistole                                     | W.     | —        | 4        | 8          | 4   | 7      | 8 1.6      |
| The new Seville double pistole                              | W.     | —        | 8        | 16         | —   | —      | —          |
| The new Seville pistole                                     | W.     | —        | 4        | 8          | —   | —      | —          |
| The half and quarter in proportion                          | W.     | —        | 4        | 8          | —   | —      | —          |
| The doppia moneda, or double moneda of Portugal, new coined | W.     | —        | 6        | 22         | 6   | 21     | 12 26 10.4 |
| The doppia moneda as they come into England                 | W.     | —        | 6        | 21         | 6   | 21     | 7 26 9.9   |
| The moneda of Portugal                                      | W.     | —        | 3        | 11         | 3   | 10     | 16 13 5.1  |
| The half moneda                                             | W.     | —        | 1        | 17         | 1   | 17     | 8 8 8.5    |
| The Hungary ducat                                           | B.     | 12       | 2        | 5          | 2   | 9      | 7 9 3.6    |
| The ducat of Holland coined <i>ad legem imperii</i>         | B.     | 12       | 2        | 5          | 2   | 9      | 3 9 3.2    |
| The ducat of Camper in Holland                              | B.     | 12       | 2        | 5          | 2   | 9      | 3 9 3.2    |
| The ducat of the bishop of Bamberg                          | B.     | 12       | 2        | 5          | 2   | 9      | 3 9 3.2    |
| The double ducat of the duke of Hanover                     | B.     | 12       | 4        | 10         | 4   | 17     | 9 18 4.8   |
| The ducat of the duke of Hanover                            | B.     | 12       | 2        | 5          | 2   | 9      | 8 9 3.2    |
| The ducat of Brandenburg                                    | B.     | 12       | 2        | 5          | 2   | 9      | 8 9 3.2    |
| The ducat of Sweden                                         | B.     | 12       | 2        | 5          | 2   | 9      | 8 9 3.2    |



|                                                                                                       |        |    |    |    |    |    |    |
|-------------------------------------------------------------------------------------------------------|--------|----|----|----|----|----|----|
| The ducat of Denmark                                                                                  | B. 1 2 | 2  | 5  | 2  | 3  | 9  | 32 |
| The ducat of Poland                                                                                   | B. 1 2 | 2  | 5  | 2  | 3  | 9  | 32 |
| The ducat of Transylvania                                                                             | B. 1 2 | 2  | 5  | 2  | 3  | 9  | 32 |
| The sequin, chequin, or zaccatien of Venice                                                           | B. 1 2 | 2  | 5  | 2  | 3  | 9  | 32 |
| The old Italian pistole                                                                               | B. 1 3 | 3  | 8  | 4  | 7  | 15 | 57 |
| The double pistole of pope Urban, 1634                                                                | W. 1   | 4  | 14 | 4  | 11 | 15 | 57 |
| The half pistole of Innocent II. 1685                                                                 |        | 2  | 10 |    |    |    |    |
| A double pistole of Placencia                                                                         |        | 3  | 10 |    |    |    |    |
| A double pistole of Genoa, 1621                                                                       |        | 3  | 16 |    |    |    |    |
| A double pistole of Milan                                                                             |        | 3  | 13 |    |    |    |    |
| A single pistole of Milan                                                                             |        | 4  | 6  |    |    |    |    |
| A pistole of Savoy, 1675                                                                              |        | 4  | 8  |    |    |    |    |
| Double ducat of Casale, Genoa, Portugal, Florence, Hungary, and Venice                                | B. 1 2 | 4  | 18 | 4  | 18 | 18 | 77 |
| Single ducats of the same places                                                                      | B. 1 2 | 2  | 9  | 2  | 9  | 9  | 38 |
| Double ducats of several towns in Germany                                                             | B. 1 1 | 4  | 11 | 4  | 17 | 18 | 4  |
| Single ducats of the same places                                                                      | B. 1 1 | 2  | 5  | 2  | 8  | 9  | 4  |
| Double ducats of Genoa                                                                                | B. 1 2 | 4  | 11 | 4  | 18 | 18 | 65 |
| Single ducats of Genoa, Besançon, and Zurich                                                          | B. 1 2 | 2  | 5  | 2  | 9  | 9  | 22 |
| Pistole of Rome, Milan, Venice, Florence, Savoy, Genoa, Orange, Trevou, and Besançon                  | W. 1   | 4  | 6  | 4  | 17 | 16 | 67 |
| A Barbary ducat, with Arabic letters on both sides, in square tables, without any effigies or emblems | W. 2   | 12 | 16 | 12 | 9  | 9  | 35 |

## ENGLAND, with HOLLAND, FLANDERS and GERMANY.

The bank of Amsterdam is the most considerable in Europe; and as business therein is negotiated by transfers, millions may be paid in a day, without the intervention of any cash; which is of the greatest consequence imaginable in expediting trade; and is productive of so great security, that bank payments is reckoned from 3 to 6 per cent. better than payments in cash, although a premium is also allowed the bank for every deposit.

The Hollanders keep their accounts in florins or guilders, stivers and pennings, or in pounds, shillings, and pence Flemish, divided as the pound sterling.

|            |            |                    |
|------------|------------|--------------------|
| 8 pennings | } make one | grot.              |
| 2 grots    |            | stiver.            |
| 6 stivers  |            | shilling.          |
| 20 stivers |            | florin or guilder. |
| 2½ florins |            | rix-dollar.        |
| 6 florins  |            | pound Flemish.     |
| 5 guilders |            | ducat.             |

Exchange is made with London from 30 to 36 shillings Flemish, for one pound sterling.

## C A S E I.

Given the sum due in one country coin, and that payable in another country coin, to find the rate of exchange.

## R U L E.

As the sum due : is to that paid or payable : so is an unit of the first : to the value of an unit of a second.

1. A merchant at Amsterdam paid 150 guilders for 13l. 15s. received by his correspondent at London; what is the value of a guilder?

As 150 guil. : 13.75l. :: 1 guil. : 1s. 10d. the answer.

2. If I receive in London 678l. 15s. 9½d. for 1173l. 14s. 10d. Flemish, due at Rotterdam; what is the rate of exchange?

As 678.790625l. ster. : 1173.7416l. Flem. :: 1 : 1l. 14s. 7d. Flemish, for 1l. sterling.

N. B.

N. B. That the country in whose money the course of exchange is reckoned, has always the greatest advantage the lower the course of exchange runs.

CASE II.

To reduce Flemish pounds, shillings and pence into guilders and stivers,

RULE,

Bring them into pence Flemish, then divide by 40 (because 40 pence make one guilder) and the quotient will be guilders, and if any thing remain, divide it by 2 (because two pence make one stiver) and the quotient will be stivers.

3. In 1173l. 14s. 10d. Flemish, how many guilders?

$$\begin{aligned} 1173\text{l. } 14\text{s. } 10\text{d.} &= 281698 \text{ pence.} \\ 40 \overline{) 281698} & (7042 \text{ guilders, } 9 \text{ stivers.} \end{aligned}$$

By PRACTICE.

|        |            |   |                                      |
|--------|------------|---|--------------------------------------|
|        | ftiv.      |   |                                      |
| 1173   | × 6 = 7038 | - | 6 guilders being 1 l. Flemish.       |
| 10s.   | = 3        | - | $\frac{1}{2}$ of 6 guilders, or 1 l. |
| 4s.    | = 1        | 4 | $\frac{1}{2}$ of 1 l.                |
| - 10d. | =          | 5 | $\frac{1}{2}$ of 10d.                |

Guilders 7042 9 stivers.

4. In 7042 guilders and 9 stivers, how many Flemish pounds?

$$\begin{array}{r} \text{guil. ftiv.} \\ 6 \overline{) 7042 \ 9} \end{array}$$

1173 -, remains 4 guilders, 9 stivers.

|            |      |    |
|------------|------|----|
|            | s.   | d. |
| 3 guilders | = 10 | -  |
| 1 guilder  | = 3  | 4  |
| 9 stivers  | = 1  | 6  |

£ 1173 14 10, the answer.

## C A S E III.

To reduce sterling into Flemish money.

## R U L E.

As one pound sterling : is to the given rate of exchange :  
so is the given sterling : to the Flemish sought.

5. If I pay in London 678l. 15s. 9d. what may I draw my bill for on Amsterdam, exchange 1l. 14s. 7d. per pound sterling?

As 1l. : 1.72916l. Flem. :: 678.790625 : 1173.7418 =  
1173l. 14s. 16d. Flemish.

By PRACTICE.

|                | l.  | s. | d.              |
|----------------|-----|----|-----------------|
| $\frac{1}{2}$  | 678 | 15 | $9\frac{1}{2}$  |
| $\frac{1}{4}$  | 339 | 7  | $10\frac{1}{4}$ |
| $\frac{1}{8}$  | 169 | 15 | $1\frac{1}{8}$  |
| $\frac{1}{16}$ | 84  | 15 | $4\frac{1}{16}$ |
| $\frac{1}{32}$ | 42  | 16 | $6\frac{1}{32}$ |

1173 14 9 $\frac{1}{2}$  Flemish money, the answer.

## C A S E IV.

To reduce Flemish money into sterling.

## R U L E.

As the given rate of exchange : is to one pound sterling :  
so is the given Flemish : to the sterling required.

## Chap. VI. EXCHANGE.

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6. Change 1173l. 14s. 10d. Flemish into sterling exchange, at 34 s. 7d. per pound sterling.

$\begin{array}{r} \text{s.} \quad \text{d.} \\ 34 \quad 7 : 1 :: 1173 \quad 14 \quad 10 \\ \times 12 \quad \quad \quad 20 \end{array}$

34 7 : 1 :: 1173 14 10

**X 12**

20

415

23474  
12

12

415) 281698 (678 15 9½

3260

3648

328

20

6560

2410

335

12

4020

285

4

1140

310

D d

7. In

7. In 1036 l. Flemish, exchange at 34 s. 4½ d. how much sterling?

| s.    | d. | l.             |
|-------|----|----------------|
| 34    | 4½ | 1              |
| 12    |    | 6              |
| <hr/> |    |                |
| 412   |    | 6216 guilders. |
| 2     |    | 20             |
| <hr/> |    |                |
| 825   |    | 12430 stivers. |

|      | l.     | s.   | d.              |
|------|--------|------|-----------------|
| 825) | 497280 | (602 | 15 3¼ sterling. |
|      | 2280   |      |                 |
|      | 630    |      |                 |
|      | 20     |      |                 |
|      | 12600  |      |                 |
|      | 4350   |      |                 |
|      | 225    |      |                 |
|      | 12     |      |                 |
|      | 2700   |      |                 |
|      | 225    |      |                 |
|      | 4      |      |                 |
|      | 900    |      |                 |

8. In 5875 florins 17 stivers banco, how many pounds sterling, exchange at 32 s. 10 d.?

| s.    | d. | l. | flor.  | ft.      |
|-------|----|----|--------|----------|
| 32    | 10 | 1  | 5875   | 17       |
| 12    |    |    | 20     |          |
| <hr/> |    |    |        |          |
| 394   |    |    | 117517 | stivers. |

|      |        |                    |
|------|--------|--------------------|
| 394) | 235034 | (596 l. 10 s. 8 d. |
|      | 3803   |                    |
|      | 2574   |                    |
|      | 210    |                    |
|      | 20     |                    |
|      | 4200   |                    |
|      | 260    |                    |
|      | 12     |                    |
|      | 3120   |                    |

Agio signifies the difference of the value of current money and bank notes in Holland, Venice, &c. which in Holland is from 3 to 6 per cent. in favour of the notes; also the reward given for the changing one coin, or species of money, for another.

C A S E V.

To turn current money into banco.

R U L E,

As 100 with the agio added to it : is to 100 :: so is any given sum current : to its value in banco.

9. In 3758 florins, 15 stivers current, agio  $5\frac{1}{2}$  per cent. how many pounds sterling, exchange at 35 s. 11 d.?

As 20 stivers make 1 florin  $\therefore \frac{1}{2}$  florin =  $12\frac{1}{2}$  stivers.

Then as  $104\ 12\frac{1}{2} : 100 :: 3785\ 15$   
 $\quad\quad 20 \qquad\quad 20$

2092  
 $\quad\quad 2$

4185 pence

75175  
 $\quad\quad 2$

150350

$4185) 15035000 (8592\ 12\ \text{banco.}$

24800  
38750  
10850

2480  
 $\quad\quad 20$

49600  
7750

*D d 2*

*Alfo,*

s. d. l. flor. stiv.  
Also, as 35 11 : 1 :: 3592 12  
12 20

431

71852  
2

431) 143704 (333 8  $4\frac{1}{2}$  s. d. answer.  
1440

1474

181

20

3620

172

12

2064

340

4

1360

67

## C A S E VI

To turn banco into current money.

As 100 : is to 100 with the agio added :: fo is any given banco : to its value current.

10. In 456 l. 8 s. sterling, how many rix-dollars current, agio  $4\frac{1}{2}$ , exchange 36 s.  $1\frac{1}{2}$  d. ?

|               |     |    |                |
|---------------|-----|----|----------------|
|               | l.  | s. |                |
| $\frac{1}{2}$ | 456 | 8  |                |
| $\frac{1}{2}$ | 228 | 4  |                |
| $\frac{1}{2}$ | 114 | 2  |                |
| $\frac{1}{2}$ | 22  | 16 | $4\frac{1}{2}$ |
| $\frac{1}{2}$ | 2   | 17 | $-\frac{1}{2}$ |

824 7  $5\frac{1}{2}$   
× 6

4946 4  $7\frac{1}{2}$   
2

5) 9892

Rix-dollars 1978 24 stivers banco.

As



$$\begin{array}{r} \text{R. D. fl.} \\ \text{As } 100 : 104\frac{1}{4} :: 1978 \text{ } 24 \\ \hline \text{50} \quad \text{8} \quad \text{50} \end{array}$$

$$\begin{array}{r} 5000 \quad 837 \quad 98924 \\ \hline 837 \end{array}$$

$$\begin{array}{r} 692468 \\ 296772 \\ \hline 791392 \end{array}$$

$$5.000) 82799.388$$

$$8) 16559.8776$$

$$\begin{array}{r} \text{R. D. fl.} \\ 2069.9847 = 2069 \text{ } 49 \text{ current.} \end{array}$$

11. If by remitting to Holland, at 31 s. 9d. Flemish per pound sterling, 5 per cent. is gained; how goes the exchange, when by remittance I clear 10 per cent.?

$$\begin{array}{r} 31 \text{ s. } 9 \text{ d. } = 1.5875 \text{ l.} \\ \text{As } 105 : 1.5875 :: 110 \\ \hline 110 \end{array}$$

105) 174.6250 (1.663095 = 1 l. Flem. 13s. 3 $\frac{1}{2}$  grots, answer.

What is said above may be sufficient for reducing the coins of any country into sterling, and to render the following examples, and business of exchange in general, obvious to every common capacity.

# H A M B U R G H.

$$\left. \begin{array}{l} 2 \text{ deniers gros} \\ 12 \text{ deniers gros or } \} \\ 6 \text{ fols lubs} \\ 16 \text{ fols lubs} \\ 2 \text{ Marks M}^k. \\ 3 \text{ Marks} \\ 7\frac{1}{2} \text{ M}^k. = 2\frac{1}{2} \text{ R. D.} \end{array} \right\} \begin{array}{l} \text{make} \\ \text{one} \end{array} \left\{ \begin{array}{l} \text{fol lubs,} \\ \text{fol gros.} \\ \text{Mark M}^k. \\ \text{dollar.} \\ \text{rix-dollar} = 48 \text{ fols lubs.} \\ \text{livre gros or pound Flemish,} \\ \text{or } 120 \text{ fols lubs} = 20 \text{ fols gros.} \end{array} \right.$$

D d 3

12. Re-

12. Reduce 1541 M<sup>k</sup>.  $14\frac{1}{2}$  fols lubs, bank money of Hamburg, into sterling money of England, exchange at  $32\frac{1}{2}$  fols gros per pound sterling.

|                             |                                            |
|-----------------------------|--------------------------------------------|
| 1.                          |                                            |
| $32\frac{1}{2}$<br><u>6</u> | : 1 :: 1541 $14\frac{1}{2}$<br><u>16</u>   |
| 194 fols lubs.<br><u>2</u>  | 9246<br><u>1541</u>                        |
| 388 deniers.<br><u>3</u>    | 24670 $\frac{1}{2}$ fols lubs.<br><u>2</u> |
| 1164 thirds of den.         | 49340 $\frac{2}{3}$ deniers.<br><u>3</u>   |
|                             | 1. s. d.                                   |
|                             | 1164) 148022 (127 3 4, the answer.         |
|                             | 3162                                       |
|                             | 8342                                       |
|                             | <u>194</u>                                 |
|                             | 20                                         |
|                             | <u>3880</u>                                |
|                             | 388                                        |
|                             | 12                                         |
|                             | <u>4656</u>                                |

13. In 127l. 3s. 4d. sterling, how many Hamburg marks, &c.?

|                                           |                                                   |
|-------------------------------------------|---------------------------------------------------|
| 1.                                        |                                                   |
| 127<br>$32\frac{1}{2}$<br><u>        </u> |                                                   |
| 254<br>381                                |                                                   |
| 42                                        | 4, being $\frac{1}{2}$ of 127.                    |
| 5                                         | 5, being $\frac{1}{5}$ of $32\frac{1}{2}$ nearly. |
| <u>20) 411.1</u>                          | 9, deniers gros.                                  |

Livres 205 11 9  
 $\frac{1}{2}$  7 $\frac{1}{2}$  marks in one livre.

1433  
 102 8, for  $\frac{1}{2}$  livres.  
 3 12, for  $\frac{1}{2}$  of 7 $\frac{1}{2}$  marks.  
 - 6, for  $\frac{1}{10}$  of 60 sols.  
 - 3, for  $\frac{1}{2}$  of 6 deniers.  
 - 1 $\frac{1}{2}$ , for  $\frac{1}{2}$  of 3 deniers.

M<sup>t</sup>. 1541 14 $\frac{1}{2}$ , sols lubs, the answer, and proof of the foregoing example.

14. In 750 l. 14 s. 7 d. sterling, exchange at 32 s. 8 d. how many rix-dollars banco of Hamburg?

|                 |    |                 |                               |
|-----------------|----|-----------------|-------------------------------|
| l.              | s. | d.              |                               |
| 750             | 14 | 7               |                               |
| 375             | 7  | 3 $\frac{1}{2}$ |                               |
| 75              | 1  | 5 $\frac{1}{2}$ |                               |
| 25              | -  | 5 $\frac{1}{2}$ |                               |
| 1226            | 3  | 9 $\frac{1}{4}$ | Flemish.                      |
| 2 $\frac{1}{2}$ |    |                 | And 3 sols gros = 18 } sols.  |
|                 |    |                 | 9 $\frac{1}{4}$ deniers = 5 } |
| 2452            |    |                 |                               |
| 613             |    |                 |                               |
|                 |    |                 | 23                            |

R. D. 3065 23 sols lubs, the answer.

15. In 3065 rix-dollars, 23 sols lubs, how many pounds sterling, exchange at 32 s. 8 d.?

|       |          |        |             |
|-------|----------|--------|-------------|
| s.    | d.       | l. st. | R. D. S. L. |
| As 32 | 8        | : 1 :: | 3065 24     |
| 12    |          |        | 3           |
| 392   | deniers. |        | 9195        |
|       |          |        | 16          |

147143 sols lubs.

392) 294286 (750 l. 14 s. 7 d. answer.

1988  
 286  
 20  
 5720  
 1800  
 232  
 12  
 2784

D d 4

16. In

16. In 584 rix-dollars, 9 fols gros slight money, agio  $4\frac{2}{5}$  per cent. exchange 35s.  $8\frac{1}{2}$  d. how many pounds sterling?  
 1 rix-dollar being 48 fols lubs.  $\therefore \frac{2}{5} = 27$  fols lubs.

R. D. S. G.

$$\begin{array}{r} 104 \ 27 : 100 :: 584 : 9 \\ \underline{48} \qquad \qquad \underline{8} \\ 832 \qquad \qquad 4681 \text{ fols gros.} \\ \underline{416} \qquad \qquad \underline{6} \\ 5019 \text{ fols lubs.} \quad 28086 \end{array}$$

$$\begin{array}{r} 100 \\ 5019 \overline{) 2808600} \quad (559 \text{ R. D. } 28 \text{ S. L. banco.} \\ \underline{29910} \end{array}$$

$$\begin{array}{r} 48150 \\ \underline{2979} \end{array}$$

$$\begin{array}{r} 48 \\ \underline{23832} \end{array}$$

$$\begin{array}{r} 11916 \\ \underline{142992} \end{array}$$

$$\begin{array}{r} 42612 \\ \underline{2460} \end{array}$$

$$\begin{array}{r} \text{s.} \quad \text{d.} \quad \text{l.} \quad \text{R. D. S. L.} \\ 35 \quad 8\frac{1}{2} : 1 :: 559 \quad 28 \end{array}$$

$$\begin{array}{r} \underline{12} \qquad \qquad \underline{48} \\ 428 \qquad \qquad 4472 \\ \underline{2} \qquad \qquad \underline{2236} \\ 857 \qquad \qquad 26860 \text{ fols lubs.} \end{array}$$

$$\begin{array}{r} 4 \\ 857 \overline{) 107440} \quad (125 \text{ l. } 7 \text{ s. } 4 \text{ d. ster. ansf.} \\ \underline{2174} \end{array}$$

$$\begin{array}{r} 4600 \\ \underline{315} \end{array}$$

$$\begin{array}{r} \times 20 \\ \underline{6300} \end{array}$$

$$\begin{array}{r} 301 \\ \underline{\times 12} \end{array}$$

$$\begin{array}{r} \underline{3612} \\ 184 \end{array}$$

17. In 1075 M<sup>k</sup>. 14 fols lubs current, agio  $8\frac{1}{2}$  per cent. and 384 dollars, 2 fols gros slight, agio  $4\frac{2}{5}$  per cent. exchange 35 s. 7 d, how many rix-dollars banco, and pounds sterling?

As

# Chap. VI. EXCHANGE.

409

M<sup>k</sup>. S. L. M<sup>k</sup>. 108  $\frac{1}{2}$  = 108 ft. 6 fols lubs.  
 As 108 6 : 100 :: 1075 14 fols lubs.  
 16 16

1734

17214  
 X 100

M<sup>k</sup>. S. L. D.

1734) 1721400 (992 11 1 banco.  
 16080

Dol. Dol. S. L.  
 104  $\frac{1}{4}$  = 104 28 1272  
 2 fols gros. = 12 16  
 30352  
 3012  
 1278

Dol. S. L. Dol. S. L.  
 104 28 : 100 :: 384 12

32  
 768  
 1152  
 12300  
 100

R. D. S. L. R. D. S. L. 3356) 1230000 (366 16 banco.  
 As 7  $\frac{1}{2}$  : 1 :: 575 11  $\frac{1}{2}$  22320 2  
 16 48 21840  
 120 4600 1740 733 M<sup>k</sup>. ban.  
 2 2300 32 992 11  $\frac{1}{2}$   
 240 27611 3) 1725 11  $\frac{1}{2}$   
 2 3408 575 11  $\frac{1}{2}$  banco.  
 5112

l. F. s. d. 54528  
 24.0) 5522.3 (230 1 6 ban. 20968

s. d. l. l. F. s. d. 832  
 35 7 : 1 :: 230 1 6  
 12 20  
 427 4601  
 12

427) 55218 (129 l. 6 s. -  $\frac{1}{2}$  d. sterling, answer.  
 Amsterdam,

410

Amsterdam, January 10, 1764.  
 Invoice, or factory of 10 pieces of Holland, 10 pieces of cambrick, 9 pieces of Ghentish cloth, laden by me  
 Jonas Diligent, on board the Josias, Thomas Cook, master, for the proper account and risque of Henry Portez,  
 merchant, in London, under the mark per margin: contents, cost and charges, viz.

10 pieces of Holland

N<sup>o</sup> 6 33 $\frac{1}{2}$  5 pieces, qt. 158 $\frac{1}{2}$   
 N<sup>o</sup> 1 qt. 31 $\frac{1}{2}$  32 ditto - 163  
 to 32 to 34 in all 32 $\frac{1}{2}$  ells, at 1 gil. 11 ft. per ell - 497 18  
 31 31 $\frac{1}{2}$   
 5 32 $\frac{1}{2}$  to 32 $\frac{1}{2}$  9 pieces of cambrick, qt. 124 $\frac{1}{2}$  ells Flemish, at 1 gil. 3 fliv. per ell - 143 10  
 9 pieces of Ghenting, qt. 105 $\frac{1}{2}$  ells Flemish, at 19 flivers per ell - 100 4

# EXCHANGE.

Book II.

## C H A R G E S.

To custom and brokerage of the Hollands, 3 gil. per piece - 30 0  
 To charges in buying - - - - - 2 5  
 To custom of cambrick and Ghenting - - - - - 19 12  
 To sledage and boatage - - - - - 3 16  
 To warehouse room - - - - - 4 3  
 To average and portage - - - - - 1 11

To my commission at 2 $\frac{1}{2}$  per cent. - - - - - 20 1 5

Errors excepted.

From your humble servant,

JONAS DILIGENT.

18. What sterling doth the invoice on the other side amount to; viz. 823 guilders, 6 pennings, at 34 s. 6 d. Flemish for 1 l. sterling?

| s.    | d. | l.     | Flem.                                 |
|-------|----|--------|---------------------------------------|
| 34    | 6  | : 1 :: | 823                                   |
| 12    |    |        | 20                                    |
| <hr/> |    |        | <hr/>                                 |
| 414   |    |        | 16460                                 |
|       |    |        | <hr/>                                 |
|       |    |        | 414) 32921 (79 l. 10 s. 4½ d. the an- |
|       |    |        | 3942                                  |
|       |    |        | <hr/>                                 |
|       |    |        | 215                                   |
|       |    |        | 20                                    |
|       |    |        | <hr/>                                 |
|       |    |        | 4306                                  |
|       |    |        | <hr/>                                 |
|       |    |        | 160                                   |
|       |    |        | × 12                                  |
|       |    |        | <hr/>                                 |
|       |    |        | 1920                                  |
|       |    |        | <hr/>                                 |
|       |    |        | 264                                   |
|       |    |        | <hr/>                                 |

F R A N C E.

England exchanges with France on the crown of three livres Tournois, or 60 sols French, and gives pence, sterling, more or less, for this exchange crown.

|            |            |                    |
|------------|------------|--------------------|
| 12 deniers | } make one | { sol.             |
| 20 sols    |            |                    |
| 3 livres   |            |                    |
|            |            | { livre of France. |
|            |            | { ecu or crown.    |

The exchange between France and other countries are more variable than any, owing to the frequent alteration of their coin; which is so great, that Mr. Postlethwayt affirms he has known, in the space of a few years, the crown or ecu of three livres from 5 d. to near 60 d. English; but that the first indeed was payable in their bank-notes then (viz. anno 1720) in great discredit: so that there can be no other way of ascertaining the par of exchange with that kingdom, but by an actual assay, and weighing their species at the times.

19. In 27 l. 16 s. 8 d. sterling, exchange at  $31\frac{1}{2}$  d. per ecu; how many livres Tournois?

d. liv. l. s. d.  
As  $31\frac{1}{2} : 3 :: 27 \ 16 \ 8$

20  
556  
12  
6680  
2  
13360  
3

63) 40080 (636 liv. 3 fol.  $9\frac{1}{2}$  den. answer.  
228

390  
12  
20  
240  
51  
12  
612  
45

20. In 5731 crowns, 45 fols, how much sterling, exchange at  $31\frac{1}{2}$  per crown?

crown. crowns. fols.

1 :  $31\frac{1}{2}$  :: 5731 45  
8 60

1 crown = 60 deniers.

249 343905  
249  
3095145  
1375620  
687810

60) 85632345  
8) 1427205  
12) 178400 $\frac{1}{2}$ , pence sterling.  
20) 14866  $8\frac{1}{2}$

£ 743 6  $8\frac{1}{2}$  sterling, answer.



21. Suppose Paris owes London 4186 livres, 7 fols, 5 deniers, and remits the same sum to London at  $31\frac{1}{2}$  per crown.

|                        |                                                   | liv. fols. den. |
|------------------------|---------------------------------------------------|-----------------|
| 3 : $31\frac{1}{2}$ :: | 4186                                              | 7 5             |
| 8                      | $\times 20$                                       |                 |
| <hr/>                  | <hr/>                                             |                 |
| 253                    | 83727                                             |                 |
|                        | $\times 12$                                       |                 |
| <hr/>                  | <hr/>                                             |                 |
|                        | 1004729                                           |                 |
|                        | 253                                               |                 |
| <hr/>                  | <hr/>                                             |                 |
|                        | 3014187                                           |                 |
|                        | 5023645                                           |                 |
|                        | 2009458                                           |                 |
| <hr/>                  | <hr/>                                             |                 |
| 80                     | 254196437                                         |                 |
| <hr/>                  | <hr/>                                             |                 |
| 9                      | 3177455                                           |                 |
| <hr/>                  | <hr/>                                             |                 |
| 8                      | 353050                                            |                 |
| <hr/>                  | <hr/>                                             |                 |
| 12                     | 44131 $\frac{1}{2}$ , pence sterling.             |                 |
| <hr/>                  | <hr/>                                             |                 |
| 20                     | 3677 $7\frac{1}{2}$                               |                 |
| <hr/>                  | <hr/>                                             |                 |
|                        | $\pounds$ 183 17 $7\frac{1}{2}$ sterling, answer. |                 |

22. What comes 175.96 quintals to, at 2 liv. 17 fol. 7 den. per quintal, of 100 lb. per invoice on the other side ?

|                | quint.    |                 |
|----------------|-----------|-----------------|
|                | 175.96    |                 |
|                | 2         |                 |
| <hr/>          | <hr/>     |                 |
| $\frac{1}{2}$  | 351.92    |                 |
| $\frac{1}{4}$  | 87.98     |                 |
| $\frac{1}{8}$  | 43.99     |                 |
| $\frac{1}{16}$ | 17.598    |                 |
| $\frac{1}{32}$ | 4.3995    |                 |
| $\frac{1}{64}$ | .73325    |                 |
| <hr/>          | <hr/>     |                 |
|                | 506.62075 | liv. fols. den. |
|                | =         | 506 12 5        |

Bourdéaux,

Bordeaux, November 22, 1764. 4

Invoice of half a tun of wine, and 20 parcels of prunes, shipped on the Canary-Merchant, John King, master, for account of Valentine Aulin, merchant, in London, marked as in the margin.

To 2 hogheads of Graves claret, at 50 crowns per tun - - - - - liv. 50. 3s. 6d.  
 To 20 punchions of prunes, bought of Mr. Tart and company; viz. - - - - - 75 - - -

# EXCHANGE

Book II

| N <sup>o</sup> | lb.  | N <sup>o</sup> | l.   | Tare 97½ lb. per cask.                          |
|----------------|------|----------------|------|-------------------------------------------------|
| 1              | 1000 | 11             | 955  | 20                                              |
| 2              | 1000 | 12             | 960  |                                                 |
| 3              | 1000 | 13             | 960  | 1950                                            |
| 4              | 1005 | 14             | 955  |                                                 |
| 5              | 990  | 15             | 900  |                                                 |
| 6              | 995  | 16             | 925  | 9990 ten punchions.                             |
| 7              | 905  | 17             | 950  | 9556 ditto.                                     |
| 8              | 1045 | 18             | 981  |                                                 |
| 9              | 1000 | 19             | 930  | Gr. 19546.20 punchions.                         |
| 10             | 1000 | 20             | 1040 | Tare 1950                                       |
|                |      |                |      | liv. fol. den.                                  |
|                |      |                |      | 17596 lb. net, at 2 17 7 per quintal - 506 12 5 |

VA

C H A R G E S.

|                                                            | liv. | sol. | den. |
|------------------------------------------------------------|------|------|------|
| To custom and brokerage of wine, 20 livres per tun         | -    | 10   | -    |
| To charges in buying, 15 sols per tun                      | -    | -    | 6    |
| To sledage and boitage of the said wine                    | -    | -    | 7    |
| To custom of prunes, 4 livres, 15 sols per ps. or punchion | -    | -    | 15   |
| To sledage and boitage, 9 sols per ps.                     | -    | 95   | -    |
| To the ship-broker for the prunes, 10 sols per ton         | -    | 9    | -    |
| To average poors-box, 27 sols per ten                      | -    | 4    | 17   |
|                                                            | -    | 13   | 3    |
|                                                            | -    | 10   |      |

133 4 1

714 16 6

17 17 5

Livres 732 13 11

To my commission, at  $2\frac{1}{2}$  per cent.

Errors excepted.

SAMUEL KENT.

23. Three hogheads of Graves claret, at 50 crowns per tun, per invoice.

crowns.

50  
× 3 livres in one crown.

2) 150

75  
20 punchions of prunes 506 12 5  
Charges - - - - - 133 4 1

What comes livres 714 16 6 to, at  $2\frac{1}{2}$  per cent.

1429 13 -  
357 8 3

17.87 1 3  
20

17.51  
12

6.15 Answer, 17 17 6

liv. fol. den.

24. What ought the 175.96 quintals of prunes to weigh in London, one quintal at Bourdeaux being 110 lb.?

quint.

175.96  
110

112) 19355.6 (172 3 7, weight in London.

815  
315

28) 91

7

25. What

25. What comes 732 livres, 13 sols, 11 deniers to in London, at  $57\frac{1}{2}$  d. per crown at Bourdeaux?

$$\begin{array}{r}
 \text{liv. cr. den.} \\
 60 : 57\frac{1}{2} :: 732 \text{ 13 11} \\
 \hline
 12 \qquad \qquad 20 \\
 \hline
 720 \qquad \qquad 14653 \\
 \qquad \qquad \qquad 12 \\
 \hline
 \qquad \qquad \qquad 175847 \\
 \qquad \qquad \qquad 57\frac{1}{2} \\
 \hline
 \qquad \qquad \qquad 1230929 \\
 \qquad \qquad \qquad 879235 \\
 \qquad \qquad \qquad 87923.5 \\
 \hline
 \qquad \qquad \qquad \qquad \qquad 12) \\
 720) 1011120.25 (14043\frac{1}{2} \\
 \qquad \qquad \qquad 291 \\
 \qquad \qquad \qquad 312 \quad 20) 1170 \quad 3\frac{1}{2} \\
 \qquad \qquad \qquad 240 \\
 \qquad \qquad \qquad 24 \quad \pounds 58 \quad 10 \quad 3\frac{1}{2} \\
 \qquad \qquad \qquad 4 \\
 \hline
 \qquad \qquad \qquad 96 \\
 \hline
 \qquad \qquad \qquad 24 \\
 \hline
 \hline
 \end{array}$$

S P A I N.

|                 |                      |              |                             |
|-----------------|----------------------|--------------|-----------------------------|
| 4               | Maravedis vellon, or | } make one { | Quartas.                    |
| 2 $\frac{1}{2}$ | Maravedis plate      |              | Rial vellon.                |
| 8 $\frac{1}{2}$ | Quartas, or - - -    |              | Rial of plate.              |
| 34              | Maravedis vellon     |              | Piso, piafter, piece of     |
| 16              | Quartas, or - - -    |              | $\frac{1}{16}$ , or dollar. |
| 34              | Maravedis plate      |              |                             |
| 8               | Rials of plate       |              |                             |

N. B. A rial vellon is  $\frac{1}{16}$  of a rial of plate, and  $\frac{1}{16}$  of a piafter.

26. Reduce 1387 piafters, 3 rials, 3 maravedis of Spain, into pounds, &c. Sterling, of England, exchange at  $45\frac{1}{2}$  Sterling piafter?

piast. ri. mar.

1387 3 3

45 $\frac{1}{2}$

6935

5548

693 $\frac{1}{2}$  =  $\frac{1}{2}$  } 1387 piafters.

173 $\frac{1}{2}$  =  $\frac{1}{2}$  }

11 $\frac{1}{2}$  = 2 rials or  $\frac{1}{2}$  } of the exchange.

5 $\frac{1}{2}$  = 1 rial or  $\frac{1}{2}$  }

$\frac{1}{2}$  = 3 maravedis,

12 | 64209 $\frac{1}{2}$

20 | 5374 11 $\frac{1}{2}$

£ 263 14 11 $\frac{1}{2}$ , the answer.

27. In 572l. 18s. 9d. how many pieces of  $\frac{3}{4}$ , exchange at  $42\frac{1}{2}$  per piafter?

l. s. d.

42 $\frac{1}{2}$  : 1 :: 572 18 9

8

20

341

11458

12

137505

8

piast. ri. mar.

341) 1100040 (3225 7 13, the answer,

770

884

2620

315

8

1500

133

34

532

399

4522

1182

89

28. In 274 dollars, 4 rials, 7 quartas, how many pounds sterling, exchange at 48 d. per dollar?

$$\begin{array}{r}
 \text{dol. ri. qr.} \\
 274 \quad 4 \quad 7 \\
 48 \\
 \hline
 2192 \\
 1096 \\
 24 = 4 \text{ rials or } \frac{1}{2} \text{ exchange} \\
 2\frac{1}{2} = 7 \text{ quartas.} \\
 \hline
 \begin{array}{r}
 12 \overline{) 13178\frac{1}{2}} \\
 20 \overline{) 1098 \quad 2\frac{1}{2}} \\
 \hline
 \text{£ } 54 \quad 18 \quad 2\frac{1}{2}
 \end{array}
 \end{array}$$

29. In 58794 quartas, how many pounds sterling, exchange at 40 $\frac{1}{2}$  d. per piafter?

$$\begin{array}{r}
 \text{ps. } \frac{1}{2} \text{ d. quartas.} \\
 8 : 40\frac{1}{2} :: 58794 \\
 16 \quad 8 \quad 321 \\
 \hline
 128 \quad 321 \quad 58794 \\
 117588 \\
 176382 \\
 \hline
 128) 16872874 (147444 \\
 607 \\
 952 \\
 568 \\
 567 \\
 554 \\
 42 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 8 \overline{) 147444} \\
 12 \overline{) 18430\frac{1}{2}} \\
 20 \overline{) 1535 \quad 10\frac{1}{2}} \\
 \hline
 \text{Answer, £ } 76 \quad 15 \quad 10\frac{1}{2}
 \end{array}$$

30. What is the brokerage of 15066 rials of plate in the invoice following, at  $\frac{1}{4}$  per cent.?

$$\begin{array}{r}
 \text{rials.} \\
 4) 15066 \\
 \hline
 37\frac{1}{2}, \text{ brokerage.}
 \end{array}$$

31. What is the commiffion of 151475 rials of plate, at  $2\frac{1}{2}$  per cent. ?

$$\begin{array}{r}
 \text{rials.} \\
 151475 \\
 \underline{2.5} \\
 757375 \\
 302950 \\
 \hline
 378.6875 \text{ or } 378\frac{1}{2} \text{ rials commiffion.}
 \end{array}$$

32. What fterling money does the whole 15526 rials of plate in the invoice following amount to, exchange at 52d. fterling per piece of  $\frac{8}{11}$  ?

$$\begin{array}{r}
 15526 \text{ rials of plate.} \\
 \underline{52} \\
 31052 \\
 77630 \\
 \hline
 8 \overline{) 807352} \\
 \hline
 12 \overline{) 100919} \text{ d.} \\
 \hline
 20 \overline{) 8409} \text{ 11} \\
 \hline
 \text{420 9 11 fterling, the anfwer.}
 \end{array}$$



Laus Deo, in Cadiz, November 23, 1764.

Invoice of one barrel, cont. one feron of cascarilla, shipped on board the Seville-merchant, Captain John Tatam, commander, for account and risque, as per advice of Henry-Eustace Johnson, Esq; merchant, in London; the mark as per margin.

HE

EXCHANGE.

|                                                                                      |           |       |
|--------------------------------------------------------------------------------------|-----------|-------|
| One feron, quantity net 209½ lb. of cascarilla, at 9 pieces of $\frac{3}{8}$ per lb. | Ry. Pl.   |       |
| To dispatch, 4 pieces of $\frac{3}{8}$ , is                                          | - - - - - | 15066 |
| To portrage to the house and boat                                                    | - - - - - | 32    |
| To boat-hire abroad                                                                  | - - - - - | 4     |
| To brokerage, at $\frac{1}{4}$ per cent.                                             | - - - - - | 8     |
|                                                                                      | - - - - - | 37½   |
| To my commission, at 2½ per cent.                                                    | - - - - - | 81½   |
|                                                                                      | - - - - - | 378½  |

R. Plate 15526

Errors excepted.

JOHN HICKSON.

## P O R T U G A L.

Portugal exchanges with London on the milrea, and London gives from 60 to 66 pence sterling for the same.

400 reas - - - - } make one { crusadoe.  
1000 reas, or  $2\frac{1}{2}$  crusadoes } milrea.

33. In 2729 crusadoes, 372 reas, how much sterling money, exchange 62 d. per milrea?

crus. reas.

2729 372

400

1091.972

5 s. 2 d.

5455

181 10 d. being  $\frac{1}{2}$  of 1091 milreas.

2 7 =  $3\frac{1}{2}$  d. being  $\frac{1}{2}$  exchange for 500 } reas.

2  $\frac{1}{2}$  =  $\frac{1}{2}$  or double exchange for 400 }

$2\frac{1}{2}$  =  $\frac{1}{2}$  of 400 = 40 }

$1\frac{1}{2}$  =  $\frac{1}{2}$  of 40 = 20 } reas.

$\frac{1}{2}$  =  $\frac{1}{2}$  of 20 = 10 }

20) 5641 10

£ 282 1 10 d. sterling, the answer.

34. In 754 l. 18 s. 6 d. sterling, how many crusadoes, exchange  $64\frac{1}{2}$  d. per milrea?

d. mil. l. s. d.

$64\frac{1}{2}$  : 1 :: 754 18 6

2

20

129

15098

12

181182

2

129) 362364 (2809 milreas.

2809

1043

$2\frac{1}{2}$

1164

3

5618

1404 $\frac{1}{2}$

7022 $\frac{1}{2}$  crusadoes, the answer.

35. What

35. What sterling money does the invoice following, viz. 187 milreas, 686 reas, amount to, at 40 reas for 3 d. ?

$$\begin{array}{r}
 \text{mil. reas.} \\
 40 \overline{) 187.686} \quad \text{Rem. } 6 = \frac{1}{5} \text{ of } 3 \text{ d. or } \frac{2}{5} \text{ of } 1 \text{ d.} \\
 \underline{4692 \frac{1}{4}} \\
 52 \quad 13 \quad \frac{1}{4} \text{ sterling, the answer.}
 \end{array}$$

Also 2) 175.150, at  $\frac{1}{2}$  per cent.  
is .875, brokerage.

And 176.025, at 3 per cent.  
 $\times 3$   
is 5.280, commission.

Oporto, November 23, 1764. 424

Invoice of wine. laden by Charles Colby, on board the Savanna, Richard Delamore, master, for William Blaydwin and company, and consigned to Spelman Swain, Elq; in Dantzick.

|                                                                             |   | E X C H A N G E. |         |
|-----------------------------------------------------------------------------|---|------------------|---------|
|                                                                             |   | M. R.            | —       |
| To cost for 10 pipes of wine, bought of Anthony de Minas, at 16 M. per pipe | - | 160              | —       |
| To custom, at 1055 reas per pipe                                            | - | 10               | 550     |
| To trimming, &c. at 400 reas                                                | - | 4                | —       |
| No 1 To primage, at 60 reas per pipe                                        | - | -                | 600     |
| to 10 To brokerage, at $\frac{1}{2}$ per cent.                              | - | -                | 876     |
| To commission, at 3 per cent.                                               | - | 5                | 280     |
| To port charges of the said ship                                            | - | 6                | 380     |
|                                                                             |   | M. R.            | 187 686 |

Errors excepted.

GENOA.

CHARLES COLBY. Book II.

G E N O A.

In St. George's bank at Genoa, accounts are kept in piafters, or pezzoes, which are divided into solidi and denari, as the pound sterling.

But some merchants keep their accounts in lires, or liras solidi, and denari, divided as before, which money is only  $\frac{1}{3}$  in value of the bank money.

The exchange runs from 45 to 54 d. per piafter.

36 In 784 pez. 19 s. 6d. lire money, how much money of exchange?

$$\begin{array}{r} \text{pez. s. d.} \\ 5) 784 \quad 19 \quad 6, \text{ lire money.} \end{array}$$

$$\underline{156 \quad 19 \quad 10\frac{2}{3}}, \text{ exchange money, answer.}$$

37. Reduce 156 pez. 19 s. 10 $\frac{2}{3}$ , exchange money, to livres.

$$\begin{array}{r} \text{pez. s. d.} \\ 156 \quad 19 \quad 10\frac{2}{3} \\ \underline{5} \end{array}$$

$$\underline{784 \quad 19 \quad 6}, \text{ lire money, answer.}$$

38. London is indebted to Genoa in 1710l. 16 s. 4 d.; for how many pezzoes may Genoa value on London, the exchange at 47 $\frac{1}{2}$  d.?

$$\begin{array}{r} \text{d.} \quad \frac{1}{2} \text{P. pez.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \quad \frac{1}{2} \text{P.} \\ 47\frac{1}{2} = 95 : 1 :: 1710 \quad 16 \quad 4 = 821192 \end{array}$$

$$\begin{array}{r} \text{s. d.} \\ \text{pezzoes } 8644 \quad 2 \quad 6, \text{ answer.} \end{array}$$

39. Genoa is indebted to London in 8644 pez. 2 s. 6d. for how much sterling may London value on Genoa, the exchange at 47 $\frac{1}{2}$  per pezzoe?

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 6) 8644 \quad 2 \quad 6 \end{array}$$

$$8) 1440 \quad 13 \quad 9, \text{ for } 40 \text{ d.}$$

$$2) 180 \quad 1 \quad 8\frac{1}{2}, \text{ for } 5.$$

$$90 - 10\frac{1}{2}, \text{ for } 2\frac{1}{2}.$$

$$\underline{1710 \quad 16 \quad 4}, \text{ answer.}$$

40. London draws on Genoa for 1710l. 16s. 4d. sterling; how much lire money will pay the draught, exchange at 48 d. per piafter?

|         |       |    |    |                                   |
|---------|-------|----|----|-----------------------------------|
|         | l.    | s. | d. |                                   |
|         | 1710  | 16 | 4  |                                   |
|         |       |    |    | 5 times 4 s. in a pound sterling. |
| Pezzoes | 8554  | 1  | 8  | of exchange.                      |
|         |       |    |    | 5 livres in a pezzoe.             |
|         | 42770 | 8  | 4  | lire money, answer.               |

## L E G H O R N.

N. B. At Leghorn a dollar is valued at 6 livres, at Genoa but five.

|            |   |          |   |                              |
|------------|---|----------|---|------------------------------|
| 12 denarii | } | make one | { | foldi.                       |
| 20 foldi   |   |          |   | lira, or piafter of Leghorn. |
| 5½ foldi   |   |          |   | grosfi.                      |
| 24 grosfi  |   |          |   | ducat.                       |

In Leghorn accounts are kept in piafters, foldi, and denari, divided as at Genoa. Some likewise keep their accounts in liras, or lires, divided as the piafter; but this money is only  $\frac{1}{2}$  of the money of exchange.

41. In 278l. 17s. 9d. sterling, how many pezzoes of Leghorn, exchange at  $47\frac{1}{2}$  d. per pezzoe?

|            |        |              |    |    |
|------------|--------|--------------|----|----|
| d.         | pez.   | l.           | s. | d. |
| 47½        | : 1 :: | 278          | 17 | 9  |
| 8          |        | 20           |    |    |
| <u>379</u> |        | <u>5577</u>  |    |    |
|            |        | 12           |    |    |
|            |        | <u>66933</u> |    |    |
|            |        | 8            |    |    |

379) 935464 (1412 pez. 16 fol. 9 den. answer.

|             |
|-------------|
| 1564        |
| 486         |
| <u>1074</u> |
| 316         |
| <u>× 20</u> |
| 6320        |
| <u>2590</u> |
| 314         |
| <u>12</u>   |
| <u>3768</u> |
| 357         |

42 London is indebted to Leghorn in 7456 piast. 9 s. 6d. lire money; what sterling stands as an equivalent in the London merchant's books, the exchange being at 49½ d. per piaster?

piast. s. d.  
6) 7456 9 9

6) 1089 4 11, money of exchange.

|   |      |    |    |          |
|---|------|----|----|----------|
| 5 | 181  | 10 | 9½ | at 40 d. |
| 8 | 36   | 6  | 1½ |          |
| 2 | 4    | 10 | 9½ |          |
| 2 | 2    | 4  | 4½ |          |
|   | 1    | 2  | 8½ |          |
|   | - 11 | 4  |    |          |

£ 226 7 8½, answer.

FACTORY of the cost and charges of one hundred barrels of anchovies, shipped on board the Tortoise, to captain William Rastal, for account of Mr. John Hourd, of London, merchant, and consigned to himself, as under mark per margin.

|                                                                                          |                 |   |
|------------------------------------------------------------------------------------------|-----------------|---|
| To prime cost of the said 100 barrels of anchovies, at piece of $\frac{1}{8}$ per barrel | 2 $\frac{7}{8}$ | - |
| To portage and warehouse                                                                 | Liv. 6          | - |
| To jefling, with coopers pains                                                           | 15              | - |
| To warehouse room and Leviathan                                                          | 25              | - |
| To portage and boitage abroad                                                            | 20              | - |
| To brokerage, $\frac{1}{8}$ per cent.                                                    | 8               | 5 |
| To primage                                                                               | 5               | - |

**To my provision, 3 per cent.**

**Livres 1781 26**

## EXCHANGE

## Book II.

Per WILLIAM FODKIL.

**Leghorn, November 23, 1764.**

## Errors excepted.



43. At 115 fols per piece of  $\frac{8}{7}$ , viz. the foregoing invoice, how much sterling may Mr. Hourd credit his factor, exchange at 4s. 6d. sterling per piece of  $\frac{8}{7}$ ?

$$\begin{array}{r}
 \text{liv. fols. den.} \\
 1781 \quad 2 \quad 6 \\
 \underline{20} \\
 115) 35622.5 \text{ (309.76087 pieces of } \frac{8}{7} \text{)} \\
 \underline{1122} \\
 875 \\
 \underline{700} \\
 1000 \\
 \underline{800} \\
 309.76087 \\
 \hline
 61.95217 \\
 7.74402 \\
 \hline
 69.6962 = 69 \text{ l. } 13 \text{ s. } 11 \text{ d. sterling, the answer.}
 \end{array}$$

Provision of 1729 liv. 5 fols, at 3 per cent.

$$\begin{array}{r}
 \text{liv.} \\
 17.2925 \\
 \underline{3} \\
 51.8775 = 51 \text{ liv. } 17 \text{ fols. } 6 \text{ den. provision.}
 \end{array}$$

# V E N I C E.

Money of exchange is always understood to be that of ducats in bank, which is imaginary; 100 whereof make 120 ducats current money; so that the difference betwixt bank and current money is an agio of 20 per cent. Though the brokers have invented another agio to be added, which is more or less, according to bargain.

The course of exchange of a ducat of the bank of Venice is from 45 to 50 d. sterling.



When London exchanges on the piece of foreign money, as the French crown, Venetian ducat, &c. London ought to remit when the exchange is low, and draw when it is high, to negotiate with advantage. The reason will be obvious, for 100 l. will go farther in purchasing ducats, crowns, milreas, &c. when the course of exchange is at 40 d. than when it is at 50 d. and 100 crowns will go farther in paying a debt due by France to London, when the exchange is at 32 d. than when it is only at par.

POLAND AND PRUSSIA.

|                               |                    |   |   |               |                               |
|-------------------------------|--------------------|---|---|---------------|-------------------------------|
| 3 shillings, or 18 penningsen |                    |   |   |               |                               |
| 3                             | groſch             | - | - | } make<br>one | groſch.                       |
| 2                             | ditkins            | - | - |               | ditkin.                       |
| 3                             | fixers             | - | - |               | fixer.                        |
| 7½                            | groſch             | - | - |               | tymph.                        |
| 4                             | ach de halbers     | - | - |               | ach de halbers.               |
| 3                             | ſtorins or gilders | - | - | }             | ſtorin or gilder.             |
| 4                             | gilders            | - | - |               | current } dollar.<br>ſpecie } |

Dantzick and Koningsberg exchange with London by way of Amſterdam and Hamburgh; 270 Poliſh groſch being = 1 l. groſs banco in Holland, 410 Poliſh groſch being = 1 rix-dollar banco of Hamburgh.

47. Let 5850 ſtorins be changed into ſterling money, 270 groſchi Poli per pound Flemiſh, and 33s. 4 d. Flemiſh per pound ſterling.

G. P.      ſtor.  
270 : 1 :: 5850  
            30

30 | 175500  
9 | 5850

£ 650 Flemiſh.

33s. 4 d. : 1 :: 650  
12                      20  

---

400                      13000  
X 12  

---

400) 156000

£ 390 ſterling, the answer.

R U S-

## R U S S I A.

|               |   |   |              |             |
|---------------|---|---|--------------|-------------|
| 3 copecs      | - | - | } make one { | altine.     |
| 10 copecs     | - | - |              | grievener.  |
| 25 copecs     | - | - |              | polpolitin. |
| 2 polpolitons | - | - |              | poltin.     |
| 2 poltins     | - | - |              | rubble.     |
| 2 rubbles     | - | - |              | ducat.      |

The Russian rubbles are converted into florins current money of Amsterdam, and the current into bank money, according to the agio of three or five per cent. and bank money into sterling, according to the course of exchange between England and Amsterdam.

48. In 4675 rubbles, 46 copecs, exchange 122 copecs per rix-dollar current, agio three per cent. and 34 s. 7 d. Flemish per pound sterling, how much sterling money?

|                                                            |                            |
|------------------------------------------------------------|----------------------------|
| rub.                                                       | cop.                       |
| 4675                                                       | 40                         |
| 100                                                        |                            |
| ----- rix-dollars.                                         |                            |
| 122) 467546                                                | (3832.34426                |
|                                                            | X 2.5                      |
| -----                                                      |                            |
| 1916172130                                                 |                            |
| 766468852                                                  |                            |
| 9580.86065                                                 | florins current.           |
| 103 : 100 :: 9580.86065                                    |                            |
| 103) 958086.065                                            | (9301.80645 florins banco, |
| 310                                                        | 40                         |
| 186                                                        | -----                      |
| 830                                                        | 372072.258                 |
| 665                                                        |                            |
| 470                                                        |                            |
| 585                                                        | -----                      |
| 34 s. 7 d. = 415) 372072.258 (896 l. 11 s. 2½ d. the ansr. |                            |
| 4007                                                       |                            |
| 2722                                                       |                            |
| -----                                                      |                            |
| 232.2                                                      |                            |
| 20                                                         |                            |
| -----                                                      |                            |
| 4644                                                       |                            |
| &c.                                                        |                            |

I R E L A N D.

In Ireland accounts are kept in pounds, shillings, and pence Irish, divided as in England; but having no coins of their own, they are supplied by the different countries, with which they traffic.

The par of exchange between England and Ireland is 100 l. sterling for 108 l. 6 s. 8 d. Irish, or 1 s. English = 13 d. Irish.

The course of exchange is from five to 12 per cent. according to the balance of trade.

49. London remits to Ireland 787 l. 15 s. sterling; how much Irish must London be credited, exchange at  $11\frac{1}{2}$  per cent.?

$$\begin{array}{r} \text{l.} \quad \text{s.} \\ \frac{1}{10} \quad 787 \quad 15 \\ \frac{1}{10} \quad 78 \quad 15 \quad 6 \\ \frac{1}{2} \quad 7 \quad 17 \quad 6\frac{1}{2} \\ \frac{1}{4} \quad 3 \quad 18 \quad 9\frac{1}{4} \\ \hline \quad \quad \quad 19 \quad 8\frac{1}{4} \end{array} \left. \vphantom{\begin{array}{r} \text{l.} \quad \text{s.} \\ \frac{1}{10} \quad 787 \quad 15 \\ \frac{1}{10} \quad 78 \quad 15 \quad 6 \\ \frac{1}{2} \quad 7 \quad 17 \quad 6\frac{1}{2} \\ \frac{1}{4} \quad 3 \quad 18 \quad 9\frac{1}{4} \\ \hline \quad \quad \quad 19 \quad 8\frac{1}{4} \end{array}} \right\} \text{for } \left\{ \begin{array}{l} 10 \\ 1 \\ 1\frac{1}{2} \\ 1 \\ 1 \end{array} \right\} \text{per cent.}$$

£ 879 6 6, the answer.

50. Dublin draws upon London for 879 l. 6 s. 6 d. Irish, exchange at  $11\frac{1}{2}$  per cent. how much sterling must London pay Dublin, to discharge this bill?

$$\begin{array}{l} 111.625 : 100 :: 879.326041 \text{ l.} \\ 111.625) 87932.6041 \text{ l.} (787.75 = 787 \text{ l. } 15 \text{ s.} \\ \quad 979510 \\ \quad \quad 865104 \\ \quad \quad \quad 837291 \\ \quad \quad \quad \quad 559166 \\ \hline \end{array}$$

A M E R I C A AND THE WEST-INDIES.

In exchange with our colonies in America and the West-Indies, accounts are kept in pounds, shillings, and pence, divided as in England, and their money is called currency.

The scarcity of cash obliges them to substitute a paper currency for carrying on their trade; which being subject to casualties, suffers a very great discount for sterling in the purchase of bills of exchange.

51. Philadelphia is indebted to London 1575 l. 14 s. 9 d. currency; what sterling may London reckon to be remitted, when the exchange is 75 per cent. ?

As 175 : 100 :: 1575 l. 14 s. 9 d.

By dividing the two first terms by 25.

|    |          |     |    |           |
|----|----------|-----|----|-----------|
|    | cur. ft. | l.  | s. | d.        |
| As | 7        | : 4 | :: | 1575 14 9 |
|    |          |     |    | 4         |

7) 6302 19 -

Answer, 900 8 5 $\frac{1}{2}$ , sterling.

52. London receives a bill of exchange from Philadelphia for 900 l. 8 s. 5 $\frac{1}{2}$  d. sterling; for how much currency was London indebted, exchange being at 75 per cent. ?

|               |     |    |                 |                 |
|---------------|-----|----|-----------------|-----------------|
|               | l.  | s. | d.              |                 |
| $\frac{1}{2}$ | 900 | 8  | 5 $\frac{1}{2}$ |                 |
| $\frac{1}{2}$ | 450 | 4  | 2 $\frac{1}{2}$ | at 50 per cent. |
| $\frac{1}{2}$ | 225 | 2  | 1 $\frac{1}{2}$ | at 25 per cent. |

Answer, £ 1575 14 9, currency.

53. London consigns to Virginia goods, per invoice, amounting to 578 l. 19 s. 6 d. which are sold for 847 l. 15 s. 6 d. currency; what sterling ought the factor to remit, deducting five per cent. for commission and charges; and what does London gain per cent. upon the adventure, supposing the exchange at 30 per cent. ?

$130 + 5 = 135 = 5 \times 9 \times 3$   
 $135 : 100 :: 847 \quad 15 \quad 6$

8477 15 6  
 10

5) 84777 10  
 3) 16955 10  
 9) 5651 16 8

£ 627 19 7 $\frac{1}{2}$ , to be remitted.

578 19 6, consigned.

£ 49 - 1 $\frac{1}{2}$  sterling, gained.

578.975) 4900.625 (8.464c3 = 8 l. 9 s. 3 $\frac{1}{2}$  d. per cent.

54. Vir-

## Chap. VI. SIMPLE ARBITRATION of EXCHANGE. 435

54. Virginia is indebted to London 575 l. 19 s. 6 d. sterling; with how much currency will London be credited at Virginia, when the exchange is  $33\frac{1}{2}$  per cent.?

$$\begin{array}{r} \text{l. s. d.} \\ \frac{1}{2} | 575 \ 19 \ 6 \\ \hline 191 \ 19 \ 10 \end{array}$$

Answer, £ 767 19 4, currency.



### S E C T. II.

#### SIMPLE ARBITRATION of EXCHANGE.

**W**HEN a factor has orders from his employers to remit a certain sum of money to any place, and then draws upon the last place to some other; as the par of exchange is continually fluctuating, there may happen to be a loss in the executing one part of the commission, and a gain in the other part thereof; which the skilful factor should endeavour (if possible) so to improve to the benefit of his employer, to make the gain superior to the loss; or in case the negotiation would be to his constituent's loss, he may write to him for new orders, or wait till the course of exchange be more in his favour.

Arbitration of exchange may be performed by one or more operations in the rule of three.

1. V, of Amsterdam, draws upon X, of Hamburg, at 67 d. Flemish per dollar of 32 sols Lubeck; and on Y, of Nuremberg, at 70 d. Flemish per florin of 65 crutzers current. If V has orders to draw on X, in order to remit to Y at the said prices, how would run the exchange between Hamburg and Nuremberg?

67 d. : 32 ster. :: 70 d. :  $30\frac{2}{3}$  sols Lubeck per florin.

2. M, of Amsterdam, orders N, of London, to remit to O, of Paris, at 54 d. sterling, and to draw on P, of Antwerp, for the value, at  $33\frac{1}{2}$  shillings Flemish per pound sterling; but as soon as N received the commission, the exchange was on Paris at  $54\frac{1}{2}$  d. per crown: pray at what rate of

F f 2

ex-

# 436 SIMPLE ARBITRATION of EXCHANGE. Book II.

exchange ought N to draw on P, to execute his orders, and be no loser?

$$\begin{array}{ccc} \text{d.} & \text{s.} & \text{d.} \\ \text{Reciprocally, } 54 & : 33.5 & :: 54.5 \end{array}$$

$$\begin{array}{r} 54 \\ 1340 \\ 1675 \end{array}$$

$$54.5) 1809.0 (33\text{s. } 2\frac{2}{3}\text{d. Flem. the answer.}$$

$$\begin{array}{r} 1740 \\ 105 \\ 12 \\ 1260 \\ 170 \end{array}$$

3. London changes with Amsterdam on par at 33½ s. Flemish, for one pound sterling; Amsterdam changes on Middleburgh, at 2 per cent. advance: how stands the exchange between London and Middleburgh?

Flem. l.

$$\text{As } 100 : 102 :: 33\frac{1}{2} = 1.6$$

$$\begin{array}{r} 1.6 \\ 9) 612 \\ 680 \\ 102 \end{array}$$

$$100) 170.0 (1.7\text{l.} = 1\text{l. } 14\text{s. Fl. per lb. ster.}$$

4. Amsterdam changes on London at 34 s. 4 d. per pound sterling, and on Lisbon at 52 d. Flemish for 400 reas; how then ought the exchange to go between London and Lisbon?

$$\begin{array}{ccc} \text{s.} & \text{d.} & \text{d.} \\ \text{As } 34 & 4 = 412 & : 240 :: 52 \end{array}$$

$$\begin{array}{r} 52 \\ 480 \\ 1200 \end{array}$$

$$412) 12480 (30\frac{10}{103}\text{ pence for 400 reas.}$$

$$120$$

$$\therefore 30\frac{10}{103} \times 2\frac{1}{2} = 75\frac{7\frac{1}{2}}{103}\text{d. sterling for 1000 reas.}$$

5. Q



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5. Q<sub>2</sub> of Amsterdam, remits to R, of Paris, 2000 crowns, 91 pence Flemish per crown, at double usance, or two months, and pays  $\frac{1}{10}$  per cent. brokerage; with orders to remit him again the value at 93 d. per crown, allowing at the same time  $\frac{1}{10}$  per cent. for provision: what is gained per cent. per annum, by a remittance thus managed?

$$\begin{aligned} 100\frac{3}{10} : 100 :: 91 : 90\frac{1710}{1000} &= 90\frac{3078}{1000} \\ 100\frac{1}{10} : 100 :: 93 : 92\frac{90}{100} &= 92\frac{162}{100} \\ \text{Thus } 92\frac{162}{100} - 90\frac{3078}{1000} &= 1\frac{483}{1000} \\ \therefore 2 : 1\frac{483}{1000} :: 12 : 10\frac{81}{1000}, &\text{ the answer.} \end{aligned}$$

6. A, of Paris, draws on B, of London, 1200 crowns, at 55 d. sterling per crown; for the value whereof B draws again on A, at 56 d. sterling, besides reckoning half per cent. did A get or lose by this transaction, and what?

|                         |    |                   |    |      |                          |
|-------------------------|----|-------------------|----|------|--------------------------|
| cr.                     | s. | d.                | l. | l.   | l.                       |
| $\frac{1}{5}$ 1200 at 4 | 7  | As 100 : 5 :: 275 |    |      |                          |
|                         |    |                   |    |      | .5                       |
| $\frac{1}{8}$ 240       |    |                   |    |      |                          |
| $\frac{1}{6}$ 30        |    |                   |    | 100) | 137.5 = 11. 7s. 6d. com. |
| 5                       |    |                   |    |      |                          |

|   |     |   |   |                                |        |                        |
|---|-----|---|---|--------------------------------|--------|------------------------|
| £ | 275 | - | - | d.                             | cr.    | d.                     |
|   | 1   | 7 | 6 | commission. As 56 : 1 :: 66330 |        |                        |
|   |     |   |   | 56)                            | 166330 | (1184 $\frac{1}{4}$ ). |
| £ | 276 | 7 | 6 | = 66330 pence.                 |        |                        |

∴ 1200 — 1184 $\frac{1}{4}$  = 15 $\frac{3}{4}$ , A's gain by this transaction.

7. A, of Amsterdam, owes B, of Paris, 2000 florins of current specie, which he is to remit him, by order, the exchange 90 $\frac{1}{2}$  d. per crown of 60 sols Turnois, the agio of the bank being four per cent. better than specie; but when this was to be negotiated the exchange was down at 89 $\frac{1}{2}$  d. per crown, and the agio raised to five per cent. what did B get by this turn of affairs?

Florins 2000 × 40 = 80000 Flemish pence.

90.5) 80000 (883.9778 } crowns Turnois.  
89.5) 80000 (893.8558 }

As 105 : 100 :: 893.8558 : 851.2912 }  
104 : 100 :: 883.9787 : 849.9787 } agio accounted.

cr. fol. den.  
Difference 1.3125 = 1 18 9 in fa-  
vour of B. Q.E.F.  
F f 3 8. But

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8. But arbitration of exchange may commonly be more readily performed by a numerical equation; viz. Let us suppose that the exchange between London and Amsterdam is at 34 s. 6 d. for 1 l. sterling; and between London and France  $31\frac{1}{2}$  d. sterling, for 1 ecu or crown.

To find the proportional arbitrated price between Amsterdam and Paris,

Make the following numerical equation;

viz. 1 crown Paris =  $31\frac{1}{2}$  d. sterling,

And 240 sterling = 34 s. 6 d. = 414 d. Flemish.

The right-hand numbers constitute (by being multiplied continually into one another) a general dividend; the left-hand a general divisor, the quotient of which will give a true solution to the question.

But these may be reduced in lower terms, or less proportional numbers, by observing the axiom in reduction of vulgar fractions.

$$\begin{array}{r|l}
 1 & 1 \text{ cr.} = 31\frac{1}{2} \\
 2 & 240 \text{ d.} = 34 \times 6 \text{ d.} = 414 \\
 \hline
 1 \times 4 & 3 \quad 4 = 127 \\
 2 \div 6 & 4 \quad 40 = 69 \\
 & 5 \quad \frac{127 \times 69}{4 \times 40} = \frac{8763}{160} = 54\frac{123}{160} \text{ Flem. pence.} \\
 & \quad \quad \quad \text{the answer.}
 \end{array}$$

This operation is thus performed:

$31\frac{1}{2} \times 4 = 127$ , which place under the line on the same side, and place 4 on the other side to balance it.

Divide 240 and 414 each by six, and the quotes will be 40 and 69; which place on the same side with their dividends, cancelling all numbers as they are done with.

The rest are so plain and easy, it needs no explanation.

9. Again, suppose the exchange between Paris and Amsterdam is at  $54\frac{123}{160}$ , and on London  $31\frac{1}{2}$ ; the proportional arbitrated price between London and Amsterdam is required?

$$\begin{array}{rcl}
 & \text{l. ster.} & \text{d.} \\
 & 1 & = 31\frac{1}{2} \\
 & \frac{31\frac{1}{2}}{1} & = \frac{64\frac{123}{160}}{1} \\
 & 127 & \\
 & 4 & \\
 \therefore \frac{8763 \times 6}{127} & = \frac{52578}{127} & = 414 \text{ Flem. pence} = 34 \text{ s. } 6 \text{ d.}
 \end{array}$$

The

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The foregoing operation is performed thus :

$54\frac{1}{2} \times 160 = 8763$  placed underneath, and 160 set on the other side to balance.

Then  $31\frac{1}{2} \times 4 = 127$ , to balance which place 4 on the other side.

Then I perceive, that 160 and 240 are each divisible by 40, the quotes whereof are 4 and 6.

Lastly, finding 4 on each side, they cancel each other.

You are desired, as before directed, to cancel every figure as it is done with.

10. Lastly, exchange Amsterdam on Paris at  $54\frac{1}{2}$ , and Amsterdam on London at 34 s. 6 d. what is the arbitrated price between London and Paris ?

$$\begin{array}{l} \text{1 crown Paris} = 84\frac{1}{2} \\ 34 \text{ s. } 6 \text{ d. or } 44 \text{ d.} = 44 \end{array} \quad \therefore \frac{2921}{92} = 31\frac{6}{2} = 31\frac{1}{2} \text{ d.}$$

$$\begin{array}{r} 100 \\ 2 \\ 138 \\ 46 \end{array}$$

$$\begin{array}{r} 3103 \\ 8 \\ 2921 \\ 1 \end{array}$$

The common measure of the fraction being 23.

11. London exchanges on Amsterdam at 34 s. 9 d. for 1 l. sterling, and on Lisbon at 5 s. 5 $\frac{1}{2}$  d. per milrea ; what is the arbitrated price between Amsterdam and Lisbon ?

$$\begin{array}{l} \text{1 cruzado Lisbon} = 400 \text{ reas.} \\ 1000 \text{ reas} = 68\frac{1}{2} \text{ d. sterling.} \\ 40 \text{ d. sterling} = 34 \text{ s} = 417 \text{ Flemish pence.} \end{array}$$

$$\begin{array}{r} 4 \\ 8 \\ 40 \\ 1 \\ 4 \\ 16 \end{array}$$

$$\begin{array}{r} 878 \\ 7 \\ 108 \\ 77 \\ 77 \\ 1 \\ 7 \end{array}$$

$$\therefore \frac{417 \times 7}{16 \times 4} = \frac{2919}{64} = 45\frac{9}{4} \text{ d. Flemish, for one cruzado Lisbon.}$$

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12. Amsterdam exchanges on Lisbon at  $45\frac{1}{2}$  Flemish pence, and on London at 34 s. 9 d. what is the arbitrated price of exchange between London and Lisbon?

$$\begin{aligned} 1 \text{ milrea} &= 1000 \text{ reas.} \\ 1 \text{ crusado} &= 400 \text{ reas} = 45\frac{1}{2} \text{ d. Amsterdam.} \\ 417 \text{ d. Fl.} &= 740 \text{ d. sterling.} \end{aligned}$$

$$\begin{array}{r} 84 \\ 7 \\ 139 \\ 8 \\ \hline 2919 \\ 5 \\ 40 \\ 5 \end{array}$$

$$\therefore \frac{2919 \times 5 \times 5}{139 \times 8} = \frac{72975}{1112} = 65\frac{1}{2} \text{ d. sterling per milrea.}$$

N. B. The common measure of the fraction being 139.

13. Lisbon exchanges on Amsterdam at  $45\frac{1}{2}$  per crusado, on London at 5 s.  $5\frac{1}{2}$  d. per milrea; what is the arbitrated price between London and Amsterdam?

$$\begin{aligned} 1 \text{ l. sterling} &= 740 \text{ d. sterling.} \\ 68\frac{1}{2} \text{ sterling} &= 1000 \text{ reas Portugal.} \\ 400 \text{ reas} &= 45\frac{1}{2} \text{ Flemish pence.} \end{aligned}$$

$$\begin{array}{r} 571 \\ 7 \\ 92 \\ 8 \\ 208 \\ 71 \\ \hline 3 \\ 417 \end{array}$$

Here the answer comes out exactly 417 Flemish pence, or 34 s. 9 d. of Amsterdam; and this will frequently happen, and the operation performed on the thumb-nail by the expert accountant.

These examples prove the truth of this method in regard to each other.

14. Amsterdam hath orders to remit a certain sum to Cadiz; at the time of this order Amsterdam can remit to Cadiz at  $94\frac{1}{2}$  d. per ducat of 375 maravedis, and London to Cadiz at 38 d. per piafter of 272 maravedis. Quere, which will be most advantageous to Amsterdam, to remit directly

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 to Cadiz, or by London, the exchange between Amsterdam  
 and London being 35  $\text{ft.}$  10  $\text{guil.}$  per pound sterling ?

$$\begin{array}{r} 111 \text{ maravedis} = 38 \text{ d. sterling.} \\ 111 \text{ d. sterling} = 28 \text{ s. } 10 \text{ guil.} = 122 \text{ d. Amsterdam.} \\ \hline 136 \\ 111 \\ \hline 19 \\ 43 \\ \hline 125 \end{array}$$

$\therefore \frac{19 \times 43 \times 125}{136 \times 8} = \frac{102125}{1088} = 93.86489$  pence Amsterdam  
 for 375 maravedis ; which is 18 s.  $8\frac{1}{4}$  d. for every 100 l.  
 sterling in favour of Amsterdam ; viz.

$$\frac{941}{1088} = .86489, \text{ nearly.}$$

As 94.75 : 93.86489 :: 100 : 99.06584 = 99 l. 1 s.  $3\frac{1}{4}$  d.  
 Then 100 l. — 99 l. 1 s.  $3\frac{1}{4}$  d. = 18 s.  $8\frac{1}{4}$  d. as above.

~~~~~

S E C T. III.

'COMPOUND ARBITRATION of EXCHANGE.

WHEN the price of exchange is given betwixt one
 country and another, betwixt that second and a third,
 and betwixt that third and a fourth, &c. to find the arbi-
 trated price between the first and the last, observe the
 following

R U L E.

Place the antecedents in one column, and the conse-
 quents in another, to the right of the antecedents, so as to
 form a numerical equation in the algebraic way of analysis,
 in which the first antecedent and the last consequent, to
 which an antecedent is required, must always be of the
 same denomination or species ; the first consequent must be
 of the same denomination with the second antecedent ; the
 second

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second consequent with the third antecedent, &c. throughout. If a fraction is annexed to any of the numbers, both the antecedent and consequent must be multiplied into the denominator of that fraction, and the proportion will still be the same. The terms being thus disposed, cancel the quantities that are the same on both sides of the equation, and abridge such quantities as are commensurable; then multiply all the antecedents into one another for a general divisor, and all the consequents for a general dividend, and the quotient will be the answer, or value of the antecedent required.

1. Suppose London to remit 500 l. to Spain, by the way of Holland, at 35 s. per pound; thence, by the way of France, at 58 grotes per crown; thence to Venice, at 100 crowns per 60 ducats banco; and from Venice to Spain, at 360 maravedis per ducat banco; how many piafters of 272 maravedis will it amount to in Spain, exclusive of charges?

anteced.	conseq.
1 pound	= 48d. Flemish.
58 grotes	= 1 crown.
100 crowns	= 60 ducats.
1 ducat	= 360 maravedis.
272 maraved.	= 1 piafter.
How many piafters for 500 l.?	

These reduced, will be,

1	-	-	21
29	-	-	1
1	-	-	3
1	-	-	45
17	-	-	1
			500

$$\frac{21 \times 3 \times 45 \times 500}{29 \times 17} = 2875\frac{1}{2} = 2875\frac{1}{2}, \text{ nearly, the answer.}$$

2. A banker in Paris remits to his factor in Amsterdam 455 crowns Tournois; first to London, at 30 d. per crown; from London to Rome, at 65 d. per stamp crown; from Rome to Venice, at 100 stamp crowns for 140 ducats banco; from Venice to Leghorn, at 100 ducats banco for 100 piafters of Leghorn; and from Leghorn to Amsterdam, at 86 Fle-

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86 Flemish pence per piafter, how many guilders banco will be received at Amsterdam ?

anteced.	conseq.
1 crown Paris	= 30 d. sterling.
8 d. sterling	= 1 crown Rome.
100 crowns Rome	= 140 ducats Venice.
100 ducats Venice	= 100 piafters Leghorn.
1 piafter Leghorn	= 86 pence Flemish.
	488 crowns Tournois.

<u>13</u>	<u>6</u>
8	7
	91

$$\therefore \frac{86 \times 6 \times 7 \times 91}{13} = \frac{328692}{13} = 25284 \text{ Flemish pence.}$$

And 40) 25284

Answer, 632 guilders, 2 stivers.

3. A merchant of London hath credit for 1360 piafters of Leghorn, from which there is advice that a remittance can be made at 50d. per piafter. The London merchant, finding he could make no more by drawing for them, orders them to be remitted in the following manner; viz. first to Venice, at 94 piafters for 100 ducats banco; thence to Cadiz, at 320 maravedis per ducat; thence to Lisbon, at 630 reas per piafter of 272 maravedis; thence to Amsterdam, at 50 grotes per crusado of 400 reas; from thence to Paris, at 56 grotes per crown; and lastly, he brings them home at $31\frac{1}{2}$ d. per crown: what will be the arbitrated price per piafter between London and Leghorn, and how much will be received at London, without reckoning charges?

anteced.	conseq.
84 piafters	= 100 ducats banco.
1 ducat	= 320 maravedis.
272 maravedis	= 400 reas.
400 reas	= 50 grotes.
56 grotes	= 1 crown.
3 crowns	= 84 pence sterling = $31\frac{1}{2} \times 3$.
What	= 1 piafter ?

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I	-	-	-	-	I
I	-	-	-	-	5
34	-	-	-	-	15
I	-	-	-	-	25
I	-	-	-	-	I
I	-	-	-	-	I

$$\therefore \frac{5 \times 15 \times 25}{34} = \frac{1875}{34} = 55\frac{5}{4} \text{ d. per piafter.}$$

		l.	s.	d.
1360 piafters, at $55\frac{5}{4}$ d. per piafter	=	312	10	-
Ditto at 50 d. per piafter	- - - -	283	6	8

Gained by the negotiation £ 29 3 4

4. Amfterdam being to remit to London 750 l. Flemifh, he firft fends it to France, at 54 d. per crown; from thence to Venice, at 100 crowns for 56 ducats banco; from thence to Hamburgh, at 100 grotes per ducat; from thence to Portugal, at 45 grotes per cruſado of 400 reas; and from Portugal to London, at 5 s. 3 d. for 1000 reas; and fuppofe the commiſſion, &c. at each place be half per cent. quere, how much ſterling money muſt be received in London; and whether more or leſs, than if it was remitted direſtly from Amfterdam to London, at 35 s. 6 d. Flemifh per pound ſterling?

4 pence	=	1 crown.
100 crowns	=	56 ducats banco of Venice.
1 ducat V.	=	100 grotes Hamburgh.
45 gr. Ham.	=	400 reas Portugal.
1000 reas	=	63
	£	750 Flemifh, at $35\frac{1}{2}$ s. per pound ſterl.

5	2
5	7
77	28
9	28
	10

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$$\therefore \frac{2 \times 7 \times 18 \times 10}{9} = \frac{2520}{9} = 435 \text{ l. sterling.}$$

$$\times .005 = \frac{1}{2} \text{ per cent. at Port.}$$

$$35 \frac{1}{2} \text{ s.} = 1.775 \text{ Flemish} - 2.17$$

$$\begin{array}{r} 1.775 \text{ } 750.000 \text{ (422.5352; } \times .005 = \frac{1}{2} \text{ per cent. Hamb.} \\ \text{had it been remitted di-} \\ \text{rectly to London - - } \end{array} \left. \begin{array}{r} 433.37 \\ 2.1668 \end{array} \right\}$$

$$431.2108$$

$$\times .005 = \frac{1}{2} \text{ per cent. Venice.}$$

$$2.156$$

$$429.0548$$

$$\times .005 = \frac{1}{2} \text{ per cent. Paris.}$$

$$2.1453$$

$$426.9095 \text{ received in London.}$$

$$422.5352 \text{ direct remittance.}$$

Answer, $4.3743 = 4 \text{ l. } 7 \text{ s. } 5 \frac{1}{2} \text{ d.}$ London gains by the remittance above.

S E C T. IV.

Comparison of WEIGHTS and MEASURES.

IT is a very necessary way (of great importance to the merchant) to be acquainted with the weights and measures of the different countries where he deals; to facilitate which knowledge, I have in the following pages exhibited authentic tables of the conformity which weights and measures in the most noted trading places in Europe have with one another.

1. Suppose 100 lb. of Amsterdam be equal to 100 lb. of Paris; and 100 lb. of Paris to be 150 lb. in Genoa; and 100 lb. of Genoa to be 70 lb. in Leipfick; and 100 lb. of Leipfick

Leipfick to be 160 lb. in Milan; how many Milan pounds will equiponderate 548 lb. of Amsterdam?

lb.		lb.
100 Amsterdam	=	100 Paris.
100 Paris	=	100 Genoa.
100 Genoa	=	70 Leipfick.
100 Leipfick	=	100 Milan.
Quere, Milan	=	548 Amsterdam?

7	-	-	-	-	-	-	3
25	-	-	-	-	-	-	4
							7
							2

$$548 \times 3 \times 7 \times 2 = 23016$$

$$25) 23016 (920\frac{1}{5} \text{ Milan, the answer.}$$

2. If 7 aunes of Paris make 9 yards of London, 36 yards of London 49 aunes of Holland, 7 aunes of Holland 9 braces of Milan, 3 braces of Milan 2 vares of Aragon, 5 vares of Aragon 2 canes of Montpelier, 9 canes of Montpelier 10 canes of Thouloufe, and 4 canes of Thouloufe 9 canes of Troyes, in Champaigne; how many aunes of Troyes will measure 100 aunes of Paris?

7 aunes of Paris	=	9 yards of London.
36 yards of London	=	49 aunes of Holland.
7 aunes of Holland	=	9 braces of Milan.
3 braces of Milan	=	2 vares of Aragon.
5 vares of Aragon	=	2 canes of Montpelier.
9 canes of Montpelier	=	10 canes of Thouloufe.
4 canes of Thouloufe	=	9 aunes of Troyes.
How many aunes of Troyes	=	100 aunes of Paris?

1	-	-	-	-	-	-	3
4	-	-	-	-	-	-	7
2	-	-	-	-	-	-	1

$$10 \times 3 = 300 \text{ dividend. } 2) 300 (150 \text{ aunes of Troyes.}$$

A TABLE representing the conformity which the weights of the principal trading cities of Europe have with each other, taken from Poffthouy's Dictionary.

A	B	C	D	E	F	G	H	I	K
Eng-land, and Ireland.	Amster- dam, Paris, &c.	Ant- werp, or Bra- bant.	Rouen, the vic- county weight.	Lyons, the city weight.	Ro- chelle.	Thou- loufe, and Upper Lan- guedoc.	Mar- feilles, and Pro- vence.	Gene- va.	Ham- burgh.
100	91 8	96 8	88	106	90 9	107 11	113	81 7	93 5
109 8	100	105 8	96 4	116	99	118	123 8	89	102
103 2	94 12	100	91 4	110	93 13	111 12	117	84 5	96 10
113 14	104	109 12	100	120 8	102 15	122 11	128 8	92 9	106
94 3	86	90 12	82 12	100	85 2	101 8	106 4	76 8	87 12
110 9	101	106 8	97 3	117	100	119 3	124 12	89 14	103
92 6	84 12	89 6	81 8	98 5	83 15	100	105 4	75 7	86 7
88 11	81	85 8	78	94	80 3	95 9	100	72	82 10
123	112 6	118 8	108	130	5 111 6	132 9	128 4	100	114 10
107 5	98	103 6	94 4	111 10	97	115 10	121	87 4	100
112 11	102	107 8	98 3	118 5	101	120 6	126	84 12	104
104 5	95 4	100	91 12	110 8	94 4	112 6	117 12	82 12	92 2
73	66 3	70 5	64 5	77 5	66 5	78 10	82 5	59 5	68 6
75 8	69	72 12	66 6	80	68 5	81 6	85 4	53 6	60 1
65 3	59 8	62 12	57 4	69	58 14	70 3	73 8	53 6	61 3
65 11	60	63 6	57 12	69 10	59 6	70 13	74 2	53 8	60 2
64 10	59	62 4	57	68 7	58 6	69 10	72 14	52 2	59 6
103 7	94 8	99 12	91	109 10	93 9	111 8	116 11	84 2	96 6
95 4	87 8	92	84 4	101 8	86 10	103 4	108	77 14	89 4
104	95	100 3	91 7	110 3	94	112	117 5	84 8	96 14

As the weight of Amsterdam, Paris, Bourdeaux, Befançon, and several other places, have but a very trifling difference, they are comprehended under those of Amsterdam, as those of Nuremberg are under Frankfort, and others in the same manner.

A	B	C	D	E	F	G	H	I	K
Eng-land, and Ireland.	Amster- dam, Paris, &c.	Ant- werp, or Bra- bant.	Rouen, the vic- county weight.	Lyons, the city weight.	Ro- chelle.	Thou- loufe, and Upper Lan- guedoc.	Mar- feilles, and Pro- vence.	Gene- va.	Ham- burgh.
100	91 8	96 8	88	106	90 9	107 11	113	81 7	93 5
109 8	100	105 8	96 4	116	99	118	123 8	89	102
103 2	94 12	100	91 4	110	93 13	111 12	117	84 5	96 10
113 14	104	109 12	100	120 8	102 15	122 11	128 8	92 9	106
94 3	86	90 12	82 12	100	85 2	101 8	106 4	76 8	87 12
110 9	101	106 8	97 3	117	100	119 3	124 12	89 14	103
92 6	84 12	89 6	81 8	98 5	83 15	100	105 4	75 7	86 7
88 11	81	85 8	78	94	80 3	95 9	100	72	82 10
123	112 6	118 8	108	130	5 111 6	132 9	128 4	100	114 10
107 5	98	103 6	94 4	111 10	97	115 10	121	87 4	100
112 11	102	107 8	98 3	118 5	101	120 6	126	84 12	104
104 5	95 4	100	91 12	110 8	94 4	112 6	117 12	82 12	92 2
73	66 3	70 5	64 5	77 5	66 5	78 10	82 5	59 5	68 6
75 8	69	72 12	66 6	80	68 5	81 6	85 4	53 6	60 1
65 3	59 8	62 12	57 4	69	58 14	70 3	73 8	53 6	61 3
65 11	60	63 6	57 12	69 10	59 6	70 13	74 2	53 8	60 2
64 10	59	62 4	57	68 7	58 6	69 10	72 14	52 2	59 6
103 7	94 8	99 12	91	109 10	93 9	111 8	116 11	84 2	96 6
95 4	87 8	92	84 4	101 8	86 10	103 4	108	77 14	89 4
104	95	100 3	91 7	110 3	94	112	117 5	84 8	96 14

Eng- land, &c.	* L Frank- fort, &c.	M Leip- sic, &c.	N Genoa.	O Leg- horn.	P Milan.	Q Venice.	R Naples.	S Seville, Cadiz, &c.	T Portu- gal.	U Liege
100	89	96	137	132	153	152	154	97	104	96
109	98	105	150	145	168	166	169	106	114	105
103	112	92	142	137	159	157	166	108	108	99
113	114	102	156	152	174	172	175	110	119	109
94	84	90	129	124	144	142	145	91	98	90
110	99	106	129	126	159	167	170	107	115	106
92	83	89	127	124	142	140	143	89	97	89
88	79	85	121	117	136	134	136	85	92	85
123	110	118	168	163	188	186	189	119	128	118
107	5	102	154	148	164	162	165	103	112	103
111	11	100	153	147	171	169	172	108	116	107
104	5	100	148	143	158	156	161	101	109	100
73	65	70	100	96	112	110	112	70	76	70
75	67	72	103	98	116	114	116	73	79	72
65	58	62	89	86	100	98	100	63	68	62
65	58	63	90	87	100	98	101	63	68	63
64	57	62	88	85	99	98	100	62	67	62
103	7	92	141	137	158	156	159	100	108	99
95	4	91	131	126	147	145	148	92	100	92
104	92	99	142	137	159	157	160	100	108	100

Continued.

A 100 lb. of England, &c.	-	-	-	-	-	-	-	-	-	-
B 100 lb. of Amsterdam, &c.	-	-	-	-	-	-	-	-	-	-
C 100 lb. of Antwerp, &c.	-	-	-	-	-	-	-	-	-	-
D 100 lb. of Rouen, &c.	-	-	-	-	-	-	-	-	-	-
E 100 lb. of Lyons, &c.	-	-	-	-	-	-	-	-	-	-
F 100 lb. of Rochelle	-	-	-	-	-	-	-	-	-	-
G 100 lb. of Thoulouse, &c.	-	-	-	-	-	-	-	-	-	-
H 100 lb. of Marseilles, &c.	-	-	-	-	-	-	-	-	-	-
I 100 lb. of Geneva	-	-	-	-	-	-	-	-	-	-
K 100 lb. of Hamburgh	-	-	-	-	-	-	-	-	-	-
L 100 lb. of Frankfurt	-	-	-	-	-	-	-	-	-	-
M 100 lb. of Leipsick	-	-	-	-	-	-	-	-	-	-
N 100 lb. of Genoa	-	-	-	-	-	-	-	-	-	-
O 100 lb. of Leghorn	-	-	-	-	-	-	-	-	-	-
P 100 lb. of Milan	-	-	-	-	-	-	-	-	-	-
Q 100 lb. of Venice	-	-	-	-	-	-	-	-	-	-
R 100 lb. of Naples	-	-	-	-	-	-	-	-	-	-
S 100 lb. of Seville, &c.	-	-	-	-	-	-	-	-	-	-
T 100 lb. of Portugal	-	-	-	-	-	-	-	-	-	-
U 100 lb. of Liege	-	-	-	-	-	-	-	-	-	-

N. B. By means of this TABLE may be easily discerned, at one view, the conformity which the weights of one place therein exhibited have with those of another: for example, if you would know how many pounds 100 lb. weight English make at Amsterdam, look for England in the first column, and from thence pass your eye along the line till you come to the column under the title of Amsterdam at the top, and you will find that 91 lb. 8 oz. 16 gr. to the pound, equal 100 lb. English; and in like manner you may find the agreement between any other weight of its place specified in the Table.

A TABLE representing the conformity which the long with each other, taken from Pofflethwayt's Commercial

The ells of Amsterdam, Haerlem, Leyden, the Hague, Rotterdam, and other cities of Holland, as well as the ell of Nuremberg, are equal among themselves. They are also comprehended under the ell of Amsterdam, as that of Osnaburgh is under that of France and England; and the ell of Bern and Basil under that of Hamburg, Frankfort, and Leipfick.		A Yards of England, Scotland, and Ireland.	B Ells of France and England.	C Ells of Holland and Amsterdam.	D Ells of Antwerp and Brussels.
A 100 yards of Engl. Scotl. and Ireland	} make	100	78	133 $\frac{1}{2}$	131 $\frac{1}{2}$
B 100 ells of France and England - -		128 $\frac{1}{4}$	100	173 $\frac{1}{2}$	166 $\frac{1}{2}$
C 100 ells of Holland or Amsterdam -		75	57 $\frac{3}{4}$	100	98 $\frac{1}{4}$
D 100 ells of Antwerp and Brussels - -		76	60	101 $\frac{1}{4}$	100
E 100 ells of Hamburg, Frankfort, &c.		62 $\frac{1}{2}$	48 $\frac{3}{4}$	83 $\frac{1}{2}$	82 $\frac{1}{2}$
F 100 ells of Breslau, in Silesia - - -		60	46 $\frac{3}{4}$	80	79
G 100 ells of Dantzick - - - - -		66 $\frac{3}{4}$	52	89	87 $\frac{3}{4}$
H 100 ells of Bergue and Drontheim -		67 $\frac{1}{2}$	52 $\frac{1}{4}$	90	89
I 100 ells of Sweden or Stockholm -		65 $\frac{3}{4}$	51 $\frac{1}{4}$	87 $\frac{1}{2}$	86 $\frac{1}{2}$
K 100 ells of St. Gall, for linen - -		87	67 $\frac{3}{4}$	116	114 $\frac{1}{2}$
L 100 ells of St. Gall, for cloth - -		67	52 $\frac{1}{2}$	89 $\frac{1}{2}$	88 $\frac{3}{4}$
M 100 ells of Geneva - - - - -		124 $\frac{3}{4}$	97 $\frac{1}{2}$	166 $\frac{3}{4}$	164 $\frac{3}{4}$
N 100 canes of Marfeilles and Montpelier		214 $\frac{1}{2}$	167 $\frac{1}{2}$	286	282 $\frac{1}{2}$
O 100 canes of Thoul, and Upper Lang.		199 $\frac{3}{4}$	156	266 $\frac{3}{4}$	263 $\frac{1}{2}$
P 100 canes of Genoa, of 9 palmos -		245 $\frac{1}{2}$	191 $\frac{1}{2}$	327	323
Q 100 canes of Rome - - - - -		227 $\frac{1}{4}$	177 $\frac{1}{4}$	303	299 $\frac{1}{2}$
R 100 vares of Castille and Biscay -		93 $\frac{3}{4}$	73 $\frac{1}{2}$	125	123 $\frac{3}{4}$
S 100 vares of Cadix and Andalusia -		91 $\frac{3}{4}$	71 $\frac{3}{4}$	122 $\frac{1}{4}$	119
T 100 vares of Portugal or Lisbon -		123	96	164	162
V 100 covedos of Portugal or Lisbon -		74	58 $\frac{1}{2}$	100	98 $\frac{1}{2}$
W 100 brasses of Venice - - - - -		73 $\frac{1}{2}$	57 $\frac{1}{2}$	98	96 $\frac{1}{2}$
X 100 brasses of Bergamo, &c. - - -		72 $\frac{1}{2}$	55 $\frac{3}{4}$	95	93 $\frac{3}{4}$
Y 100 brasses of Florence, Leghorn, &c.		65 $\frac{1}{2}$	50	85 $\frac{1}{2}$	84 $\frac{3}{4}$
Z 100 brasses of Milan - - - - -		58 $\frac{1}{2}$	45 $\frac{3}{4}$	78	77

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	E Ells of Hamburg, Frank- fort, Leipzick and Cologn.	F Ells of Breslau, in Si- lesia.	G Ells of Dantzick.	H Ells of Bergue and Dron- theim.	I Ells of Sweden or Stock- holm.	K Ells of St. Gall, for linen.	L Ells of St. Gall, for cloth.	M Ells of Geneva.
A	160	166 $\frac{1}{4}$	150	146 $\frac{1}{4}$	154	114 $\frac{1}{4}$	149 $\frac{1}{4}$	80
B	205 $\frac{1}{4}$	213 $\frac{1}{4}$	192 $\frac{1}{4}$	188	195 $\frac{1}{4}$	147	191 $\frac{1}{4}$	102 $\frac{1}{4}$
C	120	125	112 $\frac{1}{4}$	110	114 $\frac{1}{4}$	86	112	60
D	121 $\frac{1}{4}$	126 $\frac{1}{4}$	114	111 $\frac{1}{4}$	116	87	113 $\frac{1}{4}$	60 $\frac{3}{4}$
E	100	104 $\frac{1}{4}$	92 $\frac{1}{4}$	91 $\frac{1}{4}$	95 $\frac{1}{4}$	71 $\frac{1}{4}$	91 $\frac{1}{4}$	50
F	96	100	89 $\frac{1}{4}$	88	91 $\frac{1}{4}$	68 $\frac{1}{4}$	89 $\frac{1}{4}$	48
G	96 $\frac{1}{4}$	111 $\frac{1}{4}$	100	98	102	76 $\frac{1}{4}$	99 $\frac{1}{4}$	53 $\frac{1}{4}$
H	108	112 $\frac{1}{4}$	101 $\frac{1}{4}$	100	103	77 $\frac{1}{4}$	100 $\frac{1}{4}$	54
I	105	109 $\frac{1}{4}$	97 $\frac{1}{4}$	96 $\frac{1}{4}$	100	75 $\frac{1}{4}$	98	52 $\frac{1}{4}$
K	139 $\frac{1}{4}$	145	130 $\frac{1}{4}$	127 $\frac{1}{4}$	133	100	130	69 $\frac{1}{4}$
L	107 $\frac{1}{4}$	111 $\frac{1}{4}$	100 $\frac{1}{4}$	98 $\frac{1}{4}$	102 $\frac{1}{4}$	76 $\frac{1}{4}$	100	53 $\frac{1}{4}$
M	200	208 $\frac{1}{4}$	187 $\frac{1}{4}$	183 $\frac{1}{4}$	191	143 $\frac{1}{4}$	130 $\frac{1}{4}$	100
N	343 $\frac{1}{4}$	357 $\frac{1}{4}$	321 $\frac{1}{4}$	314 $\frac{1}{4}$	327 $\frac{1}{4}$	246	320 $\frac{1}{4}$	171 $\frac{1}{4}$
O	320	333 $\frac{1}{4}$	300	193 $\frac{1}{4}$	304	229 $\frac{1}{4}$	298 $\frac{1}{4}$	160
P	392 $\frac{1}{4}$	408 $\frac{1}{4}$	367 $\frac{1}{4}$	359 $\frac{1}{4}$	374 $\frac{1}{4}$	281 $\frac{1}{4}$	366 $\frac{1}{4}$	196 $\frac{1}{4}$
Q	363 $\frac{1}{4}$	378 $\frac{1}{4}$	340 $\frac{1}{4}$	333 $\frac{1}{4}$	347	260 $\frac{1}{4}$	339 $\frac{1}{4}$	181 $\frac{1}{4}$
R	150	156 $\frac{1}{4}$	140 $\frac{1}{4}$	137 $\frac{1}{4}$	143 $\frac{1}{4}$	107 $\frac{1}{4}$	140	75
S	146 $\frac{1}{4}$	152 $\frac{1}{4}$	138	134 $\frac{1}{4}$	139 $\frac{1}{4}$	105	137	73 $\frac{1}{4}$
T	196 $\frac{1}{4}$	205	184 $\frac{1}{4}$	180 $\frac{1}{4}$	187 $\frac{1}{4}$	141	183 $\frac{1}{4}$	94 $\frac{1}{4}$
V	120	125	112 $\frac{1}{4}$	110	114 $\frac{1}{4}$	86	112	60
W	117 $\frac{1}{4}$	122 $\frac{1}{4}$	104 $\frac{1}{4}$	107 $\frac{1}{4}$	112 $\frac{1}{4}$	84 $\frac{1}{4}$	109 $\frac{1}{4}$	58 $\frac{1}{4}$
X	114	118 $\frac{1}{4}$	106 $\frac{1}{4}$	104 $\frac{1}{4}$	108 $\frac{1}{4}$	81 $\frac{1}{4}$	106 $\frac{1}{4}$	57
Y	102 $\frac{1}{4}$	106 $\frac{1}{4}$	96 $\frac{1}{4}$	94	98	73 $\frac{1}{4}$	95 $\frac{1}{4}$	51 $\frac{1}{4}$
Z	93 $\frac{1}{4}$	97 $\frac{1}{4}$	87 $\frac{1}{4}$	85 $\frac{1}{4}$	89 $\frac{1}{4}$	67	87 $\frac{1}{4}$	46 $\frac{1}{4}$

A TABLE representing the conformity which the long
with each other, taken from *Poſſeltſwayt's Commercial*

Continued.

	N Canes of Marſailles and Montpellier.	O Canes of Thoulouſe, Al- bi, and Caffres.	P Canes of Genoa, of 9 palmoſ.	Q Canes of Rome.
A 100 yards of England, Scotland, &c.	46 $\frac{2}{3}$	50	40 $\frac{1}{2}$	44
B 100 ells of France and England - -	59 $\frac{1}{2}$	64 $\frac{1}{2}$	52 $\frac{1}{2}$	56 $\frac{1}{2}$
C 100 ells of Holland and Amſterdam -	35	37 $\frac{1}{2}$	30 $\frac{1}{2}$	33
D 100 ells of Antwerp and Bruffels - -	35 $\frac{1}{2}$	38	30 $\frac{1}{2}$	33 $\frac{1}{2}$
E 100 ells of Hamburgh, Frankfort, &c.	29 $\frac{1}{2}$	31 $\frac{1}{4}$	25 $\frac{1}{2}$	27 $\frac{1}{2}$
F 100 ells of Breſlau, in Sileſia - - -	28	30	24 $\frac{1}{2}$	26 $\frac{1}{2}$
G 100 ells of Dapſtick - - - - -	31 $\frac{1}{2}$	33 $\frac{1}{2}$	27 $\frac{1}{2}$	29 $\frac{1}{2}$
H 100 ells of Bergue and Drontheim -	31 $\frac{1}{2}$	33 $\frac{1}{2}$	27 $\frac{1}{2}$	29 $\frac{1}{2}$
I 100 ells of Sweden or Stockholm - -	30 $\frac{1}{2}$	32 $\frac{1}{2}$	26 $\frac{1}{2}$	28 $\frac{1}{2}$
K 100 ells of St. Gall, for linen - - -	40 $\frac{1}{2}$	43 $\frac{1}{2}$	35 $\frac{1}{2}$	38 $\frac{1}{2}$
L 100 ells of St. Gall, for cloth - - -	31 $\frac{1}{4}$	33 $\frac{1}{2}$	27 $\frac{1}{2}$	29 $\frac{1}{2}$
M 100 ells of Geneva - - - - -	58 $\frac{1}{2}$	62 $\frac{1}{2}$	50 $\frac{1}{2}$	55
N 100 canes of Marſailles and Montpellier	100	107 $\frac{1}{4}$	87 $\frac{1}{4}$	94 $\frac{1}{2}$
O 100 canes of Thoulouſe, &c. - - -	93 $\frac{1}{2}$	100	81 $\frac{1}{2}$	88
P 100 canes of Genoa, of 9 palmoſ - -	114 $\frac{1}{2}$	122 $\frac{1}{2}$	100	108
Q 100 canes of Rome - - - - -	116	113 $\frac{1}{2}$	92 $\frac{1}{2}$	100
R 100 vares of Caſtile and Biſcay - -	43 $\frac{3}{4}$	40 $\frac{1}{2}$	38 $\frac{1}{2}$	41 $\frac{1}{4}$
S 100 vares of Cadiz, &c. - - - -	42 $\frac{3}{4}$	45	37 $\frac{1}{4}$	40 $\frac{3}{4}$
T 100 vares of Portugal or Liſbon - -	57 $\frac{1}{2}$	61 $\frac{1}{2}$	50	54 $\frac{1}{2}$
V 100 covedos of Portugal or Liſbon -	35	37 $\frac{1}{2}$	30 $\frac{1}{2}$	33
W 100 braſſes of Venice - - - - -	34 $\frac{1}{2}$	36 $\frac{1}{2}$	29 $\frac{1}{2}$	32 $\frac{1}{2}$
X 100 braſſes of Bergamo, &c. - - -	33 $\frac{1}{4}$	35 $\frac{1}{2}$	29	31 $\frac{1}{2}$
Y 100 braſſes of Florence, &c. - - -	30	32	26 $\frac{1}{2}$	28 $\frac{1}{2}$
Z 100 braſſes of Milan - - - - -	27 $\frac{3}{4}$	29 $\frac{1}{4}$	23 $\frac{1}{2}$	25 $\frac{1}{2}$

measures of the principal trading cities of Europe have
Dictionary.

R	S	T	V	W	X	Y	Z
Vares of Castille and Biscay.	Vares of Cadiz and Andalusia.	Vares of Portugal or Lisbon.	Covados of Portugal or Lisbon.	Braffes of Venice.	Braffes of Berg, Boulogn, Modena, and Mantua.	Braffes of Florence, Leghorn, and Lucca.	Braffes of Milan.
107	109 $\frac{1}{3}$	81 $\frac{1}{4}$	133 $\frac{1}{3}$	136	104 $\frac{1}{3}$	154 $\frac{1}{4}$	171 $\frac{1}{3}$
136 $\frac{1}{4}$	140	104 $\frac{1}{3}$	171	174 $\frac{1}{3}$	179	199 $\frac{1}{3}$	219 $\frac{1}{3}$
80	81 $\frac{1}{4}$	61	100	102	105 $\frac{1}{4}$	116 $\frac{1}{4}$	128 $\frac{1}{2}$
81	84	61 $\frac{3}{4}$	101 $\frac{1}{4}$	103 $\frac{3}{4}$	106 $\frac{1}{4}$	118	130
65 $\frac{1}{4}$	68 $\frac{1}{4}$	50 $\frac{1}{4}$	83 $\frac{1}{3}$	85	88 $\frac{1}{4}$	97	107
64	65 $\frac{1}{4}$	48 $\frac{1}{4}$	80	81 $\frac{1}{4}$	84 $\frac{1}{4}$	93 $\frac{1}{4}$	102 $\frac{1}{4}$
71 $\frac{1}{4}$	72 $\frac{1}{4}$	54 $\frac{1}{2}$	89	90 $\frac{3}{4}$	93 $\frac{3}{4}$	103 $\frac{3}{4}$	114 $\frac{1}{3}$
72	74 $\frac{1}{4}$	55	90	91 $\frac{1}{4}$	94 $\frac{1}{4}$	105	115 $\frac{1}{4}$
70	71 $\frac{1}{4}$	53 $\frac{1}{4}$	87 $\frac{1}{4}$	89 $\frac{1}{4}$	92	102	112 $\frac{1}{2}$
92 $\frac{1}{4}$	95 $\frac{1}{4}$	70 $\frac{3}{4}$	116	118 $\frac{1}{4}$	122	135 $\frac{1}{3}$	149
71 $\frac{1}{2}$	73	54 $\frac{1}{4}$	89 $\frac{1}{4}$	91 $\frac{1}{4}$	94	104	114 $\frac{1}{2}$
133 $\frac{1}{3}$	136 $\frac{1}{3}$	101 $\frac{3}{4}$	166 $\frac{1}{3}$	170	172 $\frac{1}{3}$	193 $\frac{1}{3}$	214 $\frac{1}{3}$
228 $\frac{1}{3}$	234	174 $\frac{1}{3}$	286	291 $\frac{1}{3}$	301	333 $\frac{1}{3}$	367 $\frac{1}{3}$
213 $\frac{1}{3}$	218	162 $\frac{1}{3}$	266 $\frac{1}{3}$	272	280 $\frac{1}{3}$	309 $\frac{1}{3}$	342 $\frac{1}{3}$
261 $\frac{1}{3}$	268 $\frac{1}{3}$	199 $\frac{1}{3}$	327	333 $\frac{1}{3}$	344 $\frac{1}{3}$	381	420 $\frac{1}{3}$
242 $\frac{1}{3}$	245 $\frac{1}{3}$	184 $\frac{1}{3}$	303	309	319	353	389 $\frac{1}{3}$
100	102 $\frac{1}{3}$	76 $\frac{1}{4}$	125	127 $\frac{1}{4}$	131 $\frac{1}{2}$	145 $\frac{1}{3}$	159 $\frac{1}{3}$
97 $\frac{2}{3}$	100	74 $\frac{1}{3}$	122 $\frac{1}{4}$	125 $\frac{1}{4}$	179	142	157
131 $\frac{1}{3}$	134	100	164	167 $\frac{1}{4}$	172 $\frac{1}{3}$	191	210 $\frac{1}{3}$
80	81 $\frac{1}{4}$	61	100	102	105 $\frac{1}{4}$	116 $\frac{1}{4}$	128 $\frac{1}{2}$
78 $\frac{1}{3}$	80 $\frac{1}{4}$	59 $\frac{1}{4}$	98	100	103 $\frac{1}{3}$	114 $\frac{1}{3}$	126
76	78 $\frac{1}{4}$	58	95	97	100	100 $\frac{1}{4}$	122
68 $\frac{1}{3}$	70 $\frac{1}{4}$	52 $\frac{1}{3}$	85 $\frac{1}{2}$	87 $\frac{1}{3}$	95	100	109 $\frac{1}{3}$
62 $\frac{1}{3}$	63 $\frac{1}{3}$	47 $\frac{1}{2}$	78	79 $\frac{1}{2}$	82 $\frac{1}{3}$	91	100

3. Suppose you owe 100 anees of wheat at Lyons, and would know what quantity you would purchase at Macon to replace them, and have no other means of knowledge but the following; viz.

3 anees of Lyons - - = 4 fetiers of Paris.
 1 fetier of Paris - - = 2 bushels of Bourdeaux.
 26 bushels of Bourdeaux = 77 muds of Amsterdam.
 77 muds of Amsterdam = 48 fanegas of Cadiz.
 48 fanegas of Cadiz - = 87 anees of Macon.
 How many of Macon - = 100 of Lyons?

9	-	-	-	-	-	-	-	1
1	-	-	-	-	-	-	-	19
8	-	-	-	-	-	-	-	1
1	-	-	-	-	-	-	-	20

$$2 \times 19 \times 20 = 760$$

9) 760 (84 $\frac{2}{3}$ anees of Macon, the answer.

4. Suppose a merchant of Hamburg, not knowing the proportion between the ell of that place and yard of London, and having orders to procure 81 yards of cloth, of which 7 ells of Hamburg must be had for 3 l. sterling; how shall he discover how many pounds sterling the 81 yards will amount to, only by knowing that 7 ells of France make 9 yards of London, and 7 ells of Holland make 4 ells of France, and that 1 ell of Holland make 1 $\frac{1}{3}$ of Hamburg?

Note, since $1 = 1\frac{1}{3}$; consequently, $3 = 6$, which dispatches the fraction.

8 yards of London = 7 ells of France.
 4 ells of France - = 7 ells of Holland.
 5 ditto of Holland = 8 ditto of Hamburg.
 7 ditto of Hamburg = 3 pound sterling.

How much sterling for 81 yards?

3	-	-	-	-	-	-	-	1
2	-	-	-	-	-	-	-	3

$$81 \times 7 = 567, \text{ divisor.}$$

$$2 \times 5 = 10, \text{ dividend.}$$

∴ 56.7 = 56 l. 14 s. the answer required.

The END of the SECOND BOOK.

Arithmetical Collections

AND IMPROVEMENTS.

BOOK III.

Containing the more abstruse and curious part of
ARITHMETICK.

CHAPTER I.

ALLIGATION.

WHEN corn, wine, spices, metal, &c. are required to be mixed together, the method of proportioning such mixtures is called the rule of alligation.

SECT. I.

ALLIGATION MEDIAL.

BY alligation medial the mean rate or price of any mixture is found, when the particular quantities and their prices are given.

RULE.

First find the sum of all the quantities proposed to be mixed, and also the sum of their particular rates; then as the sum of all the quantities : is to the sum of all the rates :: so is any part of the mixture : to the mean rate or price of that part.

1. A vintner mixeth $31\frac{1}{2}$ gallons of Malaga sack, worth 7 s. 6 d. a gallon, with 18 gallons of Canary, at 6 s. 9 d. a gallon; $13\frac{1}{2}$ gallons of cherry, at 5 s. a gallon; and 27 gallons of white wine, at 4 s. 3 d. a gallon; what is a gallon of this mixture worth?

gal.		s.	d.		l.	s.	d.
$31\frac{1}{2}$	- - sack	- - at	7 6	- - =	11	16	3
18	- - Canary	- at	6 9	- - =	6	1	6
$13\frac{1}{2}$	- - cherry	- at	5 -	- - =	3	7	6
27	- - whitewine	- at	4 3	- - =	5	14	9
G g 4					27 l.		

$$\begin{array}{r}
 1. \\
 10 \times 9 = 90. \\
 10 \overline{) 27} \\
 \underline{9} \quad \text{s. d.} \\
 2 \quad 14 \quad - \\
 \hline
 - \quad 6 \quad - \text{ per gallon, the answer.}
 \end{array}$$

2. With 13 gallons of Canary, at 6s. 8 d. a gallon, I mixed 20 gallons of white wine, at 5s. a gallon, and to these added 10 gallons of cyder, at 3s. a gallon; at what rate must I sell a quart of this mixture, so as to clear 10 per cent.?

gal.		s. d.		l. s. d.
13	- - Canary, - at	6 8	- -	4 6 8
20	- - white wine at	5 -	- -	5 - -
10	- - cyder - - at	3 -	- -	1 10 -
<hr/>				
43	= 172 quarts - - - -	£ 10 16 8 = 10.8s		

$$\begin{array}{r}
 10) 10.833 \\
 \underline{1.083}
 \end{array}$$

$$172) 11.916 (.06928 = 1s. 4.627d. \text{ the answer.}$$



SECT. II.

ALLIGATION ALTERNATE,

IS when the particular rate of every ingredient, and the mean rate, are given, to discover the particular quantity of each ingredient concerned in a mixture.

RULE.

Place the mean rate so, that it may be easily compared with the particular rates; setting down the differences between the mean rate and the particular rates, alternately, and they will be the quantities required.

I. A

1. A grocer would mix a quantity of sugar, at 10d. per pound, with other sugars of $7\frac{1}{2}$ d. 5d. and $4\frac{1}{2}$ d. per pound, intending to make up a commodity worth 6d. per pound; in what proportions is he to take of those sugars?

$$\begin{array}{c} (1) \\ 6 \left\{ \begin{array}{l} 10 \\ 7\frac{1}{2} \\ 5 \\ 4\frac{1}{2} \end{array} \right\} \left| \begin{array}{l} 1\frac{1}{2} \\ 1 \\ 1\frac{1}{2} \\ 4 \end{array} \right| \end{array} \quad \begin{array}{c} (2) \\ 6 \left\{ \begin{array}{l} 10 \\ 7\frac{1}{2} \\ 5 \\ 4\frac{1}{2} \end{array} \right\} \left| \begin{array}{l} 1\frac{1}{2} \\ 1\frac{1}{2} \\ 4 \\ 1\frac{1}{2} \end{array} \right| \end{array}$$

When one branch is linked to two or more other branches, the differences ought to be as often transcribed as it is so diversly linked.

$$\begin{array}{c} (3) \\ 6 \left\{ \begin{array}{l} 10 \\ 7\frac{1}{2} \\ 5 \\ 4\frac{1}{2} \end{array} \right\} \left| \begin{array}{l} 1\frac{1}{2} \\ 1\frac{1}{2} + 1 \\ 1\frac{1}{2} \\ 4 + 1\frac{1}{2} \end{array} \right| \left| \begin{array}{l} 1\frac{1}{2} \\ 2\frac{1}{2} \\ 1\frac{1}{2} \\ 5\frac{1}{2} \end{array} \right| \end{array} \quad \begin{array}{c} (4) \\ 6 \left\{ \begin{array}{l} 10 \\ 7\frac{1}{2} \\ 5 \\ 4\frac{1}{2} \end{array} \right\} \left| \begin{array}{l} 1\frac{1}{2} + 1 \\ 1 \\ 4 + 1\frac{1}{2} \\ 4 \end{array} \right| \left| \begin{array}{l} 2\frac{1}{2} \\ 1 \\ 5\frac{1}{2} \\ 4 \end{array} \right| \end{array}$$

$$\begin{array}{c} (5) \\ 6 \left\{ \begin{array}{l} 10 \\ 7\frac{1}{2} \\ 5 \\ 4\frac{1}{2} \end{array} \right\} \left| \begin{array}{l} 1\frac{1}{2} + 1 \\ 1\frac{1}{2} \\ 4 \\ 4 + 1\frac{1}{2} \end{array} \right| \left| \begin{array}{l} 2\frac{1}{2} \\ 1\frac{1}{2} \\ 4 \\ 5\frac{1}{2} \end{array} \right| \end{array} \quad \begin{array}{c} (6) \\ 6 \left\{ \begin{array}{l} 10 \\ 7\frac{1}{2} \\ 5 \\ 4\frac{1}{2} \end{array} \right\} \left| \begin{array}{l} 1 \\ 1 + 1\frac{1}{2} \\ 4 + 1\frac{1}{2} \\ 1\frac{1}{2} \end{array} \right| \left| \begin{array}{l} 1 \\ 2\frac{1}{2} \\ 5\frac{1}{2} \\ 5\frac{1}{2} \end{array} \right| \end{array}$$

$$\begin{array}{c} (7) \\ 6 \left\{ \begin{array}{l} 10 \\ 7\frac{1}{2} \\ 5 \\ 4\frac{1}{2} \end{array} \right\} \left| \begin{array}{l} 1 + 1\frac{1}{2} \\ 1 + 1\frac{1}{2} \\ 4 + 1\frac{1}{2} \\ 4 + 1\frac{1}{2} \end{array} \right| \left| \begin{array}{l} 2\frac{1}{2} \\ 2\frac{1}{2} \\ 5\frac{1}{2} \\ 5\frac{1}{2} \end{array} \right| \end{array}$$

2. A proveditor for the army intending to mix wheat at 4s. a bushel, with rye at 3s. a bushel, with barley at 2s. a bushel, with pease at 1s. 4d. a bushel, and with oats at 12d. a bushel, is desirous to know in what proportion to mix them, so that the mass may be worth 1s. 8d. per bushel?

There are divers ways of alligating or linking these numbers together; viz.

$$\begin{array}{c} \text{or,} \\ 20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right\} \left| \begin{array}{l} 8 \\ 4 \\ 4 \\ 16 + 4 \\ 28 \end{array} \right| \left| \begin{array}{l} 8 \\ 4 \\ 4 \\ 20 \\ 28 \end{array} \right| \end{array} \quad \begin{array}{c} \text{or,} \\ 20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right\} \left| \begin{array}{l} 4 \\ 8 \\ 4 \\ 28 + 4 \\ 16 \end{array} \right| \left| \begin{array}{l} 4 \\ 8 \\ 4 \\ 32 \\ 16 \end{array} \right| \end{array}$$

W
R
B
P
O
or,

or,

$$20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 8 \\ 4 \\ 8 \\ 16 \\ 28 + 4 \end{array} \right| \left| \begin{array}{l} 8|2 \\ 4|1 \\ 8|2 \\ 16|4 \\ 32|8 \end{array} \right| 20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 8 \\ 8 \\ 4 \\ 4 \\ 28 + 16 \end{array} \right| \left| \begin{array}{l} 8|2 \\ 8|2 \\ 4|1 \\ 4|1 \\ 44|11 \end{array} \right| \begin{array}{l} W \\ R \\ B \\ P \\ O \end{array}$$

$$20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 4 \\ 8 \\ 8 \\ 28 \\ 16 + 4 \end{array} \right| \left| \begin{array}{l} 4|1 \\ 8|2 \\ 8|2 \\ 28|7 \\ 20|5 \end{array} \right| 20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 8 \\ 4 + 8 \\ 4 \\ 16 + 4 \\ 28 + 16 \end{array} \right| \left| \begin{array}{l} 8|2 \\ 12|3 \\ 4|1 \\ 20|5 \\ 44|11 \end{array} \right| \begin{array}{l} W \\ R \\ B \\ P \\ O \end{array}$$

$$20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 4 \\ 4 \\ 8 \\ 28 + 16 \\ 4 \end{array} \right| \left| \begin{array}{l} 4|1 \\ 4|1 \\ 8|2 \\ 44|11 \\ 4|1 \end{array} \right| 20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 4 + 8 \\ 4 \\ 8 \\ 28 + 16 \\ 28 + 4 \end{array} \right| \left| \begin{array}{l} 12|3 \\ 4|1 \\ 8|2 \\ 44|11 \\ 32|8 \end{array} \right| \begin{array}{l} W \\ R \\ B \\ P \\ O \end{array}$$

$$20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 8 \\ 4 \\ 8 + 4 \\ 16 + 4 \\ 28 + 4 \end{array} \right| \left| \begin{array}{l} 8|2 \\ 4|1 \\ 12|3 \\ 20|5 \\ 32|8 \end{array} \right| 20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 4 + 8 \\ 4 \\ 4 \\ 28 + 16 + 4 \\ 28 \end{array} \right| \left| \begin{array}{l} 12|3 \\ 4|1 \\ 4|1 \\ 48|12 \\ 28|7 \end{array} \right| \begin{array}{l} W \\ R \\ B \\ P \\ O \end{array}$$

$$20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 4 \\ 4 + 8 \\ 4 \\ 28 + 16 + 4 \\ 16 \end{array} \right| \left| \begin{array}{l} 4|1 \\ 12|3 \\ 4|1 \\ 48|12 \\ 16|4 \end{array} \right| 20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 4 \\ 4 \\ 8 + 4 \\ 28 + 16 + 4 \\ 4 \end{array} \right| \left| \begin{array}{l} 4|1 \\ 4|1 \\ 12|3 \\ 48|12 \\ 4|1 \end{array} \right| \begin{array}{l} W \\ R \\ B \\ P \\ O \end{array}$$

$$20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 8 \\ 8 \\ 8 + 4 \\ 4 \\ 28 + 16 + 4 \end{array} \right| \left| \begin{array}{l} 8|2 \\ 8|2 \\ 12|3 \\ 4|1 \\ 48|12 \end{array} \right| 20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 8 \\ 4 + 8 \\ 8 \\ 16 \\ 28 + 16 + 4 \end{array} \right| \left| \begin{array}{l} 8|2 \\ 12|3 \\ 8|2 \\ 16|4 \\ 48|12 \end{array} \right| \begin{array}{l} W \\ R \\ B \\ P \\ O \end{array}$$

$$20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 4 + 8 \\ 8 \\ 8 \\ 28 \\ 28 + 16 + 4 \end{array} \right| \left| \begin{array}{l} 12|3 \\ 8|2 \\ 8|2 \\ 28|7 \\ 48|12 \end{array} \right| 20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 8 + 4 \\ 8 \\ 8 + 4 \\ 28 + 4 \\ 28 + 16 + 4 \end{array} \right| \left| \begin{array}{l} 12|3 \\ 8|2 \\ 12|3 \\ 32|8 \\ 48|12 \end{array} \right| \begin{array}{l} W \\ R \\ B \\ P \\ O \end{array}$$

$$20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 4 + 8 \\ 4 + 8 \\ 8 \\ 28 + 16 \\ 28 + 16 + 4 \end{array} \right| \left| \begin{array}{l} 12|3 \\ 12|3 \\ 8|2 \\ 44|11 \\ 48|12 \end{array} \right| 20 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 8 \\ 8 + 4 \\ 8 + 4 \\ 16 + 4 \\ 28 + 16 + 4 \end{array} \right| \left| \begin{array}{l} 8|2 \\ 12|3 \\ 12|3 \\ 20|5 \\ 48|12 \end{array} \right| \begin{array}{l} W \\ R \\ B \\ P \\ O \end{array}$$

or,

or,

$$\begin{array}{l}
 \text{20} \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 8+4 \\ 8+4 \\ 4 \\ 28+16+4 \\ 28+16 \end{array} \right| \left| \begin{array}{l} 12 \\ 12 \\ 4 \\ 48 \\ 44 \end{array} \right| \left| \begin{array}{l} 3 \\ 3 \\ 1 \\ 12 \\ 11 \end{array} \right| \\
 \text{20} \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 4+8 \\ 4 \\ 4+8 \\ 21+16+4 \\ 28+4 \end{array} \right| \left| \begin{array}{l} 12 \\ 4 \\ 12 \\ 48 \\ 32 \end{array} \right| \left| \begin{array}{l} 3 \\ 4 \\ 3 \\ 12 \\ 8 \end{array} \right| \begin{array}{l} W \\ R \\ B \\ P \\ O \end{array}
 \end{array}$$

$$\begin{array}{l}
 \text{20} \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 4 \\ 8+4 \\ 8+4 \\ 28+16+4 \\ 16+4 \end{array} \right| \left| \begin{array}{l} 4 \\ 12 \\ 12 \\ 48 \\ 20 \end{array} \right| \left| \begin{array}{l} 1 \\ 3 \\ 3 \\ 12 \\ 5 \end{array} \right| \\
 \text{20} \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} \right. \left| \begin{array}{l} 4+8 \\ 4+8 \\ 4+8 \\ 28+16+4 \\ 28+16+4 \end{array} \right| \left| \begin{array}{l} 12 \\ 12 \\ 12 \\ 48 \\ 48 \end{array} \right| \left| \begin{array}{l} 1 \\ 1 \\ 1 \\ 4 \\ 4 \end{array} \right| \begin{array}{l} W \\ R \\ B \\ P \\ O \end{array}
 \end{array}$$

Here you have 24 different answers by the various ways of alligating or linking the prices together, which may be increased infinitely by doubling, tripling, &c. the quantities; or they may be lessened by making the pecks, pints, or any lesser quantity.

The reason of these combinations, and the alternate placing of their differences, will appear from this plain consideration, viz. that whatsoever is lost by selling any quantity whose price exceeds the mean, is gained again on the quantity alligated thereto, whose given price is less than the mean.

When two kinds of things only are given to be mixed, the rule of alligation will give but one answer.

3. Suppose it is required to mix brandy, at 8 s. per gallon, with cyder, at 1 s. per gallon?

$$5 \left\{ \begin{array}{l} 8 \\ 1 \end{array} \right. \left| \begin{array}{l} 4 \text{ gallons of brandy.} \\ 3 \text{ gallons of cyder.} \end{array} \right.$$

If three kinds of things are given to be mixed, the rule of alligation will give but one answer; but then (as might have been observed in mixture of two things) all numbers that are in the same proportion between themselves, and the number which composes that answer, will also satisfy the question.

But by an artifice explained by the ingenious Mr. James Dodson, in the 18th edition of Wingate's Arithmetic, innumerable other answers may be obtained, composed of numbers in a different proportion.

4. Let it be required to mix brandy, at 8 s. per gallon, with wine at 7 s. per gallon, and cyder at 1 s. per gallon;

gallon; so that the mixture may be worth 5's. per gallon?

$$\begin{array}{c|c|c} 8 & 4 & 4 \text{ brandy.} \\ 7 & 4 & 4 \text{ wine.} \\ 1 & 3 + 2 & 5 \text{ cyder.} \end{array}$$

Now suppose, that if it be determined to use five gallons of cyder in the mixture; but to use any quantity of brandy and wine that will answer the question.

Then may the quantity of brandy be increased or diminished by 2; the difference between the prices of the wine and mixture, if at the same time the quantity of wine be diminished or increased by 3, the difference of the prices of the brandy and mixture.

Thus, $4 + 2 = 6$ brandy, and $4 - 3 = 1$ wine; so that six gallons of brandy, one gallon of wine, and five gallons of cyder, will also answer the question, as may be easily proved by alligation medial.

Again, $4 - 2 = 2$ brandy, and $4 + 3 = 7$ wine.

So that two gallons of brandy, seven gallons of wine, and five gallons of cyder, will also answer the question, as may be proved.

But instead of the numbers of the first answer, 4, 4 and 5, larger numbers in the same proportion, viz. 12, 12 and 15 were taken, the following eight answers would be found by increasing and diminishing the quantities of brandy and wine, as above directed, the quantity of cyder remaining constantly 15.

Brandy	18	16	14	12	10	8	6	4	2
Wine	3	6	9	12	15	18	21	24	27
Cyder	15	15	15	15	15	15	15	15	15

And if instead of these still larger numbers in that proportion, or in proportion to any of the last found answers, be assumed, a greater number of other answers may be found.

But if instead of supposing the quantity of cyder invariable, the quantity of brandy be taken for such; then an infinite number of answers may be found, by continually increasing the quantity of wine by 4, the difference between the prices of the cyder and mixture; and the quantity of cyder by 2, the difference between the prices of the wine and mixture.

Thus, assuming the second answer, 6, 1 and 5, and making the six gallons of brandy invariable:

Brandy

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Brandy 6 . 6 . 6 . 6 . 6 . 6 . 6 . 6 . 6, &c.
Wine 1 . 5 . 9 . 13 . 17 . 21 . 25 . 29 . 33, &c.
Cyder 5 . 7 . 9 . 11 . 13 . 15 . 17 . 19 . 21, &c.

Or by taking the third answer, 2, 7 and 5, as the basis, and making the seven gallons of wine invariable; increasing the quantity of brandy by 4, the difference between the price of the cyder and mixture; and the quantity of cyder by 3, the difference of the prices of the brandy and mixture:

Brandy 2 . 6 . 10 . 14 . 14 . 22 . 26 . 30, &c.
Wine 7 . 7 . 7 . 7 . 7 . 7 . 7 . 7, &c.
Cyder 5 . 8 . 11 . 14 . 17 . 20 . 23 . 26, &c.

When there are four kinds of things to be mixed, and two of them of greater value, and the other two of lesser value than the mixture, the rule of alligation will give seven answers, as may be observed by question 1, in this rule; with any of which, or with any numbers in the same proportion, innumerable other answers may be found, consisting of numbers in different proportion among themselves, by making any two invariable, and changing the rest in the manner as above, observing also the following

R U L E.

The numbers by which the quantity of any simple is to be varied, is always the difference between the price of the mixture and the price of the other simple, which in any operation is considered as variable.

Secondly, That if the simples, which in any operation are considered as variable, be both of greater, or both of less value than the mixture, then, while the one is increased, the other must be diminished; but if one be of greater value than the mixture, and the other of less, then they must both be increased, or both diminished.

5. Let it be required to mix brandy, at 8s. wine, at 7s. cyder, at 1s. and water, at nothing per gallon, together; so that the mixture may be worth 5s. per gallon?

I shall only alligate the several values of the simples together by the following method:

5 {	8 7 1 0)	5	+	4	9	brandy.
			5	+	4	9	wine.
			3	+	2	5	cyder.
			3	+	2	5	water.

Now

Now making the wine and cyder invariable :

Brandy	9	.	14	.	19	.	24	.	29	.	34	.	39	.	&c.
Wine	9	.	9	.	9	.	9	.	9	.	9	.	9	.	&c.
Cyder	5	.	5	.	5	.	5	.	5	.	5	.	5	.	&c.
Water	5	.	8	.	11	.	14	.	17	.	20	.	23	.	&c.

Making the brandy and cyder invariable :

Brandy	9	.	9	.	9	.	9	.	9	.	9	.	9	.	&c.
Wine	9	.	14	.	19	.	24	.	29	.	34	.	39	.	&c.
Cyder	5	.	5	.	5	.	5	.	5	.	5	.	5	.	&c.
Water	5	.	7	.	9	.	11	.	13	.	15	.	17	.	&c.

Making the wine and water invariable :

Brandy	9	.	13	.	17	.	21	.	25	.	29	.	33	.	&c.
Wine	9	.	9	.	9	.	9	.	9	.	9	.	9	.	&c.
Cyder	5	.	8	.	11	.	14	.	17	.	20	.	23	.	&c.
Water	5	.	5	.	5	.	5	.	5	.	5	.	5	.	&c.

Making the brandy and wine invariable :

Brandy	9	.	9	.	9	.	9	.	9	.	9	.	9	.	&c.
Wine	9	.	9	.	9	.	9	.	9	.	9	.	9	.	&c.
Cyder	10	.	5	.	5	.	5	.	5	.	5	.	5	.	&c.
Water	1	.	5	.	5	.	5	.	5	.	5	.	5	.	&c.

Or taking four other numbers in the same proportion :

As 9 . 9 . 5 and 5, viz. 36 . 36 . 20 and 20.

Brandy	36	.	36	.	36	.	36	.	36	.	36	.	36	.	36
Wine	36	.	36	.	36	.	36	.	36	.	36	.	36	.	36
Cyder	40	.	35	.	30	.	25	.	20	.	15	.	10	.	5
Water	4	.	8	.	12	.	16	.	20	.	24	.	28	.	32

Lastly, making the cyder and water invariable :

Brandy, &c.	44	.	42	.	40	.	38	.	36	.	34	.	32	.	30	.	28, &c.
Wine, &c.	24	.	27	.	30	.	33	.	36	.	39	.	42	.	45	.	48, &c.
Cyder, &c.	20	.	20	.	20	.	20	.	20	.	20	.	20	.	20	.	20, &c.
Water, &c.	20	.	20	.	20	.	20	.	20	.	20	.	20	.	20	.	20, &c.

Not only the sets of numbers thus found, but their sums and differences, will also be answers.

Thus

bran. wine. cyd. wat.
Thus from or to 42 . 27 . 20 . 20
Take or add - 9 . 9 . 10 . 1

The remainder 33 . 18 . 10 . 19 } will be answers to
And sum - - 51 . 36 . 30 . 21 } the question.

These answers may all be proved by alligation medial ; I shall only prove the last, viz. the difference, and leave the rest to exercise the young arithmetician ;

	gal.	s.		l.	s.	d.
viz.	33	brandy, at 8	- - - -	13	4	-
	18	wine, at 7	- - - -	6	6	-
	10	cyder, at 1	- - - -	-	10	-
	19	water, at -	- - - -	-	-	-
	80			£ 20	-	-

80 gal. : 20 l. :: 1 gal. : 5 s.



SECT. III.

ALLIGATION PARTIAL.

ALLIGATION PARTIAL is when, having the several rates of divers ingredients and the quantity of one of them given, we discover the several quantities of the rest in such sort, that the quantities so found, being mixed with the quantity given, that mixture may bear a certain rate proposed.

Having set down the mean rate, the particular rates and their differences, as before, say,

RULE,

As the difference opposite to the known quantity is to : the known quantity, so is :: any other difference : to the quantity of its opposite name.

1. Let it be required to mix brandy, at 8s. per gallon, and wine, at 7 s. per gallon, with 10 gallons of cyder, at 1 s.

1 s. per gallon, so that the mixture may be worth 5 s. per gallon.

$$\begin{array}{c|c|c} \begin{array}{c} 8 \\ 7 \\ 1 \end{array} & \begin{array}{c} 4 \\ 4 \\ 3 + 2 \\ \hline 13 \end{array} & \begin{array}{c} 4 \\ 4 \\ 5 \end{array} \end{array}$$

5 : 10 :: 4 : 8 gallons each of brandy and wine.

5 : 10 :: 13 : 26 gallons, the whole mixture.

Now, having found one answer by the above proportion, others may be found by the method before delivered.

Brandy 12 . 10 . 8 . 6 . 4 . 2

Wine 2 . 5 . 8 . 11 . 14 . 17

Cyder 10 . 10 . 10 . 10 . 10 . 10

By which means five other answers are obtained.

2. A tobacconist has by him 120 lb. of fine Oroonoko tobacco, worth 2 s. 6 d. a pound; to this he would put as much York-river ditto, at 20 d. with other inferior tobacco, at 18 d. and 15 d. a pound, as will make up a mixture answerable to 2 s. a pound; what will this parcel weigh?

$$\begin{array}{c|c|c} \begin{array}{c} 30 \\ 20 \\ 18 \\ 15 \end{array} & \begin{array}{c} 4 + 6 + 9 \\ 6 \\ 6 \\ 6 \end{array} & \begin{array}{c} 19 \\ 6 \\ 6 \\ 6 \end{array} \end{array}$$

Then $19 + 6 + 6 + 6 = 37$.

∴ $19 : 37 :: 120 : 233\frac{1}{3}$, the answer required.

But as some answers in whole numbers may also be obtained by the foregoing method, putting 38, 12, 12 and 12 instead of those found by alligation, the two last being invariable.

38 . 42 . 46 . 50 . 54 . 58 . 62
12 . 18 . 24 . 30 . 36 . 42 . 48
12 . 12 . 12 . 12 . 12 . 12 . 12
12 . 12 . 12 . 12 . 12 . 12 . 12

Now, taking the sum of the two last sets of numbers,

at 2 s. 6 d. at 1 s. 8 d. at 1 s. 6 d. at 1 s. 3 d.

viz. 58 . 42 . 12 . 12
62 . 48 . 12 . 12

$$\begin{array}{r} 120 + 90 + 24 + 24 = 258, \end{array}$$

being a second answer.

By

By making the second and last invariable :

32	.	38	.	44	.	50	.
12	.	12	.	12	.	12	.
6	.	12	.	18	.	24	.
12	.	12	.	12	.	12	.
at 2s. 6d.		at 1s. 8d.		at 1s. 6d.		at 1s. 3d.	
32	.	12	.	6	.	12	.
38	.	12	.	12	.	12	.
50	.	12	.	24	.	12	.
<hr/>							
120	+	36	+	42	+	36	= 234
<hr/>							
the third answer.							

Lastly, making the second and third invariable :

11	.	20	.	29	.	38	.	47	.	56	.	65	.	74	.	83	.	92	.	101	.
12	.	12	.	12	.	12	.	12	.	12	.	12	.	12	.	12	.	12	.	12	.
12	.	12	.	12	.	12	.	12	.	12	.	12	.	12	.	12	.	12	.	12	.
— 6	.	0	.	6	.	12	.	18	.	24	.	30	.	36	.	42	.	48	.	54	.
2s. 6d.				1s. 8d.				1s. 6d.				1s. 3d.									
101	.			12	.			12	.			54	.								
101	.			12	.			12	.			54	.								
47	.			12	.			12	.			18	.								
11	.			12	.			12	.			— 6	.								
<hr/>																					
260	.			48	.			48	.			120	.								
subtr. 20	.			12	.			12	.			—	.								
<hr/>																					
2) 240	.			36	.			36	.			120	.								
<hr/>																					
120	+			18	+			18	+			60	= 216								
<hr/>																					
the fourth answer.																					

These last three answers may each be made the basis of divers others in different proportion, by making the first term with any one of the others invariable ; and the other two variable to their utmost limits, which I shall leave for the practice of young students in arithmetic ; having (I think) been copious enough upon this subject.

S E C T. IV.

A L L I G A T I O N T O T A L.

ALLIGATION TOTAL is so called, when the particular rates, the mean rate, and the whole quantity of the ingredients to be mixed, are given, and the particular quantity of each ingredient is required. To find which, observe the following

R U L E.

Having found the several differences as before directed, say, as the sum of all the differences : is to the whole quantity of the mixture :: so is each particular difference : to its particular quantity.

I. Let it be required to mix brandy, at 8s. wine, at 7s. and cyder, at 1s. per gallon together; so that the mixture may contain 26 gallons, and be worth 5s. per gallon.

$$\begin{array}{rcl}
 5 \left\{ \begin{array}{l} 8 \\ 7 \\ 1 \end{array} \right. & \left| \begin{array}{l} 4 \\ 4 \\ 3 \end{array} \right. + 2 & \left| \begin{array}{l} 4 \\ 4 \\ 5 \end{array} \right| \\
 & & \hline
 & & 13 \\
 13 : 26 :: & \left\{ \begin{array}{l} 4 \\ 4 \\ 5 \end{array} \right. & : \begin{array}{l} 8 \text{ brandy.} \\ 8 \text{ wine.} \\ 10 \text{ cyder.} \end{array} \\
 & & \hline
 & & 26
 \end{array}$$

One answer being thus obtained, the rest may be found by the following

R U L E.

I. Let the quantity of that ingredient, whose value alone is greater or less than the value of the mixture, be increased or diminished by the difference or differences between the prices of the other two ingredients, and the price of the mixture.

II. Of the remaining two ingredients, let the quantity of that ingredient, whose value is farthest from the value of the mixture, be increased or decreased (according as the former is) by the sum of the differences between the prices of the other two ingredients, and that of the mixture.

I

III. Let

III. Let the quantity of the remaining ingredient be decreased or increased, also, by the sum of the differences between the prices of the other two ingredients, and that of the mixture; but observe, that the quantity of this ingredient is to be decreased, when those of the two former are increased; and the contrary.

I. The value of the cyder alone is less than the value of the mixture.

Also, $8 - 5 = 3$, $7 - 5 = 2$, and $3 - 2 = 1$, the difference of those differences.

$\therefore 10 + 1 = 11$, and $10 - 1 = 9$, are the quantities of cyder.

II. Of the other two the value of the brandy is furthest from that of the mixture.

Also, $7 - 5 = 2$, $5 - 1 = 4$, and $2 + 4 = 6$, sum of their differences.

$\therefore 8 + 6 = 14$, and $8 - 6 = 2$, are the quantities of brandy.

Lastly, $8 - 5 = 3$, $5 - 1 = 4$, and $4 + 3 = 7$.

$\therefore 8 - 7 = 1$, and $8 + 7 = 15$, are the quantities of the wine.

Thus we have obtained two answers more, which make in all three different answers to this question.

Brandy	14	.	8	.	2	} all which may be proved by alli-
Wine	1	.	8	.	15	
Cyder	11	.	10	.	9	

gation medial.

But if there be four or more ingredients out of which the mixture is to be compounded, then one or more of them must be considered as invariable; so that there may be only three variable, and those so, that one of them will be of a contrary value, with respect to the price of the mixture, from the other two.

2. It is required to mix such a quantity of brandy, at 8s. wine, at 7s. cyder, at 1s. and water at nothing per gallon, as will make a hoghead, or 63 gallons of the mixture, worth 5s. per gallon.

Then by the process in alligation alternate, question 5th, the two following proportions may be found; viz.

bran. wine. cyd. wat.

Among the first found answers 9 . 9 . 5 . 5

Among the third - - - 9 . 14 . 5 . 7

$$18 + 23 + 10 + 12 = 63.$$

Then making the water invariable, we have, by the foregoing rule,

Brandy	36	.	30	.	24	.	18	.	12	.	6
Wine -	2	.	9	.	16	.	23	.	30	.	37
Cyder	13	.	12	.	11	.	10	.	9	.	8
Water	12	.	12	.	12	.	12	.	12	.	12

II. Making the cyder invariable, produceth

Brandy	32	.	25	.	18	.	11	.	4
Wine	7	.	15	.	23	.	31	.	39
Cyder	10	.	10	.	10	.	10	.	10
Water	14	.	13	.	12	.	11	.	10

III. Making the wine invariable, gives

Brandy	19	.	18	.	17
Wine	23	.	23	.	23
Cyder	2	.	10	.	18
Water	19	.	12	.	15

Lastly, making the brandy invariable, we have

Brandy	18	.	18	.	18
Wine	24	.	23	.	22
Cyder	3	.	10	.	17
Water	18	.	12	.	6

If you are desirous to find more answers, you may, for

Water makes any number invariable from 5 to 19

Cyder from - - - - - 2 to 18

Wine from - - - - - 2 to 39

Brandy from - - - - - 4 to 36

But if instead of gallons you mix by pints; viz.

instead of 24 . 16 . 11 . 12 gallons,

you take 192 . 128 . 88 . 96 pints for the basis of the operation, a still greater number of answers may be produced; viz.

B. 192. 191. 190. 189. 188. 187. 186. 185. 184. 183. 182. 181. 180. 179

W. 128. 128. 128. 128. 128. 128. 128. 128. 128. 128. 128. 128. 128. 128

C. 88. 96. 104. 112. 120. 128. 136. 144. 152. 160. 168. 176. 184. 192

W. 96. 89. 82. 75. 68. 61. 54. 47. 40. 33. 26. 19. 12. 5

And,

B. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202

W. 128. 128. 128. 128. 128. 128. 128. 128. 128. 128. 128. 128. 128

C. 88. 80. 72. 64. 56. 48. 40. 32. 24. 16. 8

W. 96. 103. 110. 117. 124. 131. 138. 145. 152. 159. 166

C H A P.

CHAPTER II.

SPECIFIC GRAVITY *of* METALS, &c.

THE specific gravity of a body, is the relation the weight of a body of one kind hath to the weight of an equal magnitude of a body of another kind.

Gold is the heaviest of all known bodies, the most maleable and ductile of all metals; is incapable of rust, and not sonorous when struck upon; requires a strong fire to melt it, is the most divisible of all bodies; and its ductility is such, that wire-drawers can extend a leaf of gold to the 1200000th part of an inch in thinness, over a flatted silver wire, which will be perfectly covered, though viewed with a microscope; by which means an ounce of gold may be made to reach more than $155\frac{1}{2}$ miles.

Silver is the finest, purest, most ductile, and most precious metal, according to its natural properties, except gold.

Lead is the heaviest of all metals next to gold; it is the softest of all, is least sonorous, except gold, very ductile, and the most ready fusible of all, except tin.

Tin is a white shining metal, of so pliable a nature, that it may be bent into any form; its hardness is between silver and lead, and is the lightest of all metals.

Copper is an hard sonorous metal, difficult in fusion, and is mixed with gold and silver, in order to harden them, and render them more useful either in coin or utensils, which would otherwise be too soft and flexible.

Iron is the least heavy of all metals, except tin, but considerably the hardest of them all; fire renders it more ductile, being most of all maleable when hottest; when wrought into steel, is less maleable; it is more capable of rust than any other metal; it is very sonorous, and requires the strongest fire of all the metals to melt it.

In the comparison of the weights of bodies, it will be the most convenient to consider one body the standard or unit to which others are to be compared.

Rain-water is nearly alike in all places, a cubic foot of which hath, by repeated experiments, been found to weigh $62\frac{1}{2}$ pounds averdupoise.

*A TABLE shewing the specific gravity to rain-water, of
inch of each in parts of a pound averdupoise, taken from
of an ounce from Ward: the deficiencies in both authors*

Bodies.	sp. gra.	wt. lb. av.	wt. oz. tr.
Fine gold - - -	19.640	0.7103587	10.359273
Standard gold - - -	19.520	0.7060185	9.962625
Coast gold - - -	18.888	0.6828703	9.911707
Quicksilver - - -	13.762	0.4976574	7.384411
Lead - - - - -	11.313	0.4091696	5.984010
Fine silver - - -	11.091	0.4011501	5.850035
Standard silver - - -	10.629	0.3844400	5.556769
Cast silver - - -	10.528	0.3807870	5.503967
Copper - - - - -	8.769	0.3171658	4.747121
Plate brads - - -	8.350	0.2942593	4.404273
Cast brads - - -	8.104	0.2929832	4.272409
Steel - - - - -	7.850	0.2839265	4.142127
Bar iron - - - -	7.764	0.2808159	4.031361
Block tin - - - -	7.238	0.2617901	3.861519
Cast iron - - - -	7.135	0.2580647	3.806568
Loadstone - - - -	5.106	0.1846788	2.724083
Blue slate - - - -	3.500	0.1264914	1.867272
Veined marble - - -	2.702	0.0977286	1.429411
Common glass - - -	2.600	0.0940393	1.360841
Flint stone - - - -	2.582	0.0933883	1.351419
Portland stone - - -	2.570	0.0929543	1.345139
Freestone - - - -	2.352	0.0915788	1.231038
Brick - - - - -	2.000	0.0723379	1.046801
Alabaster - - - -	1.888	0.0683061	0.988456
Ivory } - - - - -	1.832	0.0662606	0.958489
Horn }			

*metals and other bodies; and the weight of a cubic
Robinson's Mensuration, and of ounces troy, and parts
supplied.*

Bodies.	sp. gra.	wt. lb. av.	wt. oz. tr.
Brimstone - - -	1.800	0.0651042	0.949424
Clay - - - - -	1.712	0.0619213	0.902498
Lignum vitæ - - -	1.327	0.0479862	0.699936
Coal - - - - -	1.255	0.0453921	0.661959
Pitch - - - - -	1.150	0.0415943	0.606576
Mahogany wood - -	1.063	0.0384475	0.560691
Dry box wood - - -	1.030	0.0372530	0.543282
Milk }			
Sea water } - - -	1.033	0.0372530	0.542742
Rain water - - -	1.000	0.0361690	0.527458
Red wine - - - -	0.993	0.0359158	0.523766
Bees wax - - - -	0.995	0.0359881	0.524820
Linseed oil - - -	0.932	0.0337095	0.491591
Proof spirits }			
or brandy } - - -	0.927	0.0335503	0.489268
Dry oak - - - - -	0.915	0.0330946	0.489008
Olive oil - - - -	0.913	0.0330222	0.481569
Beech - - - - -	0.854	0.0308883	0.450449
Dry elm }			
Dry ash } - - -	0.800	0.0289352	0.421966
Dry waincot - - -	0.747	0.0270182	0.394011
Dry yellow fir - -	0.657	0.0237630	0.346539
Cedar - - - - -	0.613	0.0221715	0.323332
Dry white deal - -	0.569	0.0205801	0.300123
Cork - - - - -	0.240	0.0186805	0.126590
Air - - - - -	0.0012	0.0000434	0.000633

When a heavy body is weighed in any fluid, it loses therein so much of its weight, as an equal bulk of that fluid is found to weigh: as for instance,

A cubic inch of lead = 5.284010 } ounces troy, &c.
A cubic inch of water = 0.542742 }

Their difference is = 5.441268, the weight of a cubic inch of lead in the water, &c.

1. An irregular piece of lead ore, taken from the Yorkshire pit, weighs in the scale just 12 ounces; but weighed in water loses 5 ounces of that weight; so that a quantity of water of the bigness of the ore weighs just 5 ounces: from the Derbyshire pit a rough fragment of ore weighs, out of water, $14\frac{1}{2}$ ounces; and in water 9 ounces: the comparative or the specific weight of these two ores is required?

$14\frac{1}{2} - 9 = 5\frac{1}{2}$ lb. weight of water of an equal bulk.

Then $14\frac{1}{2} \times 5 = 72\frac{1}{2}$ Derbyshire } ore's gravity. Q.

And $12 \times 5\frac{1}{2} = 66$ to Yorkshire } E. F.

2. An irregular fragment of glass in the scale weighs 171 grains; another of magnet 102 grains: in water the first fetches up no more than 120 grains, and the other 79: then 51 and 23 are the several weights of their comparative bulks of water: what then will their specific gravities turn out to be?

$171 \times 23 = 3933$ glass to }
 $102 \times 51 = 5202$ magnet } or as 437 to 578.

The solidity of any body, multiplied by the tabular weight corresponding, will give the weight in pounds averdupoise, or ounces troy.

3. What is the weight of a piece of oak, of a rectangular form, whose length is 56 inches, breadth 18, and depth 12 inches?

First, $56 \times 18 \sqrt{12} = 12096$ cubic inches.

Then $12096 \times .0330946 = 400.3122816$ lb. Q. E. F.

4. What is the diameter of an iron shot, weighing 42 pounds averdupoise?

First, $.2580647) 42.0000000 (162.7499.$

Then $.5236) 162.7499 (310.84778457.$

$\sqrt[3]{310.84778457} = 6.7743$, the diameter required.

5. What is the weight of an iron bombshell of three inches thick, the greatest diameter being 16 inches?

First, $16 - 6 = 10$, the diameter of the concavity.

Also $16 \times 16 \times 16 = 4096.$

And $4096 \times .5236 = 2144.6656,$

Again, $10 \times 10 \times 10 = 1000,$

Also

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Also $1000 \times .5236 = 523.6$.

Then $2144.6656 - 523.6 = 1621.0656$, the solidity of the shell.

$\therefore 1621.0656 \times .2580647 = 418.3398$ lb. the weight required.

6. In the walls of Balbeck, in Turkey, there are three stones laid end to end, now in sight, that measure in length 61 yards; one of which in particular is 63 feet long, 12 feet thick, and four yards over: now, if this block was marble, what power would balance it, so as to prepare it for moving?

Firstly, $63 \times 12 \times 12 = 9072$ solid feet.

Also $9072 \times 1728 = 15676416$ cubic inches.

Then $15676416 \times .0977286 = 1532034.1887$ lb.

$\therefore 2240$) 1532034 (683 tun, 18 cwt. 98 lb. Q. E. F.

7. Required the weight of one of the Portland key-stones to the middle arch of Westminster-bridge; the diameter of the arch being 76 feet; the height of the key-stone five feet; the chord of its greatest breadth to the front of the arch three feet four inches; and its depth in the arch four feet?

First, $76 + 5 = 81$; also 3 f. 4 in. = 3.3, greater breadth. As $81 : 3.3 :: 76 : 3.127572$, its least breadth.

Here the chords and their arches being nearly equal, viz. so small a part of so large a circle differs very little from a right line, the figure of the key-stone may be reckoned a prismoid, and measured accordingly;

viz. $3.3 \times 4 = 13.3$; also $3.127572 \times 4 = 12.510288$.

Then $\frac{13.3 + 12.510288}{2} = 12.9218105$.

Also $12.92181 + 12.510288 + 13.3 = 38.76543$.

And $38.76543 \times \frac{1}{3} = 64.60905$ solid feet.

Then $64.60905 \times 1728 = 111644.4384$ cubic inches.

$\therefore 111644.4384 \times .0929543 = 10377.83$ lb.

Answer, 10377.83 lb. = 4 ton, 12 cwt. 2 qrs. 17.83 lb.

The weight of any body in pounds averdupoise, or ounces troy, being divided by the tabular weight corresponding, the quotient will be the solidity in cubic inches.

8. What

8. What will a block of marble, weighing 8 tons, 14 cwt. come to, at 6 s. a foot solid?

$$\begin{aligned} 8 \text{ ton, } 14 \text{ cwt.} &= 19488 \text{ lb.} \\ .0977286) 19488.000000 & (199409.4 \text{ inches.} \\ 1728) 199409.4 & (115.4 \text{ cubic feet.} \\ 115.4 \times .3 &= 34.621. = 34 \text{ l. } 12 \text{ s. } 4 \frac{1}{2} \text{ d.} \end{aligned}$$

The absolute weight of a body floating in a fluid, is precisely equal to the weight of such part of the fluid as shall be thrust away thereby, and displaced; or, in other words, to the immersed part of the body.

9. Suppose that a man of war, with all its ordnance, rigging, and appointments, draws so much water as to displace 1300 tons of sea-water, London beer measure; the weight of this vessel is required?

$$\begin{aligned} \text{First, } 1300 \times 4 &= 5200 \text{ hogheads.} \\ \text{Also } 5200 \times 15228 &= 79185600 \text{ cubic inches.} \\ \text{And } 79185600 \times .037253 &= 2949901 \text{ lb. averdupoise.} \\ \text{Answer, } 2949901 \text{ lb.} &= 26338 \text{ cwt. } 1 \text{ qr. } 17 \text{ lb.} \end{aligned}$$

10. How many inches will a cubic foot of dry elm sink in common water?

$$\begin{aligned} 1728 \times .0289352 &= 50.0000256 \text{ lb. is the weight of a} \\ &\text{foot of elm, or of the water displaced.} \\ 0.36169) 50.0000256 & (1382.4 \text{ cubic inches immersed.} \\ \therefore 144) 1382.4 & (96 \text{ inches, the answer.} \end{aligned}$$

11. Suppose a seaman hath a gallon of brandy in a glass bottle, that weighs $3\frac{1}{2}$ lb. troy on board; and to conceal it from the king's officers, throws it into the sea; if it will sink, how much force will just buoy it up?

$$\begin{aligned} \text{First, } 3\frac{1}{2} \text{ lb. troy} &= 42 \text{ ounces.} \\ \text{Also } 1.360841) 42.000000 & (30.864 \text{ cubic inches.} \\ \text{Then } 231 \times .489268 &= 113.020908 \text{ ounces brandy.} \\ \text{And } 42 + 113.020908 &= 155.020908 \text{ ounces in all.} \\ \text{Again, } 231 + 30.864 &= 261.864 \text{ inches of water.} \\ \text{Also } 261.864 \times .542742 &= 142.12462 \text{ ounces of water.} \\ \therefore 155.020908 - 142.12462 &= 12.896288 \text{ ounces heavier} \\ &\text{than the same bulk of salt water.} \end{aligned}$$

12. Another of the mariners has half an anchor of brandy; the cask suppose measures $\frac{1}{8}$ of a cubic foot; what

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what quantity of lead is just requisite to keep the cask and liquor under water ?

First, $8 \times 1728 = 13824$ cubic inches, the cask.

Also $231 \times 5 = 1155$ cubic inches of brandy.

Then $216 \times .489008 = 105.625728$ oz. wt. of the cask.

Also $1155 \times .489268 = 565.104540$, weight of the brandy.

Again, $216 + 1155 = 1371$ cubic inches.

Then $1371 \times .542742 = 744.099282$, weight of water of an equal bulk.

Also $105.625728 + 565.10454 = 670.730268$.

And $744.099282 - 670.730268 = 73.268914$.

Also one inch of lead $5.98401 - .542742 = 5.441268$, weight of one inch of lead in water.

\therefore Recip. $5.98401 : 73.268914 :: 5.441268 : 80\frac{1}{2}$ ounces troy of lead to keep the cask, with its contents, just under water. Q. E. F.

13. How thick must be the metal of a concave copper ball, six inches in its outside diameter, so as to sink to its center in common water ?

First, $6 \times 6 \times 6 = 216$, cube of the diameter.

Also $216 \times .5236 = 113.0976$ cubic inches, the solidity of the sphere.

2) 113.0976 (56.5488 cubic inches to be immersed, or of water to be removed.

$\therefore 56.5488 \times .036169 = 2.0453$ lb. weight of the copper ball.

And $.3171658 \times 2.0453000 = 6.448678$ cubic inches of copper in the ball.

Again, $6 \times 6 \times 3.1416 = 113.0976$ square inches, superficies of the ball. N. B. The solidity and superficies of this ball are equal.

$\therefore 113.0976 \times 6.44870 = .057$, or about $\frac{1}{17}$ of an inch in thickness. Q. E. F.

14. What will a chain of standard gold weigh in water, that raises a fluid an inch in a vessel three inches square, when put into it ? And supposing the workman had adulterated the said chain with $14\frac{1}{2}$ ounces of silver ; how much higher would the water, upon its immersion being raised in the vessel ?

First, $3 \times 3 = 9$ solid inches in the gold chain.

Then

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Then $9 \times 9.962625 = 89.663625$, its weight in air.

And $9 \times 0.527458 = 4.747122$, wt. of its bulk of water.

Weight of the gold 84.916503 in water. Q. E. F.

A solid inch of silver is 5.556769 ounces troy.

As $5.556769 : 1 :: 14.5 : 2.6094$ inches of silver.

Then $89.663625 - 14.5 = 75.163625$ ounces of gold.

9.962625 75.163625 7.5546 , space taken up by the gold.

2.6094 , by the silver, as above.

Sum 10.164 , by both.

Then $10.164 - 9 = 1.164$.

$\therefore 9$ 1.164 $(.1292)$. Q. E. F.

15. Hiero, king of Sicily, ordered his jeweller to make him a crown, containing 63 ounces of gold; the workman thought substituting part silver therein a proper perquisite; which taking air, Archimedes was appointed to examine it, who, on putting it into a vessel of water, found it raised the fluid, or that itself contained 8.2245 cubic inches of metal; and having discovered that the cubic inch of gold more critically weighed 10.36 ounces, and that of silver but 5.85 ounces; he, by calculation, found what part of his majesty's gold had been changed; and you are desired to repeat the process?

10.36 63.00 $(6.08108$ inches in solidity, had it been gold.

5.85 63.00 $(10.76923$ solid inches, if all silver.

Then, by alligation, $8.2245 \left\{ \begin{array}{l} 6.08108 \\ 10.76923 \end{array} \right\} \left\{ \begin{array}{l} 2.54473 \\ 2.14342 \end{array} \right.$

4.68815

4.68815 2.54473 $(.5428$, part gold.

4.68815 2.14342 $(.4572$, part silver.

$\therefore .5428 \times 63 = 34.1884 = 34$ oz. 3 dwt. $22\frac{1}{2}$ gr. of gold.

And $.4572 \times 63 = 28.8036 = 28$ 16 $1\frac{1}{2}$ of silver.

Q. E. F.

Since gold and silver are always weighed, bought and sold, by troy weight, which weights are seldom in the possession of gentlemen in the country, graziers, or farmers; therefore to prevent their being imposed on by Jews, and other itinerant traders, I shall give an example concerning the reduction of troy into averdupoise weight.

In

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In the year 1696 an experiment was made by authority (by a balance which would turn with six grains put into either scale) when it was found, that 15 pounds averdupoise weight, were equal to 18lb. 2 ozs. 15 dwts. troy = 105000 grains.

∴ 15) 105000 (7000 grains troy = 16 oz. averdupoise.

Also 16) 7000 ($437\frac{1}{2}$ grains troy = 1 oz. averdupoise.

And 480) 437.5 ($.911458\bar{3}$ oz. troy = 1 oz. averdupoise.

First, suppose a silver tankard weighs 2 lb. 11 oz. 8 dr. averdupoise, its weight in ounces troy is required?

First, 2 lb. 11 oz. 8 dr. = 43.5 ounces.

And $43.5 \times .911458\bar{3} = 39.6484375 = 39 \text{ oz. } 12 \text{ pwt. } 23\frac{1}{4} \text{ gr.}$ by the experiment above.



CHAPTER III.

POSITION; OR, THE RULE OF FALSE.

THE rule of position, or supposition, is so called, because we suppose some uncertain number, in order, that by reasoning from them we may gain the true number; and because those suppositions are taken at adventure, it is also called the rule of false.



SECT. I.

SINGLE POSITION.

BY single position are solved such questions as require only one supposition to discover the true result.

RULE.

When you have made choice of your position, work it according to the nature of the question as if it were the true number; and if by the ordering your position you find the result either too much or too little, you may then find out the number sought by this proportion;

I

viz.

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viz. as the result of your position is to : the position,
so is :: the given number to : the number sought.

1. Three persons, viz. A, B, C, thus discourse together concerning their age ; says B to A, I am as old and half as old again as you ; then says C to B, but I am twice as old as you ; A replied, I am sure the sum of all our ages is 165 : now I demand each man's age ?

Suppose $24 = A$.

Then will $24 + 12 = 36 = B$.

And $36 \times 2 = 72 = C$.

$\therefore 132 : 24 :: 165 : 30, A's$ } age. Q.
 $\text{Also } 45, B's$ }
 $\text{And } 90, C's$ } E. F.

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2. Three persons, Andrew, Benjamin and Charles, are to go a journey of 235 miles ; of this journey Andrew is to go a certain number of miles unknown ; Benjamin is to go four times as many miles as Andrew, and three miles more ; and Charles is to go twice as many miles as Benjamin, and five miles more : how many miles must each of these persons travel severally ?

miles.

Suppose - - - 10, Andrew.

Then $10 \times 4 + 3 = 43$, Benjamin.

Again, $43 \times 2 + 5 = 91$, Charles.

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Also $3 + 3 \times 2 + 5 = 14$

130 and $235 - 14 = 221$.

$\therefore 130 : 10 :: 221 : 17$ }
 $\text{Also } 17 \times 4 - 3 = 71$ } miles { Andrew.
 $\text{And } 71 \times 2 + 5 = 147$ } Benjamin.
Charles.

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3. There were in company together four persons, Adam, Edward, Charles and William ; Adam told Charles that he was older than him by two years ; Edward told them, that he was as old as both of them together, and four years older ; William, hearing them, said, I am just 96 years old, and that

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that is equal to all your ages: how old was each of them severally?

Suppose Charles 20

Then Adam be 22

And Edward - 46

Then $2 + 2 + 4 = 8$.

$$\begin{array}{r} 88 \\ 8 \\ \hline 80 \end{array}$$

And $96 - 8 = 88$

$\therefore 80 : 20 :: 88 : 22 = \text{Charles's}$

Also $22 + 2 = 24 = \text{Adam's}$

And $22 + 24 + 4 = 50 = \text{Edward's}$

Their sum $= 96 = \text{William's}$

4. The captain, lieutenant and cornet of a troop have taken among them from some enemy 478 crowns, which they agree to share in this manner; the captain is to have 24 times as much as the cornet, wanting only seven crowns; and the lieutenant is to have five times as much as the cornet, wanting three crowns; what is each officer's share?

Suppose the cornet to have 8

Then $8 \times 5 - 3 = 37$

And $8 \times 24 - 7 = 185$

Again, $7 + 3 = 10$

Also $478 + 10 = 488$

And $230 + 10 = 240$

$$\begin{array}{r} 230 \\ \hline \end{array}$$

$\therefore 240 : 8 :: 488 : 16\frac{4}{3} = \text{cornet's}$

Also $16\frac{4}{3} \times 5 - 3 = 78\frac{4}{3} = \text{lieutenant's}$

Lastly, $16\frac{4}{3} \times 24 - 7 = 383\frac{6}{3} = \text{captain's}$

} share. Q.
E. F.

5. Let 273 l. be divided amongst four persons, viz. Andrew, Bennet, Christopher and Daniel; Andrew is to have a share unknown; Bennet is to have twice as much as Andrew, and 30 l. more; Christopher is to have three times as much as Andrew, wanting 52 l. and Daniel is to have five times as much as Andrew, and 20 l. more; how must this 273 l. be divided amongst them, so that every one may have his true share?

1.

Suppose - - - 20 A

Then must $20 \times 2 + 30 = 70$ B

Likewise $20 \times 3 - 52 = 8$ C

And $20 \times 5 + 20 = 120$ D

Again, $30 + 20 - 52 = 2$

Also $273 + 2 = 275$

Likew. $218 + 2 = 220$

$$\begin{array}{r} 218 \\ \hline \end{array}$$

$220 : 20 :: 275 : 25$, Andrew's
 Then will $25 \times 2 + 30 = 80$, Bennet's
 Likewise $25 \times 3 - 52 = 23$, Christopher's
 And $25 \times 5 + 20 = 145$, Daniel's

} share.

£ 273

6. Admit three merchants build a ship, which cost 1360l. A pays a certain part unknown; B paid $2\frac{1}{2}$ as much, wanting 15.5 l. and C paid as much as both A and B, and 75.25l. over; how much did each man pay?

1.

Suppose A paid 100
 $B \ 100 \times 2.5 - 15.5 = 234.5$
 $C \ 334.5 + 75.25 = 409.75$

$75.25 - 31 = 44.25$
 $1360 - 44.25 = 1315.75$

$744.25 - 44.25 = 700$

l. s. d.

$\therefore 700 : 100 :: 1315.75 : 187.96428\theta = 187 \ 19 \ 3\frac{1}{2}$, A
 Then

$187.96428\theta \times 2.5 - 15.5 = 454.41071\theta = 454 \ 8 \ 2\frac{1}{2}$, B
 And $642.375 + 75.25 = 717.625 = 717 \ 12 \ 6$, C

} paid.

1360.

7. As I walk'd forth to take the air,
 The heavens and nature smiling were;
 A grave old shepherd there I 'spy'd,
 Close by a chrystal fountain's side.
 Unto this shepherd I did say,
 How many sheep have you I pray?
 But he reply'd, add to one half of these,
 One fourth, $\frac{1}{4}$, $\frac{1}{8}$, and, if you please,
 One tenth, $\frac{1}{10}$, and $\frac{1}{10}$ too;
 These being made one sum by you,
 Exactly to my age will be,
 In this proportion, as 15 to 3.
 What is your age, good Sir? said I.
 To which the shepherd made reply;
 One-half, one-fourth, one-fifth do take,
 One-tenth, one-twentieth, they will make;
 If added, five score and ten more,
 And now my age, Sir, pray explore?

And

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And now methinks his age I'd know,
Which I must beg of you to show;
Likewise the number of the sheep,
Which this crabb'd shepherd there did keep?

Suppose $20 =$ shepherd's age.

Then $\frac{1}{2} = 10$, $\frac{1}{4} = 5$, $\frac{1}{3} = 4$, $\frac{1}{10} = 2$, and $\frac{1}{20} = 1$.

Also $10 + 5 + 4 + 2 + 1 = 22$.

As $22 : 20 :: 110 : 100 =$ shepherd's age.

Again, suppose $40 =$ number of sheep.

Then will $\frac{1}{2} = 20$, $\frac{1}{4} = 10$, $\frac{1}{3} = 8$, $\frac{1}{8} = 5$, $\frac{1}{10} = 4$,
 $\frac{1}{20} = 2$, and $\frac{1}{40} = 1$.

Also $20 + 10 + 8 + 5 + 4 + 2 + 1 = 50$.

And $3 : 15 :: 100 : 500$, per question.

$\therefore 50 : 40 :: 500 : 400$, the number of sheep. Q. E. F.



S E C T. II.

D O U B L E P O S I T I O N.

IN double position two suppositions are used; and if we miss in both, observe the nature of the errors whether they be greater or less, and accordingly mark them with the signs $+$ or $-$; then,

R U L E,

As the difference of the errors, if alike, or their sum, if unlike : is to the difference of the suppositions :: so is either of the errors : to a fourth number; which added to, or subtracted from, its proper supposition, gives the number sought.

1. A young gentleman walking in a garden, and meeting with a bevy of young ladies, began thus to address them : Bless you all 10 fair ladies ! Sir, replies one, you are mistaken, we are not 10 ; but if we were twice as many more as we are, we should be as many above 10, as we are now below : what was their number?

Suppose 4, then $4 \times 2 + 4 = 12$.

Now as 4 is 6 less than 10, and 12 but 2 above 10 ;

$\therefore 2 - 6 = -4$, the first error.

I i

Again,

Again, suppose 7, then $7 \times 2 + 7 = 21$.

As 7 is 3 less than 10, and 21 greater by 11;

$\therefore 11 - 3 = +8$, the second error.

Then $4 + 8 = 12$, sum of the errors.

And $7 - 4 = 3$, difference of the suppositions.

Also $12 : 3 :: 4 : 1$. $\therefore 4 + 1 = 5$. Q. E. F.

Or $12 : 3 :: 8 : 2$; and $7 - 2 = 5$, the answer, as above.

2. A gentleman hath two horses of good value, and a saddle worth 50l. which set on the back of the first horse made his value double that of the second; but if set on the back of the second horse, makes his worth triple that of the first horse: I demand the value of each horse?

Suppose the first horse to be worth 24l.

Then $24 + 50 = 74$; also $\frac{74}{2} = 37$ l. value of the second.

And $37 + 50 = 87$; but $24 \times 3 = 72$, less than 87 by 15.

So that $-15 =$ first error.

Again, suppose the first horse to be worth 34l.

Then $34 + 50 = 84$; also $\frac{84}{2} = 42$, value of the second.

And $42 + 50 = 92$; also $34 \times 3 = 102$, more than 92 by 10.

Hence $+10 =$ second error.

Then $10 + 15 = 25 =$ sum of the errors.

And $34 - 24 = 10 =$ difference of the suppositions.

$25 : 10 :: 15 : 6$. $\therefore 24 + 6 = 30$. Q. E. F.

Or $25 : 10 :: 10 : 4$. $\therefore 34 - 4 = 30$, as above.

3. A lady bought tabby at four shillings a yard, and Persian at two shillings; the whole number of yards she bought were eight, and the whole price 20 shillings; how many yards had she of each sort?

s.

Suppose four yards of tabby at 16

Then must she have four of Persian at 8

Sum of their values 24

So that the first error is $+4$.

s.

Again, suppose she had three yards of tabby, value 12

Then must she have five of Persian, value - - 10

The sum of their values = 22 s.

So

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So that the second error is $+ 2$.

Then $4 - 3 = 1$, difference of the suppositions.

Also $4 - 2 = 2$, difference of the errors.

As $2 : 1 :: 4 : 2$. $\therefore 4 - 2 = 2$ yards of tabby.

And $8 - 2 = 6$ yards of Persian.

For two yards of tabby, at $4 = 8$

And six yards of Persian, at $2 = 12$

Sum 20, as was required.

4. A and B having a certain number of crowns, says B to A, give me one of your crowns, and I shall have as many as you; but says A to B, give me one of your crowns, and I shall have twice as many as you; how many had each?

Suppose A to have 5,

And B - - - - 3.

Then $3 - 1 = 2$; and $5 + 1 = 6$.

The first error being $- 2$.

Again, suppose A to have 9,

And B - - - - 7.

Then $7 - 1 = 6$; and $9 + 1 = 10$.

The last error being $+ 2$.

Then $2 + 2 = 4$, sum of the errors.

And $9 - 5 = 4$, difference of the suppositions.

As $4 : 4 :: 5 : 5$. $\therefore 5 + 2 = 7$.

Also $4 : 4 :: 9 : 9$. $\therefore 9 - 2 = 7$.

$\frac{7 + 1}{2} = 4$; and $4 + 1 = 5$, B's crowns.

For $5 + 1 = 7 - 1$; and $7 + 1 = 5 - 1 \times 2$, as was required.

5. There is a certain fish whose head is nine inches long, the tail is as long as the head, and half the body, and the body is as long as both the head and tail; I demand the whole length of the said fish?

Suppose the body be 20 inches.

Then $\frac{20}{2} + 9 = 19$, tail.

Also $19 + 9 = 28 - 20 = 8$.

So that the first error is $- 8$.

Again, suppose the body 24 inches.

$$\text{Then } \frac{24}{2} + 9 = 21, \text{ tail.}$$

$$\text{Also } 21 + 9 = 30; \text{ and } 30 - 24 = 6.$$

So that the second error is — 6.

$$\text{Then } 8 - 6 = 2, \text{ difference of the errors.}$$

$$\text{And } 24 - 20 = 4, \text{ difference of the suppositions.}$$

$$\text{Also } 2 : 4 :: 8 : 16. \quad \therefore 16 + 20 = 36 \quad \left. \vphantom{\begin{matrix} 16 \\ 20 \end{matrix}} \right\} \text{body.}$$

$$\text{Or } 2 : 4 :: 5 : 12. \quad \therefore 12 + 24 = 36$$

$$\text{Likewise } \frac{36}{2} + 9 = 27, \text{ tail.}$$

$$\text{And } 36 + 27 + 9 = 72 \text{ inches. } Q. E. F.$$

6. When first the marriage knot was ty'd,
 Betwixt my wife and me,
 My age did her's as far exceed,
 As three times three doth three :
 But after ten and half ten years
 We man and wife had been,
 Her age came up as near to mine,
 As eight is to sixteen :
 Now tell me (you who can) I pray,
 What were our ages on the wedding-day ?

Suppose the wife's age 12 years.

Then must the husband's be 36.

$$\text{And 15 years after } \left\{ \begin{array}{l} \text{wife} - 27. \\ \text{husband } 51. \end{array} \right.$$

Twice her age greater than his by 3.

\therefore The first error is + 3.

Again, suppose the wife's age 18 years.

Then must the husband's be 54.

$$\text{Also 15 years after } \left\{ \begin{array}{l} \text{wife's} - 33. \\ \text{husband's } 69. \end{array} \right.$$

And the second error — 3.

Proceeding, $3 + 3 = 6$, sum of the errors.

And $18 - 12 = 6$, difference of the suppositions.

$$\therefore 6 : 6 :: 12 : 12; \text{ also } 12 + 3 = 15 \quad \left. \vphantom{\begin{matrix} 12 \\ 3 \end{matrix}} \right\} \text{the wife's age.}$$

$$\text{Or } 6 : 6 :: 18 : 18; \text{ and } 18 - 3 = 15$$

7. A man that was idle, and minded to spend
 Both money and time, went to drink with his friend;
 He said to his host, if you'll now to me lend
 As much coin as I have, then my sixpence I'll spend.

His

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His host lent the money, his sixpence he spent,
 And, having so done, to another house went,
 Where the same he requested, and the same sum he spent. }
 He went to a third house, where, Landlord, cries he,
 Lend me as much money as I've left here you see;
 Which having receiv'd, his sixpence he spent,
 So, all being gone, home the fuddle-cap went,
 To cast up his reck'nings; but his head aching fore,
 He begs you to do't, and he'll do so no more; }
 What had he at first, and how much on score?

Suppose he had 8 d.

$$\text{Then } 8 + 8 = 16$$

$$\text{Also } 16 - 6 = 10$$

$$10 + 10 = 20$$

$$20 - 6 = 14$$

$$14 + 14 = 28$$

$$\text{And } 28 - 6 = 22, \text{ which should be nothing.}$$

∴ The first error is 22.

Again, suppose he had 7 d.

$$\text{Then } 7 + 7 = 14$$

$$14 - 6 = 8$$

$$8 + 8 = 16$$

$$16 - 6 = 10$$

$$10 + 10 = 20$$

$$\text{And } 20 - 6 = 14$$

∴ The second error is 14.

Then $22 - 14 = 8$, difference of the errors.

And $8 - 7 = 1$, difference of the suppositions.

$$\text{As } 8 : 1 :: 22 : 2\frac{1}{2}. \quad \dots 8 - 2\frac{1}{2} = 5\frac{1}{2} \text{ d. } \left. \begin{array}{l} \text{As } 8 : 1 :: 22 : 2\frac{1}{2}. \\ \text{Or } 8 : 1 :: 14 : 1\frac{3}{4}. \end{array} \right\} \text{Q. E. F.}$$

$$\text{Or } 8 : 1 :: 14 : 1\frac{3}{4}. \quad \dots 7 - 1\frac{3}{4} = 5\frac{1}{4} \text{ d.}$$

R U L E II.

Proceed as directed in the first rule, till you have found the errors and their signs; then

Multiply alternately the first supposition by the second error, and the second supposition by the first error; and divide the sum of the products by the sum of the errors, when the errors are of different kinds; or the difference of the products by the difference of the errors, when the errors are of the same kind, and the quotient is the number sought.

8. A person finding several beggars at his door, gave each of them three pence a-piece, and had five pence remaining:

I i 3

he

he would have given them four pence a-piece, but he wanted seven pence to do it; how many beggars were there?

Suppose 15 beggars. 15

3	4
<hr/>	<hr/>
45	60
+ 5	- 7
<hr/>	<hr/>
50	53
<hr/>	<hr/>

Then $53 - 50 = + 3$, the first error.

Again, suppose 13 beggars. 13

3	4
<hr/>	<hr/>
39	52
+ 5	- 7
<hr/>	<hr/>
44	45
<hr/>	<hr/>

Also $45 - 44 = + 1$, the second error.

Then $13 \times 3 = 39$

Also $15 \times 1 = 15$

And $3 - 1 = 2$ 24 (12 beggars. Q. E. F.

9. A labourer agreed to thrash 60 bushels of corn, part of it wheat, and part oats, at the rate of 2 d. per bushel for the wheat, and $1\frac{1}{2}$ d. for the oats; at last he received 8 s. for his labour; how much of each did he thrash?

First, I suppose 30 bushels of wheat, price	d.	
Then must there also be 30 bushels of oats	- -	60
		<hr/>
		105

Which should be but	- - - - -	96
		<hr/>

Therefore the first error is	- - - - -	+ 9
		<hr/>

Again, I suppose 18 bushels of wheat, price	d.	
Then also must there be 42 bushels of oats	- -	36
		<hr/>
		63

	- -	99
		<hr/>
		96

Then will the second error be	- - - - -	+ 3
		<hr/>
		Also

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$$\text{Also } 18 \times 9 = 162$$

$$\text{And } 30 \times 3 = 90$$

$\therefore 9 - 3 = 6$ 72 (12 bushels of wheat, at 2 d. 2 s.
Then will there be 48 bushels of oats, at $1\frac{1}{2}$ d. is 6 s.

$$\text{Sum } 60 \text{ bushels} - - - - - 8 \text{ s.}$$

10. Two merchants, A and B, lay out an equal sum of money in trade; A gains 126 l. and B loses 87 l. and A's money is now double to B's; what did each lay out?

Suppose each lays out	220	-	-	-	220
	126				87
	<hr/>				<hr/>
	346				133
	266				$\times 2$
	<hr/>				<hr/>
First error +	80				266

Again, suppose	350	-	-	-	350
	126				87
	<hr/>				<hr/>
	476				263

$$263 \times 2 = 526 - 476 = - 50, \text{ the second error.}$$

$$\begin{array}{l} 350 \times 80 = 28000 \\ 220 \times 50 = 11000 \end{array} \left. \vphantom{\begin{array}{l} 350 \\ 220 \end{array}} \right\} 39000$$

$$\therefore 80 + 50 = 130 \} 39000 (300 \text{ l. Q. E. F.}$$

The following rules and examples I had from the ingenious Mr. Emerson's Arithmetic, page 146, &c.

R U L E III.

" Proceed as directed in the first rule, till you have found the errors and their signs; then,

I. Multiply the difference of the supposed numbers by the least error, and divide the product by the difference of the errors, if they are alike; or by the sum, if unlike: the quotient is the correction of the number belonging to the least error.

II. Observe whether this be the lesser or greater number, as also whether the errors have like or unlike signs.

III. If it is the less number, and like signs, subtract the correction; if unlike signs, add it.

IV. If the greater number, and like signs, add the correction; if unlike signs, subtract it: so you'll have the true number required.

11. A certain man being asked what was the age of his four sons, answered, that his eldest was four years older than the second, and the second five years older than the third, and the third six years older than the fourth, which was half the age of the eldest; how old was each?

Suppose $\begin{cases} 16 \text{ eldest.} \\ 12 \text{ second.} \\ 7 \text{ third.} \\ 1 \text{ youngest.} \end{cases}$

Again, $\begin{cases} 24 \text{ eldest.} \\ 20 \text{ second.} \\ 15 \text{ third.} \\ 9 \text{ youngest.} \end{cases}$

$\frac{1}{2}$ eldest = $8 - 1 = -7$, first error.
 $\frac{1}{2}$ eldest = $12 - 9 = -3$, second error.
 Then $24 - 16 = 8$, difference of suppositions.

Also $8 \times 3 = 24$.

And $7 - 3 = 4$, difference of the errors.

$\frac{1}{4}$ 24 (6, the correction.

$\therefore 24 + 6 = 30$, the age of the eldest.

Also 26, second.

21, third.

And 15, youngest. Q. E. F.

12. There is a crown weighing 60lb. which is made of gold, brass, tin, and iron; the weight of the gold and brass together is 40lb. of the gold and tin 45 lb. of the gold and iron 36 lb. Quere, how much gold was in it?

Suppose	32 lb. of gold	- - - - -	28
	8 brass	- - - - -	12
	13 tin	- - - - -	17
	4 iron	- - - - -	8
	<hr/>		<hr/>
	57		65
	<hr/>		<hr/>

Then $60 - 57 = -3$, the first error.

And $65 - 60 = +5$, the second error.

Also $5 + 3 = 8$, sum of the errors.

Likewise $32 - 28 = 4$, difference of the suppositions.

Again, $4 \times 3 = 12$; and $\frac{12}{8} = 1\frac{1}{2}$, the correction.

$\therefore 32 - 1\frac{1}{2} = 30\frac{1}{2}$, the quantity of gold. Q. E. F.

13. Three

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13. Three companies of soldiers passing by a shepherd, the first takes half his flock, and half a sheep; the second takes half the remainder, and half a sheep; the third takes half the remainder, and half a sheep; after which the shepherd had 20 sheep remaining; how many had he at first?

$$\begin{array}{rcl}
 \text{Suppose } 60 & - & - & - & - & - & - & 80 \\
 \text{First took } 30.5 & - & - & - & - & - & - & 40.5 \\
 \hline
 & 29.5 & - & - & - & - & - & 39.5 \\
 \text{The second took } 15.25 & - & - & - & - & - & - & 20.25 \\
 \hline
 & 14.25 & - & - & - & - & - & 19.25 \\
 \text{The third took } 7.625 & - & - & - & - & - & - & 10.125 \\
 \hline
 & 6.625 & - & - & - & - & - & 9.125 \\
 \text{Then } 20 - 6.625 = 13.375, \text{ first} & & & & & & & \} \text{ error.} \\
 \text{Also } 20 - 9.125 = 10.875 = \text{second} & & & & & & & \\
 \text{Again, } 80 - 60 = 20, \text{ difference of the suppositions.} & & & & & & & \\
 \text{Also } 13.375 - 10.875 = 2.5, \text{ difference of the errors.} & & & & & & & \\
 20 \times 10.875 = 217.5. & & & & & & & \\
 2.5 \mid 217.5 (87, \text{ correction.} & & & & & & & \\
 \therefore 80 + 87 = 167 \text{ sheep. } Q. E. F. & & & & & & &
 \end{array}$$

SCHOLIUM.

By supposing one of the numbers 0, and the other 1, the work is sometimes shortened.

14. A factor delivers six French crowns, and four dollars, for 2 l. 13 s. 6 d. and at another time four French crowns, and six dollars, for 2 l. 9 s. 10 d. what was the value of each?

$$\begin{array}{l}
 \text{Suppose } 0 = \text{value of a French crown.} \\
 \text{Then will 4 dollars} = 53.5 \text{ s.} \\
 \text{Also } 4 \mid 53\frac{1}{2} (13\frac{3}{4}. \\
 \text{And 6 crowns} + 6 \text{ dollars} = 80\frac{1}{4}. \\
 \text{Then } 80\frac{1}{4} - 49\frac{1}{2} = + 30\frac{1}{4}, \text{ first error.} \\
 \text{Again, suppose 1 s. = value of a French crown.} \\
 \text{Then 6 crowns and 4 dollars} = 53\frac{1}{2}. \\
 \text{Also } 53\frac{1}{2} - 6 = 47\frac{1}{2} = 4 \text{ dollars.} \\
 4 \mid 47\frac{1}{2} (11\frac{3}{4}, \text{ value of a dollar.} \\
 \text{And 4 crowns} + 6 \text{ dollars} = 75\frac{1}{2}. \\
 \text{Then } 75\frac{1}{2} - 49\frac{1}{2} = + 25\frac{1}{2}, \text{ second error.} \\
 \text{Also } 30\frac{1}{4} - 25\frac{1}{2} = 5, \text{ difference of the errors.} \\
 \therefore 5 \mid 30\frac{1}{4} (6\frac{1}{4} = 6 \text{ s. } 1 \text{ d.} = \text{value of a crown. } Q. E. F. \\
 \text{And}
 \end{array}$$

And 6 crowns = 36 s. 6 d.

53 s. 6 d. — 36 s. 6 d. = 17 s. value of 4 dollars.

∴ 4) 17 (4 s. 3 d. = value of a dollar. Q. E. F.

In this rule it is generally presumed, that the first error is to the second, as the difference between the true and first supposed number is to the difference between the true and second supposed number. When this does not happen, the rule of false does not give the exact answer, except the two supposed numbers be taken very near the true one.

The errors are the difference between the true result, and each of the false results; so that if the errors are unlike, the true number lies between the supposed numbers; but if alike, the true number lies without both of the supposed ones.

A great many questions may be resolved by this rule, which cannot be resolved by any other rule in arithmetic; but there are many questions where it cannot be certainly known, whether they can be resolved by it or not, till they be tried."



CHAPTER IV. CONCERNING DIVISORS.

IT being often necessary, in arithmetical calculations, to find such multipliers, or numbers, which may be divided by any number of given divisors without any remainder, or to leave any assigned remainder, or remainders; by which means many pleasant questions, not reducible to any of the foregoing rules, may be solved.

First find the least number that can be divided by any number of given divisors without a remainder.

RULE.

Multiply all the prime numbers, and the roots of such as are square or cube numbers, continually; the product will be the least number required.

1. Shew me how to find what's the least number
That you can divide without a remainder,

By

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By giving divisors, as the digits nine,
For a true canon I'd give a pint o' wine?

Ladies Diary, 1719.

Divisors 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9.

But as $\sqrt{4} = 2$, that 6 may be cancelled, being composed of 2×3 ; $\sqrt[3]{8} = 2$; and $\sqrt{9} = 3$.

$\therefore 1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 7 \times 2 \times 3 = 2520$. Q.
E. F.

2. What particular least whole number is that, which being divided by 2, 3, 4, 5, 6, 7, 8, 9, shall leave a remainder of 1, 2, 3, 4, 5, 6, 7, 8, respectively?

It is plain by the question above, that 2520 is the least number that can be divided by nine digits, without a remainder. $\therefore 2520 - 1 = 2519$, the number required.

3. A country girl to town did go,
Some walnuts for to sell,
A gentleman she chanc'd to meet,
And thus it her befel:
My pretty maid, says he to her,
What number have you here?
I can't tell, Sir, says she to him,
But this I'll make appear;
I told them o'er ere I came out,
By fix's, five's, four's, three's, two's,
And, ev'ry time I number'd them,
One remain'd overplus;
I told them o'er by seven's at last,
And there were no remains;
If you can find the number out,
Pray take it for your pains.

First, the least number that can be divided by 1, 2, 3, 4, 5, 6, without a remainder; viz. $1 \times 2 \times 3 \times 2 \times 5 = 60$.
Then $60 + 1 = 61$, will leave 1, when divided by each number; but $7 \nmid 61$ (8, and 5 remains.

Also $\left. \begin{array}{l} 60 \times 2 + 1 = 121 \\ 60 \times 3 + 1 = 181 \\ 60 \times 4 + 1 = 241 \end{array} \right\} \begin{array}{l} \text{none of which are divisible by} \\ 7 \text{ without a remainder.} \end{array}$

But $60 \times 5 + 1 = 301$, is the least number which admits of the conditions of the question.

Then

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Then to find the next least number which admits of the same conditions; viz. $60 \times 6 + 1 = 361$

$$\begin{array}{l} 60 \times 7 + 1 = 421 \\ 60 \times 8 + 1 = 481 \\ 60 \times 9 + 1 = 541 \\ 60 \times 10 + 1 = 601 \\ 60 \times 11 + 1 = 661 \end{array} \left. \begin{array}{l} \text{none of which} \\ \text{are divisible} \\ \text{by 7, without} \\ \text{a remainder.} \end{array} \right\}$$

But $60 \times 12 + 1 = 721$, is the next number admitting the conditions aforesaid.

Also $721 - 301 = 420$, the common difference of all numbers answering the same conditions.

$\therefore 301, 721, 1141, 1561, 1981, 2401, 2821, \&c. ad infinitum$, will answer the conditions of this question.

4. To find the least number of guineas, which being divided by 6, 5, 4, 3 and 2 respectively, shall leave 5, 4, 3, 2 and 1, respectively remaining? *L. Diary, 1748.*

As by the foregoing question, $1 \times 2 \times 3 \times 4 \times 5 = 60$, the least number, which divided by 1, 2, 3, 4, 5 and 6, leaves no remainder.

$\therefore 60 - 1 = 59$. Q. E. F. as may be easily proved.

5. Required the least number, that being divided by 9, shall leave for a remainder 6; if divided by 8, the remainder will be 5; if divided by 7, the remainder will be 4; and so on, each time leaving for a remainder three less than the divisor, till, divided by 3, the remainder will be nothing?

Ans 2520 is the least number which can be divided by the nine digits, or by the seven highest of them, without a remainder,

$\therefore 2520 - 3 = 2517$. Q. E. F. as may be easily proved.

6. Required the three least numbers, which divided by 20 shall leave 19 for a remainder; but, if divided by 19, shall leave 18, if divided by 18, shall leave 17; and so on (always leaving one less than the divisor) to unity?

Gentlemen's Diary, 1747.

First, 1, 2, 3, 5, 7, 11, 13, 17, and 19, are prime numbers.

Also $\sqrt{4} = 2$, $\sqrt[3]{8} = 2$, $\sqrt{9} = 3$, and $\sqrt[4]{16} = 2$;

And all the rest are composite numbers.

$\therefore 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times 17 \times 18 \times 19 = 232792560$, the least number that can be divided by the

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the given divisors without a remainder; also $232792560 \times 2 = 465585120$; and $232792560 \times 3 = 698377680$, being divided by the given divisors, will leave no remainder.

$$\begin{array}{l} \therefore 232792560 - 1 = 232792559 \\ 465585120 - 1 = 465585119 \\ \text{and } 698377680 - 1 = 698377679 \end{array} \left. \vphantom{\begin{array}{l} \therefore 232792560 - 1 = 232792559 \\ 465585120 - 1 = 465585119 \\ \text{and } 698377680 - 1 = 698377679 \end{array}} \right\} \begin{array}{l} \text{the three least num-} \\ \text{bers. Q. E. F.} \end{array}$$

Agreeing with the algebraic process, by Mr. Robinson, in the Gentlemen's Diary, 1748.

7. A jolly fine girl did ride on the way,
With plums in a basket, it being market-day;
She rode on but softly, the weather being hot,
So I ask'd her what number of plums she had got:
She said, the just number she did not well know,
But I'll tell you which way will the true number show.
If you count them by two's, there will then remain one;
If you count them by three's, there rests two when
you've done;
If you count them by four's, the remainder is three;
If by five's, then just four the remainder will be;
If by six at a time you do count them again,
You'll find, when you've done, that just five will remain;
But if seven at a time, you do count them o'er all,
The remainder will be then just nothing at all:
Now what is the number, and to what do they come,
At fourteen a penny, I'd fain know the sum?

By the third question it appears, 60 is the least number that can be divided by the first six digits, without a remainder, $60 - 1 = 59$, the least number that can be divided by the said six digits, leaving each division one less than the divisor; but 59, divided by 7, leaves a remainder.

Then $60 \times 2 - 1 = 119$, the least number that answers the conditions of this question.

Also 420 is found, by Question 3, to be the common difference of numbers, answering the same conditions. $\therefore 119, 539, 959, 1379$, &c. will admit of the same conditions. Q. E. F.

$$\begin{array}{l} \therefore 14) 119 (8\frac{1}{2} \text{ d.} \\ \text{Or } 14) 539 (38\frac{1}{2} = 3 \text{ s. } 2\frac{1}{2} \text{ d.} \end{array} \left. \vphantom{\begin{array}{l} \therefore 14) 119 (8\frac{1}{2} \text{ d.} \\ \text{Or } 14) 539 (38\frac{1}{2} = 3 \text{ s. } 2\frac{1}{2} \text{ d.} \end{array}} \right\} \text{their value.}$$

8. Once old mother Gripe to a market went,
Some butter to sell it was her intent;
At a certain rate per pound she it sold,
What she got for it all, as I have been told,

Were

Were two shillings and two pence farthing just:
 Now how much butter had the old toast,
 And how she might sell her butter per pound,
 Is what is required to be found?
 Of various answers this question will admit,
 Find them all out, and they will whet thy wit.

First, 2s. 2½d. = 105 farthings, which is composed of these odd numbers; viz. $1 \times 3 \times 5 \times 7 = 105$.

So that 105 lb. at				d.	} per pound, all answer the conditions of this question. Q. E. F.
3	105	35	- - -	1¼	
5	105	21	- - -	1½	
7	105	15	- - -	1¾	
15	105	7	- - -	3½	
21	105	5	- - -	5½	
35	105	3	- - -	8¾	
Or 1 lb. at 2s. 2½				d.	

This question, and the foregoing, was taken from Tapper's Delight for the Ingenious, for July, August and September, 1711; the solutions my own.

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## CHAPTER V.

PROGRESSION, VARIATION, COMBINATION, &c.

### SECT. I.

#### ARITHMETICAL PROGRESSION.

**A**RITHMETICAL PROGRESSION is a rank or series of numbers increasing or decreasing by a common difference, or by a continual addition or subtraction of some equal numbers.

As  $\{ 1. 2. 3. 4. 5. 6. 7. 8. 9 \}$  common difference 1.

Or  $1. 3. 5. 7. 9. 11. 13$ , common difference 2.

Also  $42. 35. 28. 21. 14. 7$ , common difference 7.

In an arithmetical progression are five things; any three of which being given, the other two may be found, which admit of twenty different propositions,

1. The first term, commonly the least
2. The last term, commonly the greatest
3. The number of terms.
4. The common excess, or difference.
5. The aggregate, or sum of all the terms.

We shall only concern ourselves with some few of them, but let us premise, that,

1. If any three numbers are in arithmetical progression, the sum of the two extremes, viz. the first and last, will be equal to the double of the mean or middle number.

As in these, 3 . 8 . 13; viz.  $3 + 13 = 8 + 8$

Or 1 . 7 . 13 - - -  $1 + 13 = 7 + 7$

And 7 . 14 . 21 - - -  $7 + 21 = 14 + 14$

2. If four numbers are in arithmetical progression, the sum of the two extremes will be equal to the sum of the two means.

As 1 . 3 . 5 . 7; viz.  $1 + 7 = 3 + 5$

And 5 . 8 . 11 . 14 - - -  $5 + 14 = 8 + 11$

3. Also if many numbers be in arithmetic progression, the sum of the two extremes will be equal to the sum of any two means that are equally distant from the extremes.

7 . 9 . 11 . 13 . 15 . 17;

viz.  $7 + 17 = 9 + 15 = 11 + 13$ .

Or if the numbers be odd:

1 . 3 . 5 . 7 . 9 . 11 . 13;

viz.  $1 + 13 = 3 + 11 = 5 + 9 = 7 + 7$ .

4. Every series of numbers in arithmetic progression is composed of the excess or common difference, so often repeated as there are terms in the progression, except the first.

As in these, 2 . 5 . 8 . 11 . 14 . 17, &c.

Here the common difference being 3.

Then will  $2 + 3 = 5$  .  $5 + 3 = 8$  .  $8 + 3 = 11$  .  $11 + 3 = 14$  .  $14 + 3 = 17$ , &c.

Hence may be observed, that the difference between the two extremes (2 and 17) is composed of the common difference, multiplied into the number of all the terms, except the first.

In

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In the aforesaid progression, 2 . 5 . 8 . 11 . 14 . 17.

The number of terms without the first is 5 } multipl.  
The common difference - 3 }

The difference of the two extremes - 15

P R O P O S I T I O N L

The two extremes, and the number of terms, being given, to find the sum of all the series,

R U L E,

Multiply the sum of the two extremes into the number of terms, and divide the product by 2, the quotient will be the sum of all the series.

1. How many strokes do the clocks of Venice (which go on to 24 o'clock) strike in the compass of a natural day?

1 + 24 = 25, sum of the extremes.  
24, number of terms.

$$\begin{array}{r} 100 \\ 50 \\ \hline 2) 600 \text{ (300 strokes. Q. E. F.} \end{array}$$

2. The length of my garden is 94 feet; now if eggs be laid along the pavement a foot asunder, to be fetched up singly to a basket, removed one foot from the last, how much ground must he travel that does it?

2 + 188 = 190, sum of the extremes.  
94, number of terms.

$$\begin{array}{r} 2) 17860 \text{ (8930 feet.} \\ \text{Feet in a mile } 5280 \text{ (1 mile, 5 furl. 21 poles, } 3\frac{1}{2} \text{ feet.} \\ 660 \text{ } 3650 \\ \hline 16.5 \text{ } 350 \\ \hline 3\frac{1}{2} \end{array} \quad \begin{array}{l} \text{Q. E. F.} \end{array}$$

PROPOSITION II.

The first term, the common excess, and the number of terms being given, to find the sum of all the series.

R U L E.

From the product of the number of terms in the common excess, subtract the common excess, and to the remainder add the double of the first term; half the product of that sum multiplied by the number of terms, gives the sum of all the series.

3. A gentleman bargains with a bricklayer to sink him a well twenty fathoms deep, upon these terms; viz. to pay him three shillings for the first fathom, five for the second, seven for the third, &c. raising two shillings every fathom; what will be due to the bricklayer for completing the same?

First,  $20 \times 2 = 40$ ; also  $40 - 2 = 38$ .  
 Again,  $38 + 6 = 44$ ; and  $44 \times 20 = 880$ .  
 $\therefore 880 \div 2 = 440$  shillings = 22 l. Q. E. F.

PROPOSITION III.

The first term, number of terms, and sum of all the series given, to find the common excess.

R U L E.

Divide the double sum of all the series by the number of terms, and from the quotient subtract double the first term; divide the remainder by the number of terms lessened by unity, the quotient will be the common excess.

4. A gentleman travelled 100 leagues in eight days, and every day travelled equally farther than the preceding day; it is known that the first day he travelled two leagues, how many leagues did he travel each of the other days?

$200 \div 8 = 25$ ; also  $25 - 4 = 21$ ; and  $8 - 1 = 7$ .  
 7) 21 (3, the common difference sought.

Then 3 added to 2, and every other term respectively,

|                        |                           |
|------------------------|---------------------------|
| gives 5 for the second | } day's journey. Q. E. F. |
| 8 - - - third          |                           |
| 11 - - - fourth        |                           |
| 14 - - - fifth         |                           |
| 17 - - - sixth         |                           |
| 20 - - - seventh       |                           |
| 23 - - - eighth        |                           |

100 leagues.

PROPOSITION IV.

The two extremes, and number of terms being given, to find the common difference.

RULE.

The difference of the two extremes, divided by the number of terms less unity, the quotient will be the common excess.

5. One had 12 children that differed alike in their ages, the youngest was nine years old, the elder  $36\frac{1}{2}$ ; what was the difference of their ages, and the age of each?

Here  $36.5 - 9 = 27.5$ , difference of the extremes.

Also  $12 - 1 = 11$   $27.5 (2.5$ , common excess.

Which add to the age of the youngest, and so on continually to the rest.

|               |   |   |   |   |   |   |                 |              |
|---------------|---|---|---|---|---|---|-----------------|--------------|
| viz. youngest | - | - | - | - | - | - | 9               | } years old. |
| 11th          | - | - | - | - | - | - | $11\frac{1}{2}$ |              |
| 10th          | - | - | - | - | - | - | 14              |              |
| 9th           | - | - | - | - | - | - | $16\frac{1}{2}$ |              |
| 8th           | - | - | - | - | - | - | 19              |              |
| 7th           | - | - | - | - | - | - | $21\frac{1}{2}$ |              |
| 6th           | - | - | - | - | - | - | 24              |              |
| 5th           | - | - | - | - | - | - | $26\frac{1}{2}$ |              |
| 4th           | - | - | - | - | - | - | 29              |              |
| 3d            | - | - | - | - | - | - | $31\frac{1}{2}$ |              |
| 2d            | - | - | - | - | - | - | 34              |              |
| eldest        | - | - | - | - | - | - | $36\frac{1}{2}$ |              |

6. A debt is to be discharged at 11 several payments in arithmetic progression; the first payment to be 12 l. 10 s. and the last 63 l. what is the whole debt, and what must each payment be?

First,  $12.5 + 63 = 75.5$ , sum of the extremes.

11, number of terms.

2)  $830.5 (415.25 = 415$  l. 5 s. whole debt.

Then  $63 - 12.5 = 50.5$ , difference of the extremes.

$11 - 1 = 10$   $50.5 (5.05 = 5$  l. 1 s. common difference.

Therefore

|           | l. | s. |            |
|-----------|----|----|------------|
| Therefore | 12 | 10 | = first.   |
|           | 17 | 11 | = second   |
|           | 22 | 12 | = third    |
|           | 27 | 13 | = fourth   |
|           | 32 | 14 | = fifth    |
|           | 37 | 15 | = sixth    |
|           | 42 | 16 | = seventh  |
|           | 47 | 17 | = eighth   |
|           | 52 | 18 | = ninth    |
|           | 57 | 19 | = tenth    |
|           | 63 | —  | = eleventh |

} payment.

415 5, whole debt, as before.

7. A man is to travel from London to a certain place in ten days, and to go but two miles the first day, increasing every day's journey by an equal excess, so that the last day's journey may be 29 miles; what will each day's journey be, and how many miles is the place he goes to distant from London?

First,  $29 - 2 = 27$ , difference of the extremes.

$10 - 1 = 9$  27 (3, the common difference.

Which added to each day's journey,

|         |                 |          |
|---------|-----------------|----------|
| gives 2 | } miles for the | first.   |
| 5       |                 | second.  |
| 8       |                 | third.   |
| 11      |                 | fourth.  |
| 14      |                 | fifth.   |
| 17      |                 | sixth.   |
| 20      |                 | seventh. |
| 23      |                 | eighth.  |
| 26      |                 | ninth.   |
| 29      |                 | tenth.   |

155 miles from London.

### PROPOSITION V.

The two extremes, and the common excess given, to find the number of terms.

#### R U L E.

Divide the difference of the two extremes by the common excess, the quotient plus unity is the number of terms.

8. A man going a journey, his first day's travel was five miles, his last day's travel 35 miles; he increased his journey every day three miles; how many days did he travel?

First,  $35 - 5 = 30$ , difference of the extremes.  
Then  $3 \mid 30$  (10; and  $10 + 1 = 11$  days journey. Q. E. F.

### PROPOSITION VI.

The common excess, number of terms, and sum of all the series given, to find the first term.

### RULE.

Divide the sum of all the series by the number of terms, and from the quotient subtract half the product of the common excess into the number of terms less unity, the remainder will be the first term.

9. A man is to receive 300l. at 12 several payments, each payment to exceed the former by four pounds; he is willing to bestow the first payment on any one that can tell him what it is; what must the arithmetician have for his pains?

First,  $12 \mid 300$  (25; also  $12 - 1 = 11$ .  
Then  $11 \times 4 = 44$ ; and  $2 \mid 44$  (22.  
 $\therefore 25 - 22 = 3$ , the artist's reward. Q. E. F.

10. Suppose it 100 leagues between London and Carlisle, two couriers set out from each place on the same road; that from London towards Carlisle travelling every day two leagues more than the day before; the other from Carlisle to set off one day after, travelling every day three leagues more than the preceding one; and that they meet exactly half way, the first at the end of five days, and the other at the end of four; how many leagues did each travel each day?

First,  $5 \mid 50$  (10; also  $5 - 1 = 4$ .  
Then  $4 \times 2 = 8$ ; and  $2 \mid 8$  (4.  
 $\therefore 10 - 4 = 6$ , his first  
 $6 + 2 = 8$  - - second  
 $10$  - - third  
 $12$  - - fourth  
 $14$  - - fifth  


---

Sum 50

} day's journey.

Again,



Again, 4) 50 (12.5; also  $4 - 1 = 3$ .

Then  $3 \times 3 = 9$ ; and 2) 9 (4.5.

$$\begin{array}{rcl} \therefore 12.5 - 4.5 = 8, & \text{his first} & \\ 8 + 3 = 11 & - \text{second} & \\ 14 & - \text{third} & \\ 17 & - \text{fourth} & \\ \hline & & 50 \text{ leagues.} \\ \hline \end{array} \left. \vphantom{\begin{array}{rcl} 12.5 - 4.5 = 8, & \text{his first} & \\ 8 + 3 = 11 & - \text{second} & \\ 14 & - \text{third} & \\ 17 & - \text{fourth} & \end{array}} \right\} \text{day's journey.}$$

### PROPOSITION VII.

The last term, number of terms, and common excess given, to find the first term.

#### R U L E.

Multiply the common excess into the number of terms less unity, the product subtracted from the last term leaves the first.

11. A man in six days went from London to Manchester, every day's journey was greater than the preceding one by four miles, his last day's journey was 40 miles, what was the first?

$$\begin{array}{rcl} \text{Number of terms } 6 - 1 = 5 & & \\ \text{Common excess } 4 & & \\ \hline & & 20 \\ \hline \end{array}$$

Then  $40 - 20 = 20$ , the first day's journey. Q. E. F.

I shall now add one proposition more, exclusive of the 20 above-mentioned.

### PROPOSITION VIII.

When one person or thing moves with an equal, and another the same way by a progressive motion, to find in what time the first will be overtaken.

#### R U L E.

Add the common excess of the pursuer's day's journey to double the space gone each day by the pursued; from that sum subtract double the space that the pursuer travelled the first day, and divide the remainder by the com-

K k 3

mon

mon excess, the quotient will give the number of days in which the pursued will be overtaken by the pursuer.

12. A noted highwayman having committed a robbery, not suspecting a pursuit, fled northward at the rate of eight leagues a day; Jonathan Wyld, upon the scent, follows him, in a progressive motion, only three leagues the first day, five the next, seven the third, and so on, increasing every day two leagues; in how many days will the highwayman be overtaken?

First,  $2 + 16 = 18$ ; also  $18 - 6 = 12$ .

∴ 2) 12 (6 days. Q. E. F.

For  $6 \times 8 = 48$  leagues, the space travelled by the robber.

Then, by Prop. II.  $6 \times 2 = 12$ ; also  $12 - 2 = 10$ ; and  $10 + 6 = 16$ .

Also  $16 \times 6 = 96$ . ∴ 2) 96 (48 leagues, when the thief-taker comes up with the highwayman.

13. Y. Z. made the following bet for 1000 guineas, to be decided the Monday, Tuesday, and Wednesday, in Whitsun-week, on Barnham Downs, between the hours of eight in the morning, and eight at night. The proposer has ten choice cricketers in full exercise, who on this occasion are to be distinguished by the first 10 letters of the alphabet. These are to run and gather up, and carry singly, 1000 eggs, laid in a right line, just two yards asunder, putting them gently into a basket placed just a fathom behind the first. They are to work one at a time, in the following order: A is to fetch up the first 10 eggs, B the second, C the third, and so forward to K, whose turn it will be to fetch up the 1000th egg. After which, A sets out again for the next 10, B takes the next, and so forward alternately, till K shall have carried up the 1000th egg, at 100 eggs per man. The fellows are to have 300l. for their three days work, if they do it, and it is to be distributed in proportion to the ground each man shall in his course have gone over. It is required, first, How many miles each person will have run? Secondly, What part of the 300l. will come to his share? Thirdly, Whether if the men had been posted at proper places, they had not better have run from London to York twice, and back in the time, taking the measure 180 miles?

First, for A's race, 4, first term, 40, last term, their sum 44.

Nº of terms  $10 \times 44 = 440$ .

∴ 2)

∴ 2) 440 (220, A's first race.

Then  $901 \times 4 = 3604$ , first term of the last race.

Also  $910 \times 4 = 3640$ , last term.

$3604 + 3640 = 7244$ , their sum.

∴  $\frac{7244 \times 10}{2} = 36220$ , A's last race.

Then to find his whole part in this expedition, put 220, first term, 36220 last term, sum 36440.

∴  $\frac{36440 \times 10}{2} = 182200$ , sum of A's races.

For B's part in the expedition :

First,  $11 \times 4 = 44$  ; last term  $20 \times 4 = 80$  ;

Also  $44 + 80 = 124$  ; which  $\times 10 = 1240$ .

∴ 2) 1240 (620 yards, B's least race.

Last race  $911 \times 4 = 3644$  } their sum 7324.

Also  $920 \times 4 = 3680$  }

∴  $\frac{7324 \times 10}{2} = 36620$  yards, B's greatest race.

Then  $620 + 36620 = 37240$ , sum of the extremes.

∴  $\frac{37240 \times 10}{2} = 186200$ , sum of B's races.

Again,  $186200 - 182200 = 4000$  yards, common difference ; which added continually to each of their shares, shews that

|              | yards.   | miles. | furl. | poles. |
|--------------|----------|--------|-------|--------|
| A in all ran | 182200 = | 103    | 4     | 40     |
| B - -        | 186200 = | 105    | 6     | 80     |
| C - -        | 190200 = | 108    | -     | 120    |
| D - -        | 194200 = | 110    | 2     | 160    |
| E - -        | 198200 = | 112    | 4     | 200    |
| F - -        | 202200 = | 114    | 7     | 20     |
| G - -        | 206200 = | 117    | 1     | 60     |
| H - -        | 210200 = | 119    | 3     | 100    |
| I - -        | 214200 = | 121    | 5     | 140    |
| K - -        | 218200 = | 123    | 7     | 180    |

2002000 = 1137 4 — Q. E. F.

As 10010 : 911 :: 300 : 27 6  $-\frac{648}{10010}$ , A's } part.

Also 10010 : 931 :: 300 : 27 18  $-\frac{504}{10010}$ , B's }

Then 27 l. 18 s.  $-\frac{504}{10010}$  d. — 27 l. 6 s.  $-\frac{648}{10010}$  d. = 11 s.

$11\frac{37}{10010}$  d. common difference ; which added to each man's share,

K k 4

gives

|       |   |     | l. | s. | d.                 |            |
|-------|---|-----|----|----|--------------------|------------|
| gives | A | - - | 27 | 6  | $\frac{648}{1000}$ | } Q. E. F. |
|       | B | - - | 27 | 18 | $\frac{304}{1000}$ |            |
|       | C | - - | 28 | 10 | $\frac{160}{1000}$ |            |
|       | D | - - | 29 | 2  | $\frac{216}{1000}$ |            |
|       | E | - - | 29 | 14 | $\frac{72}{1000}$  |            |
|       | F | - - | 30 | 5  | $\frac{929}{1000}$ |            |
|       | G | - - | 30 | 17 | $\frac{785}{1000}$ |            |
|       | H | - - | 31 | 9  | $\frac{641}{1000}$ |            |
|       | I | - - | 32 | 1  | $\frac{427}{1000}$ |            |
|       | K | - - | 32 | 13 | $\frac{363}{1000}$ |            |

From London to York, suppose 180 miles.

Miles in the whole expedition  $1137\frac{1}{2}$

And  $180 \times 4 = 720$

Short of the present undertaking  $417\frac{1}{2}$  miles.

The following question I was favoured with by my esteemed friend, Major Watson, chief engineer in Lord Clive's expedition to the Indies.

14. Suppose a man to have a calf, which at the end of three years begins to breed (and afterwards) a female calf every year; and that each calf begins to breed in like manner at the end of three years, bringing forth a cow calf every year, and that these last breed in the same manner, &c. &c. to determine the owners whole stock at the end of 20 years?

By nature of this question, the number of cows that calved at the end of these years will be as follows;

viz.  $\left\{ \begin{array}{l} 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14 \\ 1. 1. 1. 2. 3. 4. 6. 9. 13. 19. 28. 41 \\ 15. 16. 17. 18. 19. 20 \text{ years.} \end{array} \right.$   
60. 88. 129. 189. 277, and 406 respectively, which are found by adding the last to the last but two.

Then of the whole series  $1 + 1 + 1 + 2 + 3 + 4 + 6 + 9 + 13 + 19 + 28 \dots + D + E + F + G$  be represented by S, when D, E, F and G denote the four last terms, we shall then have  $1 + 1 + 1 + 2 + 3 + 4 + 6 + 9 + 13 + 19 + 28 \dots + D = S - E - F - G$ , which being taken from the above, we have  $1 + 1 + 1 + 1 + 2 + 3 + 4 + 6 + 9 + 13 \dots + F = E + F + G$ ; and by adding

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adding  $G$  to both sides of the equation, we then get  $1 + 1 + 1 + 1 + 2 + 3 + 4 + 6 + 9 + 13 \dots + F + G = E + F + 2G$ ; which consequently will be the man's stock of cows and calves at the end of any number of years, which, in this case,  $E = 189 + F = 277 + 2G = 812$ , will be 1278. Q. E. F.

From the above solution it appears, that the whole stock of cows and calves, at the end of any number of years, will be equal to the number of cows that would calve at the end of three years after the given time.



### S E C T. II.

## GEOMETRICAL PROGRESSION.

**G**EOMETRICAL PROGRESSION is when any rank or series of numbers increase by one common multiplier, or decrease by one common divisor;

As  $2 \cdot 4 \cdot 8 \cdot 16 \cdot 32 \cdot 64$ ; here 2 is the common multiplier.

And  $1215 \cdot 405 \cdot 135 \cdot 45 \cdot 15 \cdot 5$ ; here 3 is the common divisor.

Note, The common multiplier, or divisor, is called the ratio.

Here note, that if three numbers are in geometrical progression, the product of the two extremes will be equal to the square of the mean or middle term, as in these,  $2 \cdot 4 \cdot 8$ .

Here  $2 \times 8 = 4 \times 4$ , each being  $= 16$ .

Also if four numbers are in geometrical progression, the product of the two means will be equal to the product of the two extremes, as in these,  $135 \cdot 45 \cdot 15 \cdot 5$ .

Here  $135 \times 5 = 45 \times 15$ , each being 675.

Hence, if ever so many numbers are in geometrical progression, the product of the two extremes are equal to the product of any two means that are equally distant from the extremes.

As in these,  $3 \cdot 9 \cdot 27 \cdot 81 \cdot 243 \cdot 729$ .

Here  $3 \times 729 = 9 \times 243 = 27 \times 81 = 2187$ .

Or

Or if the number of terms be odd, as in these :

$$3 \cdot 9 \cdot 27 \cdot 81 \cdot 243, \&c.$$

$$3 \times 243 = 9 \times 81 = 27 \times 27 = 729.$$

In any geometrical progression, the same things are to be taken notice of, as in arithmetical progression ;

viz. First, The first term, commonly the least.

Secondly, The last term, commonly the greatest.

Thirdly, The number of terms.

Fourthly, The ratio, or common multiplier, or divisor.

Fifthly, The sum of all the series.

Any three of these being known, the rest may be found.

If to any series of numbers in geometrical progression not proceeding from unity, there be assigned a series of numbers in arithmetical progression, beginning with an unit or 1, whose common difference is 1, called indices, or exponents ;

$$\text{thus } \begin{cases} 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7, \text{ indices.} \\ 2 \cdot 4 \cdot 8 \cdot 16 \cdot 32 \cdot 64 \cdot 128, \&c. \end{cases}$$

then will the addition or subtraction of those indices (or numbers in arithmetic progression) directly correspond with the product or quotient of their respective terms or series in geometric progression ;

$$\text{that is, } \begin{cases} \text{as } 3 + 4 = 7, \\ \text{so } 8 \times 16 = 128, \text{ the seventh term in } \div. \end{cases}$$

$$\text{Again, } \begin{cases} \text{also } 7 + 7 = 14, \\ \text{so } 128 \times 128 = 16384, \text{ the 14th term in } \div. \end{cases}$$

$$\text{Or, } \begin{cases} \text{as } 7 - 3 = 4, \\ \text{so } 128 \div 8 = 16, \end{cases}$$

$$\text{Or, } \begin{cases} \text{as } 6 - 1 = 5, \\ \text{so } 64 \div 2 = 32, \&c. \end{cases}$$

But if the series begin with unity, the indices must begin with a cypher.

$$\text{Thus, } \begin{cases} 0 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7, \&c. \\ 1 \cdot 2 \cdot 4 \cdot 8 \cdot 16 \cdot 32 \cdot 64 \cdot 128. \end{cases}$$

Now by these indices, and a few leading terms, the last term, or any distant one, one may be speedily found.

### PROPOSITION I.

The first term being unity, the ratio and number of terms being known, to find the last or any remote term.

R U L E.

R U L E.

Find a few of the leading terms, over which place their indices, as before directed; then multiply the last found term by itself, which will produce a term double thereto; and so proceed, till you either arrive at the term sought, or one that falls a little short of it; if so, multiply the term last found by that term, answering the difference of the indice of the last found term, and that sought; which last product will be the term required.

1. A country gentleman going to a fair, meets with a crafty youth who had a drove of 25 very good oxen; upon asking their price, was answered, he should have them for 16 pounds each, one with another; the gentleman offers him 15 pounds each, and take all: the young spark tells him it would not be taken, but if he would give him what the 20th ox would come to by beginning at the first with a single farthing, and doubling only to the 20th, he should have them all; what did they come to a head?

First,  $\begin{cases} 0 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5, & \text{indices.} \\ 1 \cdot 2 \cdot 4 \cdot 8 \cdot 16 \cdot 32, & \text{terms.} \end{cases}$

Then,  $\begin{cases} 5 + 5 = 10, & \text{also, } 4 + 5 = 9, \\ 32 \times 32 = 1024; & 16 \times 32 = 512. \\ 10 + 9 = 19. \end{cases}$

$1024 \times 512 = 524288$ , which is the 20th term, as the indices are less than the term by one.

And  $524288$  farth. =  $546$  l. 2 s. 8 d. price of the whole.  
 $\therefore 25) 546$  l. 2 s. 8 d. ( $21$  l. 16 s.  $10\frac{1}{2}$  d. Q. E. F.

But if the first term of any series be greater than unity, that and the ratio being known, to find any remote term without producing the rest,

R U L E,

Find a few of the leading terms, as before directed; then multiply the last term so found by itself, and divide the product by the first term, and this again multiplied by the term as is wanting, and divided by the first, gives the term required.

2. A nobleman dying left ten sons, to whom and to his executor he bequeathed his estate in the manner following; viz. to his executor for seeing his will performed  $1024$  pounds;

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pounds; the youngest son to have as much, and half as much, and every son to exceed the next younger in the same ratio of  $1\frac{1}{2}$ : what is the share of the eldest?

$$\begin{array}{ccccccc} 0 & . & 1 & . & 2 & . & 3 & . & 4 & . & 5, \text{ indices.} \\ 1024 & . & 1536 & . & 2304 & . & 3456 & . & 5184 & . & 7776, \text{ terms.} \\ \hline 7776 \times 7776 & = & 590491. & \text{eldest son's fortune.} & \text{Q. E. F.} \\ 1024 & & & & & & & & & & \end{array}$$

## PROPOSITION II.

The first term, ratio, and last term given, to find the sum of all the series.

### RULE.

Multiply the last term into the ratio, and from the product subtract the first term; divide the remainder by the ratio less unity, the quotient will be the sum of all the series.

3. On New-year's day a gentleman married, and received of his father-in-law a guinea, on condition that he was to have a present on the first day of every month for the first year, which should be double still to what he had the month before; what was the lady's portion?

$$\begin{array}{l} \text{First, } \left\{ \begin{array}{cccccc} 0 & . & 1 & . & 2 & . & 3 & . & 4 & . & 5, \text{ indices.} \\ 1 & . & 2 & . & 4 & . & 8 & . & 16 & . & 32, \text{ terms.} \end{array} \right. \\ \text{Then } \left\{ \begin{array}{l} 3 + 3 = 6, \\ 8 \times 8 = 64; \end{array} \right. \text{ also } \left\{ \begin{array}{l} 6 + 5 = 11 \\ 64 \times 32 = 2048, \text{ last} \end{array} \right. \\ \text{term.} \end{array}$$

$$\text{Again, } 2048 \times 2 = 4096; \text{ also } 4066 - 1 = 4095.$$

$$\begin{array}{r} 20) 4095 \\ \underline{204 \ 15} \end{array}$$

$$\underline{\underline{\pounds 4299 \ 15s.}} \text{ the lady's fortune. Q. E. F.}$$

4. One at a country fair had a mind to a string of 20 fine horses; but not caring to take them at 20 guineas per head, the jockey consented that he should, if he thought good, pay but a single farthing for the first, doubling it only to the 19th, and he would give the 20th into the bargain: this being presently accepted, how were they sold?

$$\begin{array}{l} \text{First, } \left\{ \begin{array}{cccccc} 0 & . & 1 & . & 2 & . & 3 & . & 4 & . & 5 \\ 1 & . & 2 & . & 4 & . & 8 & . & 16 & . & 32, \text{ \&c.} \end{array} \right. \\ \text{Then} \end{array}$$



Then  $\begin{cases} 5 + 5 = 10, \\ 32 \times 32 = 1024; \end{cases}$  also  $\begin{cases} 5 + 3 = 8 \\ 32 \times 8 = 252. \end{cases}$

Again,  $\begin{cases} 10 + 8 = 18 \\ 1024 \times 252 = 262144, \end{cases}$  the 19th term.

Then  $262144 \times 2 = 524288;$

also,  $524288 - 1 = 524287$  farth. =  $546\text{ l. } 2\text{ s. } 7\frac{1}{2}\text{ d.}$

$\therefore 20) 546\text{ l. } 2\text{ s. } 7\frac{1}{2}\text{ d. } (27\text{ l. } 6\text{ s. } 1\frac{1}{2}\text{ d. each. Q. E. F.}$

5. A cunning servant agreed with a master (unskilled in numbers) to serve him eleven years without any reward for his service but the produce of a wheat-corn for the first year; and that product to be sowed the second, and so on from year to year, until the end of the time, allowing the increase to be in a tenfold proportion: it is required to find the sum of the whole produce?

First,  $\begin{cases} 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \text{ years.} \\ 10 \cdot 100 \cdot 1000 \cdot 10000 \cdot 100000 \text{ corns of wheat.} \end{cases}$

Then  $\begin{cases} 4 + 2 = 6 \\ 10000 \times 100 = 1000000, \end{cases}$  the 6th year's produce.

And  $\begin{cases} 6 + 5 = 11 \\ 1000000 \times 100000 = 100000000000, \end{cases}$  the 11th or last year's produce.

Then ratio  $10 \times 100000000000 = 1000000000000;$

Also  $1000000000000 - 10 = 999999999990.$

Ratio  $10 - 1 = 9) 999999999990$  (111111111110 corns in all.

As hath been before observed, 7680 wheat-corns will fill a statute pint;

Then  $7680) 111111111110$  (14467591 pints.

In a bushel 64) 14467591 (226056 $\frac{1}{2}$  bushels, which suppose at 3 s. 4 d. per bushel.

$\frac{1}{2}) 226056\frac{1}{2}$

Answer  $\underline{\underline{\text{£ } 37676 - 4\frac{1}{2}}}$  s. d. a very ample reward. Q. E. F.

6. It is reported that one Sessa, in India, having first invented the game at chess, shewed it to his prince Shekram; the king, who being highly pleased with it, bid him ask what he would for the reward of his invention; whereupon he asked, that for the first little square of the chess-board he might have one grain of wheat given him; for the second two, and so on, doubling continually according to the number of squares on the chess-board, which were 64: the king, who intending him a noble reward, was displeased that he had asked so trifling a one; but Sessa declaring

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claring he would be contented with this, it was ordered to be given him; but the king was astonished when he found that this would raise so vast a quantity, that even the whole earth could not produce it! so you are desired to repeat the operation.

$$\begin{array}{l} \text{First, } \{ 0 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5, \text{ indices.} \\ \quad \{ 1 \cdot 2 \cdot 4 \cdot 8 \cdot 16 \cdot 32, \text{ terms, \&c.} \\ \text{Then } \{ 5 + 5 = 10, \text{ also } \{ 10 + 10 = 20 \\ \quad \{ 32 \times 32 = 1024; \text{ also } \{ 1024 \times 1024 = 1048576 \\ \text{Again, } \{ 20 + 20 = 40 \\ \quad \{ 1048576 \times 1048576 = 1099511627776. \end{array}$$

$$\begin{array}{l} \text{Also} \\ 40 + 20 = 60 \\ 109951162776 \times 1048576 = 115250695006846976 \\ \text{Lastly,} \\ 60 + 3 = 63 \\ 115250695006846976 \times 8 = 922005560054775808 \\ \text{Now, } 922005560054775808 \times 2 = 1844011120109551616. \\ \therefore 1844011120109551616 - 1 = 1844011120109551615 \\ \text{wheat-corns.} \end{array}$$

7680) 1844011120109551615 (240105614597597 pints.  
64) 240105614597597 (3751650228087 bushels.  
8) 3751650228087 (46895627851.875 quarters, which, at 1l. 7s. 6d. per quarter, amounts to 64481488296l. which is more than would pay one year's rent of all the dry land on the face of the earth, at 1l. 10s. per acre, which may thus be proved :

The circumference of the earth 360 degrees (69 English miles to a degree) = 25020 English miles, circumference of the earth.

Also,  $25020 \times 25020 \times 31832 = 199268447.328$ , area in square miles of a perfect globe.

Also,  $199268447.328 \times 640 = 127531806289.92$  acres of land and water,  $\frac{2}{3}$  of which is supposed to be water.

$\therefore 3) 127531806289.92 (42510602096.64$  acres of dry land, which, at 1l. 10s. per acre, is 63765903144.96 pounds a year; which, compared with the valuation of the wheat, as above, will be found 715585151 l. less.

## PROPOSITION III.

Of any decreasing series in  $\div$ , whose last term is a cypher, to find the sum of those series.

RULE.

R U L E.

Divide the square of the first term by the difference between the said first term and the second term in the series, the quotient will be the sum of all the series.

7. To find the sum of  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ , &c. *ad infinitum*.

Thus,  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ ; also,  $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ .  
 $\therefore \frac{1}{4} \div \frac{1}{4} = 1$ , the sum of the series required.

8. To find the sum of  $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$ , &c. *ad infinitum*.

First,  $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ ; also,  $\frac{1}{3} - \frac{1}{9} = \frac{2}{9}$ ; and  $\frac{2}{9} - \frac{1}{9} = \frac{1}{9}$ .  
 $\therefore \frac{2}{9} \div \frac{1}{9} = \frac{2}{1}$ . Q. E. F.

9. Suppose a ball to be put in motion by a force which drives it 12 miles the first hour, 10 miles the second, and so on continually, decreasing in proportion of 12 to 10, to infinity; what space would it move through?

First,  $12 \times 12 = 144$ ; also  $12 - 10 = 2$ .  
 $\therefore 2) 144 (72 \text{ miles. Q. E. F.}$

It may appear strange to some, that it should be possible to give the sum of an infinite progression in numbers; whereas, if the terms were continued, it would, after a thousand years labour, and after producing thousands of millions of terms, be never the nearer finishing.



S E C T. III.

V A R I A T I O N S.

**B**Y variation is meant the different ways any number of things may be altered or changed, in respect to their places.

To find the number of different changes that may be rung on any proposed number of bells,

R U L E,

Multiply all the terms in a series of arithmetical progressions continually, whose first term and common difference is

is unity or 1, and the last term the number of things proposed to be varied; the last product will be the number of variations required.

The changes on any number of bells not exceeding 12, are exhibited in the following TABLE.

| The number of things proposed to be varied. | The manner how their several variations are produced. | The different changes or variations every one of the proposed numbs. can admit of. |
|---------------------------------------------|-------------------------------------------------------|------------------------------------------------------------------------------------|
| 1                                           | $1 \times 1$                                          | $= 1$                                                                              |
| 2                                           | $1 \times 2$                                          | $= 2$                                                                              |
| 3                                           | $2 \times 3$                                          | $= 6$                                                                              |
| 4                                           | $6 \times 4$                                          | $= 24$                                                                             |
| 5                                           | $24 \times 5$                                         | $= 120$                                                                            |
| 6                                           | $120 \times 6$                                        | $= 720$                                                                            |
| 7                                           | $720 \times 7$                                        | $= 5040$                                                                           |
| 8                                           | $5040 \times 8$                                       | $= 40320$                                                                          |
| 9                                           | $40320 \times 9$                                      | $= 362880$                                                                         |
| 10                                          | $362880 \times 10$                                    | $= 3628800$                                                                        |
| 11                                          | $3628800 \times 11$                                   | $= 39916800$                                                                       |
| 12                                          | $39916800 \times 12$                                  | $= 479001600$                                                                      |

Let it be proposed to find the number of changes that may be rung on 12 bells, and to compute how long all these changes would take ringing once over.

First,  $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 = 479001600$ , the number of changes.

And suppose 10 changes to be rung in a minute; viz  $12 \times 10 = 120$  strokes in one minute, or two strokes in a second:

Then 10) 479001600 (47900160 minutes.

Also 1 year = 365 days, 5 hours, 49' = 525949 minutes.

525949) 47900160 (91 years, 26 days, 22 hours,

564750 41 minutes; so long would

12 bells be ringing, without

In 1 day are 1440) 38801 any intermission, to ring their different changes but once over.

1000

60) 1361

2. A young scholar, but an arithmetician, coming into a town for the convenience of a good library, demands of a gentleman, with whom he lodged, what his diet would cost for a year? The gentleman asks him 10 l. the scholar answered, he was not certain what time he might stay, and would know what he must give him for his diet so long as he could place his family (consisting of six persons besides himself) every day at dinner in a contrary position? The gentleman considering of it, and thinking it could not be long, tells him, he would allow him his diet so long for five pounds; to which the scholar assents: what did he give him for his table per annum?

First,  $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$  variations, or days.

Then  $365.26) 5040.00$  (13 years, 291 days, the answer.

$13.79725) 5.000000$  (.36239 = 7s. 3d. per annum nearly, the answer.



## SECTION IV.

### COMBINATIONS.

**C**OMBINATIONS are the various conjunctions which several things may receive without any regard to order, being taken 2 and 2, 3 and 3, &c.

To find the different combinations in any number, or quantities.

#### RULE.

Having placed the given quantity by itself, decrease it gradually by an unit, so often as there are quantities in the combination; placing them one above another, with a sign of multiplication between them, which number must be multiplied into one another for a dividend: then placing an unit with the like number of places, increasing by unity till you arrive at the number to be combined; which multiply continually for a divisor, and the quotient will be the number of combinations sought.

1. A famous general having serv'd his king  
Long time in wars, and had victorious been;

I. 1

For

For which his service (with a pleasant smile)  
 Ask'd of his king one farthing for each file  
 Of ten men in a file, which he could then  
 Make with a body of one hundred men.  
 The king, considering his brave actions past,  
 And seeming modesty of his request,  
 Gave his consent.—To what will it amount  
 In sterling money? take your pen and count.

$$\frac{100}{1} \times \frac{99}{2} \times \frac{98}{3} \times \frac{97}{4} \times \frac{96}{5} \times \frac{95}{6} \times \frac{94}{7} \times \frac{93}{8} \times \frac{92}{9} \times \frac{91}{10} =$$

$$\frac{62815650955520472000}{3628800} = 17310309456440 \text{ farthings.}$$

$\therefore 17310309456440 \text{ farth.} = 180315723501. \text{ 9s. 2d.}$   
 Q. E. F.

The number of combinations of 2 in any number of things, are 2 raised to the power of the number of things to be combined; for instance, if it was required to find the different ways 11 halfpence, huddled in a hat, may turn up; as a halfpenny hath two faces,  $\therefore 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ , or  $2^{11} = 2048$ , the different combinations required.

Now to find the different chances for any number of heads or tails, put  $a$  for the heads, and  $b$  for the tails; then,

$$a^{11} + a^{10}b + a^9b^2 + a^8b^3 + a^7b^4 + a^6b^5 + a^5b^6 + a^4b^7 + a^3b^8 + a^2b^9 + ab^{10} + b^{11}.$$

Here it is to be observed, that  $a^{11}$ , or all heads, hath but one way of turning up; the same may be observed for 11 tails.

But 10 heads and one tail, and the contrary, may come up 11 different ways each.

Also nine heads and two tails, or the contrary, may each come  $\frac{11 \times 10}{2} = 55$  different ways.

Again, eight heads and three tails, or the contrary, may each come  $\frac{11 \times 9}{3} = 165$  different ways; which numbers so found are called uncizæ, or coefficients.

From hence may be deduced this general rule.

#### R U L E.

If the index of the first letter of any term be multiplied into its own uncizæ, and that product be divided by the number

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number of terms to that place, the quotient will be the unciæ of the next succeeding term forward.

Let us proceed to find the rest of the coefficients, or chances, by the rule above;

viz. seven heads and four tails, or the contrary, may each come  $\frac{165 \times 8}{4} = 330$  different ways.

Again, for six heads and five tails, or the contrary, each may come  $\frac{330 \times 7}{5} = 462$ .

It may be observed, by proceeding as above, that the unciæ, or coefficients, do only increase until the indices of the two letters become equal, or change places, and then the rest will decrease in the same order.

Thus,  $a^{11} + 11a^{10}b + 55a^9b^2 + 165a^8b^3 + 330a^7b^4 + 462a^6b^5 + 462a^5b^6 + 330a^4b^7 + 165a^3b^8 + 55a^2b^9 + 11ab^{10} + b^{11}$ , are all the different combinations, or ways, 11 halfpence can turn up;

viz.  $1 + 11 + 55 + 165 + 330 + 462 + 462 + 330 + 165 + 55 + 11 + 1 = 2^{11} = 2048$ , as before.

Also a die, having six faces or sides, the number of combinations or different ways 11 dice may turn up, are  $6^{11} = 362797056$ .

From considerations like these I composed the following question, which was published in the Palladium 1753, and the next year was answered by three ingenious gentlemen; but they not considering, that in the same web of a goose's foot, the punch mark and slit mark cannot exist together, the last mark naturally destroying the first, which caused a mistake in their calculations; for both may combine together as well as with all the rest, and bring out the same number of combinations as eight halfpence, and four triangular dice, with three faces each, shaken together, could produce.

2. In Lincolnshire, here bounteous nature yields  
 Fat sheep and oxen, and luxuriant fields;  
 Our generous clime, replete with rosy health,  
 Choice friends afford, bright, fair, and plenteous wealth.  
 Some fenny ground have we, with flocks of geese,  
 Yielding five times a year their feather'd fleece;  
 On which, devoid of care, swains sleeping lie,  
 After repast of savoury giblet-pye.

L 1 2

One

One day, at Boston, o'er a jug of ale,  
 A goffard offer'd all his flock to sale,  
 At fifteen pence a-piece; but I propos'd  
 A different price, with which he quickly clos'd.  
 (The geese are mark'd, by cutting toe or heel,  
 The webs are pierc'd or slit with sharpen'd steel)  
 An hundred pounds for just as many geese,  
 As may be different mark'd: what's that a-piece?

Each goose having three toes, two webs, and one heel  
 on each foot, in all 12 different things to be marked.

But as the four webs may be either slit or pinched, but  
 not both together,  $\therefore 2^3 \times 3^4 = 20736$ , the number of  
 combinations.

Also,  $20736 - 1 = 20735 =$  number of marks.

And 100 l. = 24000 pence.

$\therefore 20735) 24000 (1\frac{6}{7}\frac{3}{4}\frac{3}{7}$  d. the price of a goose. Q. E. F.

3. A person, P, bets six pounds with another person, Q,  
 that in throwing up three halfpence, they shall all come up  
 the same way, viz. all heads, or all tails, once at least in  
 three trials: at the same time Q bets 10 guineas with R,  
 that in throwing up four halfpence, they shall not all come  
 up the same (i. e. all heads, or all tails) once in four trials:  
 required each person's advantage or disadvantage, with the  
 odds in each case, by an arithmetical computation only?

Since there are but two chances for three halfpence com-  
 ing all one way at a single throw, and six for the contrary,  
 it is evident the probability of missing all the three throws is  
 $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$ , and that of the contrary  $1 - \frac{1}{64} = \frac{63}{64}$ .

$\therefore$  The odds in favour of P, are as 37 to 27.

In the same manner  $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{256} =$  probability  
 of missing all the four throws with four halfpence, and that  
 of the contrary  $\frac{255}{256}$ . The odds in favour of Q, are  
 as 2401 to 1695; hence P's advantage in the first case is  
 18s. 9 d. and Q's in the second case is equal 2 l. 6s. 2  $\frac{1}{2}$  d.

4. Two desperate gamesters, A and B, agreed to throw  
 each of them two guineas at a particular point or mark;  
 and then to toss up the four guineas fairly, and each person  
 to take up the heads in the following manner as they arise;  
 that is, A's two guineas happened to light the first and sec-  
 ond, or the two nearest to the said point; so that if either  
 one or two heads arise, they must fall to A's lot; but B's  
 two guineas being the third or fourth, or the two farthest  
 from the point, so that if either three or four heads arise,  
 they are to be B's property: moreover, if two heads hap-  
 pen



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pen to arise in the first tofs, then A gets all the four guineas; and if three heads come up first to B, then will the other guinea be A's of course. Quere, what is each person's probability of winning, and how much is the value of A's chance before they begin to tofs?

*Martin's Magazine.*

First, let  $a$  represent heads, and  $b$  tails; then in the binomical  $a + b$  raised to the 4th power, the powers of  $b$  being rejected (as no winning chances on either side) will remain  $a^4 + 4a^3 + 6aa + 4a$ ; the indices representing the chances, and coefficients the number of different ways those chances may happen; the two last terms being A's, and the two first B's; in all, 15.

$\frac{6}{15} + \frac{4}{15}$  of  $\frac{1}{4} = \frac{2}{3}$ , value of A's } first winning tofs;  
 $\frac{1}{15} + \frac{1}{15}$  of  $\frac{1}{4} = \frac{1}{3}$  = value of B's } sum  $\frac{1}{3}$ .

Then  $\frac{1}{15} - \frac{1}{15} = \frac{1}{15}$ , whole value of the second tofs. Now B's chance of getting four heads must cease, so there can only remain seven winning chances, viz.  $a^3 + 3a^2 + 3a$ ; whereof only one, viz. the first, belongs to B.

Also,  $\frac{6}{7}$  of  $\frac{1}{15} = \frac{2}{35}$  } = value of { A's } second winning  
 And  $\frac{1}{7}$  of  $\frac{1}{15} = \frac{1}{105}$  } B's } tofs.

Also,  $\frac{1}{15} + \frac{2}{35} = \frac{7}{105}$ ; and  $\frac{1}{15} + \frac{1}{105} = \frac{8}{105}$ .

Consequently, A's chance to that of B's, is as 73 to 32; viz. more than 9 to 4.

$\therefore \frac{7}{15} + \frac{2}{35} = \frac{73}{105} = 2l. 8s. 4\frac{4}{5}d.$  = value of A's }  
 and  $\frac{1}{15} + \frac{1}{105} = \frac{8}{105} = 1l. 5s. 7\frac{1}{3}d.$  = value of B's } chance.

5. Two gamesters met the other day,  
 The one call'd B, the other A;  
 But having neither cards nor dice,  
 They get to hotch-cap in a trice  
 With 16 halfpence fair and flat,  
 All which they hustled in a hat.

Says A to B, all these are mine,  
 And I will lay a pint of wine,  
 That in two trials there will be  
 Nine heads or tails, as here you see.  
 No matter which, but on they play'd,  
 Till silver, brass, and gold were laid;  
 But as to B, his chance was bad,  
 For he got broke of all he had.

What were the odds, I pray declare,  
 Groom-porters fly; ladies fair?

*By the late Mr. Joseph Smith, of Fleet.*

$2^{15} = 65536$ , number of different chances on 16 halfpence.

Let  $a$  represent the heads, and  $b$  the tails.

Then  $a^{16} + 16a^{15}b + 120a^{14}b^2 + 560a^{13}b^3 + 182a^{12}b^4 + 4368a^{11}b^5 + 8008a^{10}b^6 + 11440a^9b^7$ .

Then  $11440 \times 2 = 22880$  chances for nine heads or tails to come.

$\therefore 65536 - 22880 = 42656$ , not to come the first time ; viz. 22880 to 42656, that they come nine heads or tails the first toss.

As  $65536 : 22880 :: 42656 : 14892\frac{7}{8}$ .

Then  $22880 + 14892\frac{7}{8} = 37772\frac{7}{8}$ , for } nine heads

Also  $65536 - 37772\frac{7}{8} = 30763\frac{1}{8}$ , against } or 9 tails  
turning up once in two trials.

From hence A's chance to that of B,

Is something more than four to three.

6. Two gamesters one day at dice they would play,

And being full merry in wine,

Says B unto A, what odds will you lay,

I cast not all the six faces this time?

Say A then to B, ten to one I'll lay thee,

With six dice the six faces you cast not.

Pray, gentlemen, shew, and next year let them know,

For the odds on the cast, first, they do not.

$6^6 = 46656$  different combinations.

And  $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$  variations.

Then  $46656 - 720 = 45936$  chances against A.

But as A laid 10 to 1  $\therefore 7200$  chances for B.

$\therefore$  A's chance to that of B, is as 45936 to 7200, or as 6.38 to 1.

*A TABLE shewing the probability of winning or losing any number of games together, when the gamesters are equal.*

Powers.

Odds.

|           |     |          |           |     |                                   |   |
|-----------|-----|----------|-----------|-----|-----------------------------------|---|
| $2^- = -$ | 2   | } viz. { | 2 — 1 =   | 1   | } to 1, } Games running.<br>not { | 1 |
| $2^1 =$   | 4   |          | 4 — 1 =   | 3   |                                   | 2 |
| $2^2 =$   | 8   |          | 8 — 1 =   | 7   |                                   | 3 |
| $2^3 =$   | 16  |          | 16 — 1 =  | 15  |                                   | 4 |
| $2^4 =$   | 32  |          | 32 — 1 =  | 31  |                                   | 5 |
| $2^5 =$   | 64  |          | 64 — 1 =  | 63  |                                   | 6 |
| $2^6 =$   | 128 |          | 128 — 1 = | 127 |                                   | 7 |
| $2^7 =$   | 256 |          | 256 — 1 = | 255 |                                   | 8 |

7. To find how many holes a person can make at cribbage, that has the whole pack in his hand,

First, for the sequences ;

As 4 is the number of ways an ace can be shewn, it follows that  $4^2$  will be the number of different ways that one ace and one duce can be shewn; and  $4^3$  the number of sequences with an ace, duce and tray, &c. &c.

Whence it appears, that  $4^{13}$  will be the number of sequences of 13 in each; which multiplied by 13, viz.  $4^{13} \times 13 = 872415232$ , the number of holes to which all the sequences amount.

Secondly, for the number of fifteens.

The determination of these depends upon the following cases, according to the several ways by which 15 can be made by 2, 3, 4, 5, &c. cards, the number corresponding to which cases are found from the following

THEOREM.

Let  $a$  = number of cards of one sort,  $b$  of a second, and  $c$  of a third (all the cards together making 15) then will  $\frac{1}{1} \times \frac{1}{2} (a) \times \frac{1}{3} \times \frac{1}{4} (b) \times \frac{1}{5} \times \frac{1}{6} (c)$  &c. be the number of 15's corresponding.

For example, Let there be two 5's, one (tray) or 3, and one (duce) or 2, then will the number of 15's corresponding be  $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = 96$ .

|               | N <sup>o</sup> of ways. |
|---------------|-------------------------|
| 5 - - - - -   | 64                      |
| 4 + 1 - - -   | 256                     |
| 3 + 2 - - -   | 256                     |
| 3 + 1 + 1 -   | 384                     |
| 2 + 2 + 1 -   | 384                     |
| 2 + 1 + 1 + 1 | 256                     |

Sum 1600

|                   |     |
|-------------------|-----|
| 6 - - - - -       | 16  |
| 5 + 1 - - -       | 64  |
| 4 + 2 - - -       | 64  |
| 3 + 3 - - -       | 24  |
| 4 + 1 + 1 -       | 96  |
| 3 + 2 + 1 -       | 256 |
| 2 + 2 + 2 -       | 16  |
| 3 + 1 + 1 + 1     | 64  |
| 2 + 2 + 1 + 1     | 144 |
| 2 + 1 + 1 + 1 + 1 | 16  |

Sum 760

|                       | N <sup>o</sup> of ways. |
|-----------------------|-------------------------|
| 7 + 1 - - -           | 24                      |
| 6 + 2 - - -           | 64                      |
| 5 + 3 - - -           | 64                      |
| 4 + 4 - - -           | 24                      |
| 6 + 1 + 1 -           | 96                      |
| 5 + 2 + 1 -           | 256                     |
| 4 + 3 + 1 -           | 256                     |
| 4 + 2 + 2 -           | 96                      |
| 3 + 3 + 2 -           | 96                      |
| 5 + 1 + 1 + 1         | 64                      |
| 4 + 2 + 1 + 1         | 384                     |
| 3 + 3 + 1 + 1         | 144                     |
| 3 + 2 + 2 + 1         | 384                     |
| 2 + 2 + 2 + 2         | 4                       |
| 4 + 1 + 1 + 1 + 1     | 16                      |
| 3 + 2 + 1 + 1 + 1     | 256                     |
| 2 + 2 + 2 + 1 + 1     | 96                      |
| 2 + 2 + 1 + 1 + 1 + 1 | 24                      |

Sum 2348

L 14

N<sup>o</sup>

|     |                           | N° of ways. |          |                               | N° of ways. |
|-----|---------------------------|-------------|----------|-------------------------------|-------------|
| 8 + | 7 - - - - -               | 16          | 5 +      | 5 + 5 - - - -                 | 4           |
|     | 6 + 1 - - - -             | 64          |          | 5 + 4 + 1 - -                 | 96          |
|     | 5 + 2 - - - -             | 64          |          | 5 + 3 + 2 - -                 | 96          |
|     | 4 + 3 - - - -             | 64          |          | 4 + 4 + 2 - -                 | 96          |
|     | 5 + 1 + 1 - -             | 96          |          | 4 + 3 + 3 - -                 | 96          |
|     | 4 + 2 + 1 - -             | 256         |          | 5 + 3 + 1 + 1                 | 144         |
|     | 3 + 3 + 1 - -             | 96          |          | 5 + 2 + 2 + 1                 | 144         |
|     | 3 + 2 + 2 - -             | 96          |          | 4 + 4 + 1 + 1                 | 144         |
|     | 4 + 1 + 1 + 1             | 64          |          | 4 + 3 + 2 + 1                 | 1024        |
|     | 3 + 2 + 1 + 1             | 384         |          | 4 + 2 + 2 + 2                 | 64          |
|     | 2 + 2 + 2 + 1             | 64          |          | 3 + 3 + 3 + 1                 | 64          |
|     | 3 + 1 + 1 + 1 + 1         | 16          |          | 3 + 3 + 2 + 2                 | 144         |
|     | 2 + 2 + 1 + 1 + 1         | 96          |          | 5 + 2 + 1 + 1 + 1             | 96          |
|     |                           |             |          | 4 + 3 + 1 + 1 + 1             | 256         |
|     |                           | Sum 1376    |          | 4 + 2 + 2 + 1 + 1             | 576         |
| 6 + | 6 + 3 - - - -             | 24          | 4 +      | 3 + 3 + 2 + 1 + 1             | 576         |
|     | 5 + 4 - - - -             | 84          |          | 3 + 2 + 2 + 2 + 1             | 256         |
|     | 6 + 2 + 1 - -             | 96          |          | 4 + 2 + 1 + 1 + 1 + 1         | 64          |
|     | 5 + 3 + 1 - -             | 256         |          | 3 + 3 + 1 + 1 + 1 + 1         | 24          |
|     | 5 + 2 + 2 - -             | 96          |          | 3 + 2 + 2 + 1 + 1 + 1         | 384         |
|     | 4 + 4 + 1 - -             | 96          |          | 2 + 2 + 2 + 2 + 1 + 1         | 24          |
|     | 4 + 3 + 2 - -             | 256         |          | 2 + 2 + 2 + 1 + 1 + 1 + 1     | 16          |
|     | 3 + 3 + 3 - -             | 16          |          |                               |             |
|     | 6 + 1 + 1 + 1 + 1         | 24          | Sum 4388 |                               |             |
|     | 5 + 2 + 1 + 1 + 1         | 384         | 4 +      | 4 + 4 + 3 - -                 | 16          |
|     | 4 + 3 + 1 + 1 + 1         | 384         |          | 4 + 4 + 2 + 1                 | 64          |
|     | 4 + 2 + 2 + 1 + 1         | 384         |          | 4 + 3 + 3 + 1                 | 144         |
|     | 3 + 3 + 2 + 1 + 1         | 384         |          | 4 + 3 + 2 + 2                 | 144         |
|     | 3 + 2 + 2 + 2 + 1         | 64          |          | 3 + 3 + 3 + 2                 | 64          |
|     | 5 + 1 + 1 + 1 + 1 + 1     | 16          |          | 4 + 4 + 1 + 1 + 1 + 1         | 16          |
|     | 4 + 2 + 1 + 1 + 1 + 1     | 256         |          | 4 + 3 + 2 + 1 + 1 + 1         | 576         |
|     | 3 + 3 + 1 + 1 + 1 + 1     | 96          |          | 4 + 2 + 2 + 2 + 1 + 1         | 96          |
|     | 3 + 2 + 2 + 1 + 1 + 1     | 576         |          | 3 + 3 + 3 + 1 + 1 + 1         | 96          |
|     | 2 + 2 + 2 + 1 + 1 + 1     | 16          |          | 3 + 3 + 2 + 2 + 1 + 1         | 576         |
|     | 3 + 2 + 1 + 1 + 1 + 1 + 1 | 64          |          | 3 + 2 + 2 + 2 + 2 + 1         | 16          |
|     | 2 + 2 + 2 + 1 + 1 + 1 + 1 | 64          |          | 4 + 3 + 1 + 1 + 1 + 1 + 1     | 24          |
|     |                           |             |          | 4 + 2 + 2 + 1 + 1 + 1 + 1     | 144         |
|     |                           |             |          | 3 + 3 + 2 + 1 + 1 + 1 + 1     | 384         |
|     |                           | Sum 3616    |          | 3 + 2 + 2 + 2 + 1 + 1 + 1     | 384         |
|     |                           |             |          | 3 + 2 + 2 + 1 + 1 + 1 + 1 + 1 | 96          |
|     |                           |             |          | 2 + 2 + 2 + 2 + 1 + 1 + 1 + 1 | 16          |
|     |                           |             | Sum 2856 |                               |             |

|                                 | N <sup>o</sup> of ways. |
|---------------------------------|-------------------------|
| 3 + { 3 + 3 + 3 + 2 + 1 - - - - | 16                      |
| 3 + { 3 + 3 + 2 + 2 + 2 - - - - | 16                      |
| 3 + { 3 + 3 + 3 + 1 + 1 + 1 - - | 4                       |
| 3 + { 3 + 3 + 2 + 2 + 1 + 1 - - | 144                     |
| 3 + { 3 + 2 + 2 + 2 + 2 + 1 - - | 24                      |
| 3 + { 3 + 3 + 2 + 1 + 1 + 1 + 1 | 16                      |
| 3 + { 3 + 2 + 2 + 2 + 1 + 1 + 1 | 96                      |
| Sum                             | 320                     |

Then  $1600 + 760 + 1376 + 2348 + 3616 + 4388 + 2856 + 320 = 17264$ , different ways to count 15; and  $17264 \times 2 = 34528$ , the number of holes.

Lastly, The number of prials will be 13; and  $13 \times 12 = 156$ , number of holes.

$\therefore 872415232 + 34528 + 156 = 872449916$  holes the pack will make. Q. E. F.



## SECT. V. MAGIC SQUARES.

**A** MAGIC SQUARE is a square figure composed of a series of numbers in arithmetical proportion, so disposed in parallel and equal ranks, as that the sum of each row taken either perpendicularly, horizontally, or diagonally, are equal.

In ignorant ages, when mathematics passed for magic, these squares were made use of by conjurers, for the construction of talismans.

However, they have since become the serious research among mathematicians; not that they are of any real or solid use, or advantage, but only as a kind of play, where the difficulty makes the merit, as it may chance to produce some new views of numbers, which mathematicians will not lose the occasion of.

1. The numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9, being given to form them in a magic square, viz. counting each rank perpendicularly, horizontally, or diagonally, that those ranks may be equal to each other.

Suppose

Suppose it done, and represented in its proper form, by the following symbols thus placed, viz.

$$\begin{array}{ccc} a & b & c \\ d & e & f \\ g & h & i \end{array}$$

First, the sum of the progression numbers are 45.

Then  $\frac{45}{3} =$  number of rows.

Also  $\frac{45}{3} = 15 =$  sum of each side or rank.

And  $\frac{15}{3} = 5 = e$ , the middle number.

Again, to find the corner figures, and first to find the figure represented by  $a$ .

Beginning with 1, I find the corner letter  $a$ , or any other corner letter, cannot be 1; for if  $a$  was  $= 1$ , then  $i$  must be 9; and  $b + c = 15 - 1 = 14$ ; as also  $d + g = 15 - 1 = 14$ . But there remains no two numbers after 5, 1, and 9, whose sum is 14, but 6 and 8:  $\therefore$  if any of those figures were  $b$ , the other would be  $c$ ; and then no figures would remain for the value of either  $d$  or  $g$ ; wherefore  $a$  is not equal 1, nor any corner letter equal 1 or 9.

3 cannot be  $= a$ ; for if it were, then  $i$  should be  $= 7$ ; and  $b + c = 15 - 3 = 12$ ; as also  $d + g = 12$ : but there remains no two numbers after 5, 3, and 7, whose sum is 12, but 8 + 4, which cannot answer to  $b$  and  $c$ , and  $d$  and  $g$ ; wherefore  $a$ , or any other corner letter, is not  $= 3$ ; neither is  $i$ , nor any other corner letter,  $= 7$ .

From what hath been said, it is plain, that (if the question proposed is capable of being solved) the corner letters are all even numbers; wherefore, if  $a = 2$ ,  $i$  will be  $= 8$ , and  $e$  must be either 4, or 6. Let  $e = 4$ ; then  $g = 6$ ,  $b = 9$ ,  $d = 7$ ,  $f = 3$ , and  $h = 1$ ; and so the square is completed as required.

$$\begin{array}{ccc} 2 & 9 & 4 \\ 7 & 5 & 3 \\ 6 & 1 & 8 \end{array}$$

But if  $e$  were equal 6 ( $a$  being  $= 2$ ); then  $g = 4$ ,  $b = 7$ ,  $d = 9$ ,  $f = 1$ , and  $h = 3$ , and then the squares will stand thus:

|   |   |   |
|---|---|---|
| 2 | 7 | 6 |
| 9 | 5 | 1 |
| 4 | 3 | 8 |

Or they may be found mechanically thus : set them all down progressively, about which draw a square cornerways.



Then set the four angular figures at the corners, and put the outermost alternately.

|   |   |   |
|---|---|---|
| 2 | . | 6 |
| . | 5 | . |
| 4 | . | 8 |

|   |   |   |
|---|---|---|
| 2 | 7 | 6 |
| 9 | 5 | 1 |
| 4 | 3 | 8 |

2. Let it be required to form the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16, into a magic square, viz. so that counting each rank from one hand to the other, as also up, down, and diagonal-wise, those ranks may be equal to each other.

Suppose it done, and represented in its proper form by the following symbols, viz.

|          |          |          |          |
|----------|----------|----------|----------|
| <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
| <i>e</i> | <i>f</i> | <i>g</i> | <i>h</i> |
| <i>i</i> | <i>k</i> | <i>l</i> | <i>m</i> |
| <i>n</i> | <i>o</i> | <i>p</i> | <i>q</i> |

The sum of the said progression figures are 136.

Also 4 = number of rows.

Then  $\frac{136}{4} = 34 =$  sum of each side, or rank.

Now beginning with the least of those numbers, put  $a = 1$ .

Then the other corner letter  $n$  cannot be  $= 2$ ; for if it was,  $b + m = a + n = 1 + 2$  would be 3; but there are no remaining two numbers of the given ones, whose sum is 3, therefore  $n$  cannot be  $= 2$ ,  $a$  being  $= 1$ .

Neither can  $n = 3$ ; for supposing  $n = 3$ , then  $b + m = a + n = 1 + 3 = 4$ ; but there are no remaining two numbers, whose sum is 4.

Now putting  $n = 4$ ; then  $b + m = a + n = 1 + 4 = 5$ ; that is,  $b, m = 2, 3$ , which are the only two numbers

bers remaining, whose sum is 5:  $\therefore d + g = 34 - 5 = 29$ ; that is,  $d, g$  are 13, 16, or 14, 15, for no other couple amounts to 29.

Suppose again  $g = 13$ ; then must  $d = 16$ , when we design the square in part, viz.

|        |     |     |     |      |
|--------|-----|-----|-----|------|
|        | $x$ | $b$ | $c$ | 16   |
|        |     | $f$ | $g$ |      |
| 14, 15 |     | $k$ | $l$ | 2, 3 |
|        | 4   | $o$ | $p$ | 13   |

As the four corner figures are fixed, and  $e, i = 14, 15$ ; also  $b, m = 2, 3$ ; it is plain  $f$  cannot be 5, 6, or 7, for if it was,  $l$  would be 15, 14 or 13, which numbers are already disposed of; therefore, supposing it 8, and then  $l = 12$ .

Again,  $g + k = 14$ , and as there remain no two numbers whose sum is 14, but only  $5 + 9$ ; but  $k + 12 = 2 + 14 = e + b$ .  $17 = 2 + 15$ , or  $3 + 14$ , or  $18 = 3 + 15$ ; consequently  $k = 4, 5$  or  $6$  (not equal to 9)  $\therefore k = 5$ . And  $k + l = 5 + 12 = 17$ , must likewise be  $= e + b$ ; which may be effected two different ways, either by putting  $e = 15$  or  $14$ , and then  $b = 2$  or  $3$ , by chusing the former;  $i = 14$ , and  $m = 3$ , and then the square will be farther designable, viz.

|  |     |     |     |    |
|--|-----|-----|-----|----|
|  | $x$ | $b$ | $c$ | 16 |
|  | 15  | 8   | 9   | 2  |
|  | 14  | 5   | 12  | 3  |
|  | 4   | $o$ | $p$ | 13 |

It remains to dispose of four numbers, 6, 7, 10 and 11, and instead of  $b, c, o$  and  $p$ , so as  $b + c$  may be  $= 17$ ; as also  $o + p = 17$ ; which may be done by coupling 6, 11; as also 7, 10: but  $c + p$  must be  $= k + f = 13$ , which will be effected by  $6 + 7$ ; from whence  $p$  being  $= 6$ ,  $c$  will be  $= 7$ ; and then  $o = 11$ , and consequently  $b = 10$ ; and then the square will be fully completed, thus:

|  |     |    |    |    |
|--|-----|----|----|----|
|  | $x$ | 10 | 7  | 16 |
|  | 15  | 8  | 9  | 2  |
|  | 14  | 5  | 12 | 3  |
|  | 4   | 11 | 6  | 13 |

Or putting  $p = 7$ ; then  $c = 6, o = 10$ , and  $b = 11$ ; and then the square will stand thus:



|    |    |    |    |
|----|----|----|----|
| 1  | 11 | 6  | 16 |
| 15 | 8  | 9  | 2  |
| 14 | 5  | 12 | 3  |
| 4  | 10 | 7  | 13 |

Or by setting down the numbers progressionally, reserving the diagonal numbers, the square may be filled up by an easy transposition of the rest, as follows :

|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 1  | .  | .  | 4  | 1  | 15 | 14 | 4  |
| .  | 6  | 7  | .  | 12 | 6  | 7  | 9  |
| .  | 10 | 11 | .  | 8  | 10 | 11 | 5  |
| 13 | .  | .  | 16 | 13 | 3  | 2  | 16 |

3. Suppose a square form of set numbers there be,  
In their natural order (as 1, 2, and 3)  
Amount to the sum, when they're added together,  
Of 62 juſt, in rank and file either :  
If alſo from corner to corner you count,  
Yet ſtill 62 ſhall be their amount ;  
What numbers are they, and how muſt they be put,  
When ſixteen there be that completely will do't ?

The ſum of one row  $62 \times 4$  (number of rows) =  $248$  =  
ſum of all thoſe progressional numbers.

And their common difference is = 1.

16)  $248$  (15.5. Then  $15.5 - 7.5 = 8$ , the firſt term.

And  $8 + 15 = 23$  = laſt term.

Then obſerving the directions given in the foregoing  
queſtion, this magic ſquare may be filled as follows ;

|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 8  | .  | .  | 11 | 8  | 22 | 21 | 11 |
| .  | 13 | 14 | .  | 19 | 13 | 14 | 16 |
| .  | 17 | 18 | .  | 15 | 17 | 18 | 12 |
| 20 | .  | .  | 23 | 20 | 10 | 9  | 23 |

4. 'Tis to you, lovely ladies, I ſue and ſubmit,  
(Who outvie Sidrophel in magic and wit)  
For ſolution of this knotty problem propos'd,  
By which undertaking my ſenſes are doz'd ;  
To find by what method thoſe ſquares you may fill,  
Which are magical call'd, and by that try your ſkill ;  
8, 9, 10, 11, 14, 15, 16, 17, 20, 21, 22, 23, 26,  
27, 28, 29.

To place all these numbers, so that the amount,  
Just half a score ways, seventy-four you may count.  
If you'll answer but this, now yourself do assure,  
I will meddle with what they call magic no more.

|    |    |    |    |
|----|----|----|----|
| 8  | .  | .  | 11 |
| .  | 15 | 16 | .  |
| .  | 21 | 22 | .  |
| 26 | .  | .  | 29 |

|    |    |    |    |
|----|----|----|----|
| 8  | 28 | 27 | 11 |
| 23 | 15 | 16 | 20 |
| 17 | 21 | 22 | 14 |
| 26 | 10 | 9  | 29 |

5. To form a magic square of the numbers 1, 2, 4, 5, &c. to 25, inclusive.

First, 5 are the number of rows;  
also 325, the sum of those progression numbers;  
and 5) 325 (65 = sum of each side or rank.

|    |    |    |    |    |
|----|----|----|----|----|
| 16 | 14 | 8  | 2  | 25 |
| 3  | 22 | 20 | 11 | 9  |
| 15 | 6  | 4  | 23 | 17 |
| 24 | 18 | 12 | 10 | 1  |
| 7  | 5  | 21 | 19 | 13 |

6. To form the progression numbers from 1 to 49, both inclusive (their common difference being 1) into a magic square.

First,  $1 + 49 = 50 \times 49 = 2450$ .  
Also, 2) 2450 (1225 = sum of those progression numbers.  
 $\therefore 7) 1225 (175 = \text{sum of each row.}$

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| 1  | 9  | 17 | 25 | 33 | 41 | 49 |
| 24 | 32 | 40 | 48 | 7  | 8  | 16 |
| 47 | 6  | 14 | 15 | 23 | 31 | 39 |
| 21 | 22 | 30 | 38 | 46 | 5  | 13 |
| 37 | 45 | 4  | 12 | 20 | 28 | 29 |
| 11 | 19 | 27 | 35 | 36 | 44 | 3  |
| 34 | 42 | 43 | 2  | 10 | 18 | 26 |

7. You that delight in figures, try your skill,  
A magic square with numbers for to fill;  
One to a hundred numbers just must be,  
Which to the numbers of the squares agree:  
But farther, you must them so just contrive,  
Twenty-two ways, make five hundred and five.

No two squares alike in numbers must be,  
But ten in breadth, and ten in length, let's see.

|     |    |    |    |    |    |    |    |    |    |
|-----|----|----|----|----|----|----|----|----|----|
| 11  | 92 | 12 | 88 | 14 | 15 | 16 | 84 | 83 | 90 |
| 100 | 82 | 26 | 27 | 67 | 35 | 59 | 58 | 50 | 1  |
| 99  | 19 | 75 | 74 | 33 | 66 | 42 | 43 | 51 | 2  |
| 2   | 20 | 76 | 73 | 34 | 36 | 60 | 57 | 49 | 98 |
| 4   | 81 | 25 | 28 | 68 | 65 | 41 | 44 | 52 | 97 |
| 94  | 21 | 77 | 72 | 32 | 37 | 61 | 56 | 48 | 7  |
| 5   | 80 | 24 | 29 | 69 | 64 | 40 | 45 | 53 | 96 |
| 6   | 79 | 23 | 30 | 70 | 38 | 62 | 55 | 47 | 95 |
| 93  | 22 | 78 | 71 | 31 | 63 | 39 | 46 | 54 | 8  |
| 91  | 9  | 89 | 13 | 87 | 86 | 85 | 17 | 18 | 10 |

Any one of the foregoing squares may be disposed many other different ways, as may be tried by those who have time and inclination for such operations.



## CHAPTER VI.

### SECT. I.

### COMPOUND INTEREST.

**C**OMPOUND INTEREST is that which ariseth not only from the use of the principal, but also from the use of the interest as it becomes due; the interest being added to the principal at the end of every year, making a new principal for the succeeding year; so that the principal and interest are continually increasing.

#### CASE I.

The principal, rate, and time given, to find the interest.

#### RULE I.

To the principal add the interest for the first year, which will be a new principal; to which add the interest for another year, for a fresh principal for the second year; and so proceed for any number of years.

#### RULE II.

Multiply the principal by the amount of one pound for one year continually for all the proposed years; the last product will be the amount as before.

I. What

1. What is the compound interest of 500 l. for four years, at five per cent.?

$$\begin{array}{r} 1. \\ 5 = \frac{1}{20} ) 500, \text{ principal} \quad - - - - - \\ \quad 25, \text{ interest for the first year} \end{array} \left. \vphantom{\begin{array}{r} 1. \\ 5 = \frac{1}{20} ) 500, \text{ principal} \quad - - - - - \\ \quad 25, \text{ interest for the first year} \end{array}} \right\} \text{add.}$$

$$\begin{array}{r} 20) 525, \text{ amount for the 1st, or principal for 2d year.} \\ \quad 26 \quad 5, \text{ interest for the second year.} \end{array}$$

$$\begin{array}{r} 20) 551 \quad 5, \text{ amount for 2d. principal for the 3d.} \\ \quad 27 \quad 11 \quad 3, \text{ interest for the 3d year.} \end{array}$$

$$\begin{array}{r} 20) 578 \quad 16 \quad 3, \text{ amount for 3d. principal for the 4th.} \\ \quad 28 \quad 18 \quad 9\frac{1}{2}, \text{ interest for the 4th year.} \end{array}$$

$$\begin{array}{r} 607 \quad 15 \quad -\frac{1}{2}, \text{ amount for the 4th year.} \\ 500 \quad - \quad -, \text{ principal.} \end{array}$$

$$\underline{\underline{\pounds 107 \quad 15 \quad -\frac{1}{2}, \text{ interest.}}}$$

### By RULE II.

The amount of 1 l. for one year, is 1.05.

Then 500

$$\begin{array}{r} 1.05 \\ 525, \text{ amount for the first year.} \end{array}$$

$$\begin{array}{r} 1.05 \\ 2625 \end{array}$$

$$\begin{array}{r} 525 \\ 551.25, \text{ amount for the second year.} \end{array}$$

$$\begin{array}{r} 1.05 \\ 275625 \end{array}$$

$$\begin{array}{r} 55125 \\ 578.8125, \text{ amount for the third year.} \end{array}$$

$$\begin{array}{r} 1.05 \\ 28940625 \end{array}$$

$$\begin{array}{r} 5788125 \\ 607.753125, \text{ amount for the fourth year.} \end{array}$$

$$\begin{array}{r} 500. \\ \text{principal.} \end{array}$$

$$\underline{\underline{107.753125, \text{ interest.}}}$$

2. What

2. What is the compound interest of 760 l. 10 s. for four years, at four per cent. per annum?

l. s.

20 =  $\frac{1}{2}$  760 10, principal.

$\frac{1}{2}$  152 2

d.

30 8

$4\frac{1}{2}$ , interest the first year.

$\frac{1}{2}$  790 18.  $4\frac{1}{2}$ , amount; principal for the 2d year.

$\frac{1}{2}$  158 3 8

31 12  $8\frac{1}{2}$ , interest the 2d year.

$\frac{1}{2}$  822 11.  $4\frac{1}{2}$ , amount; principal for the 3d year.

$\frac{1}{2}$  164 10  $2\frac{1}{2}$

32 18  $-1\frac{1}{2}$ , interest for 3d year.

$\frac{1}{2}$  855 9 2, amount; principal for the 4th year.

$\frac{1}{2}$  171 1 10

34 4  $4\frac{1}{2}$ , interest for the 4th year.

889 13  $6\frac{1}{2}$ , amount for the 4th year.  
760 10 -, principal.

£ 129 3  $6\frac{1}{2}$ , interest.

## By RULE II.

The amount of 1 l. for a year, at four per cent. is 1.04

$$\begin{array}{r}
 1. \\
 \text{Then } 760.5 \\
 \quad 1.04 \\
 \hline
 30420 \\
 7605 \\
 \hline
 790.92, \text{ amount for the first year.} \\
 \quad 1.04 \\
 \hline
 316368 \\
 79092 \\
 \hline
 822.5568, \text{ amount for the 2d year.} \\
 \quad 1.04 \\
 \hline
 82902272 \\
 8225568 \\
 \hline
 855.459072 \\
 \quad 1.04 \\
 \hline
 3421836 \\
 855459 \\
 \hline
 889.67736, \text{ amount for the 4th year.} \\
 760.5 \quad \text{principal.} \\
 \hline
 129.17736, \text{ interest} = 129 \text{ l. } 3 \text{ s. } 6\frac{1}{2} \text{ d.} \\
 \hline
 \end{array}$$

## C A S E II.

The amount, rate per cent. and time given, to find the principal or present worth.

## R U L E.

As the amount of 1 l. compound interest, at the rate and for the time given : is to 1 l. :: so is the amount given : to the present worth required.

3. What

3. What is the present worth of 889l. 13s. 6½d. due four years hence, at four per cent. per annum, compound interest?

First,  $1.04 \times 1.04 \times 1.04 \times 1.04 = 1.16985856$  :

Therefore  $1.16985856 : 1 :: 889.67736 : 760.5$

Answer, 760l. 10s. present worth.

### CASE III.

The principal, rate, and amount given, to find the time.

### RULE.

Divide the amount by the principal, and that quotient by the amount of 1l. for a year, and the next quotient by the same; and so on continually, till the last quotient be unity; the number of which divisions will be the time required.

4. In what time will 760l. 10s. amount to 889l. 13s. 6½d. compound interest, being allowed at four per cent.?

760.5) 889.677083 (1.16

|      |      |         |     |             |
|------|------|---------|-----|-------------|
| 1.04 | 1.16 | - - - - | 1st | } division. |
| 1.04 | 1.12 | - - - - | 2d  |             |
| 1.04 | 1.08 | - - - - | 3d  |             |
| 1.04 | 1.04 | - - - - | 4th |             |

Hence the term is four years.

The 4th Case is to find the rate per cent. the principal, amount, and time given; but this requires the extraction of the roots of very high powers, or the use of logarithms; which (as my book is swelled to a greater bigness than at first intended) I am obliged to omit.

And for the solving questions in compound interest with more facility, have inserted the following tables.

The construction of the first table following, shewing the amount of 1l. for years, is only by the involution of the amount of 1l. for years, to the power of the number of years.

Thus, the amount of 1 l. for two years, at five per cent. compound interest, will be  $1.05 \times 1.05 = 1.1025$ .

Also,  $1.05 \times 1.05 \times 1.05 = 1.157625$  = the amount of 1 l. for three years, at five per cent.

And the construction of the second table is by the continual multiplication of the amount of 1 l. for a day; the amount of 1 l. for a day being the root of its amount for a year, extracted to the 365th power.

The amount of 1 l. for a day, at five per cent. being 1.0001336, its amount for two days will be  $1.0001336 \times 1.0001336 = 1.0002672$  &c. and  $1.0001336 \times 1.0001336 \times 1.0001336 = 1.0004011$ , the amount of 1 l. at compound interest, for three days, at five per cent.

And thus by continually multiplying by the amount for a day, at each rate per cent. the second table is constructed; and the 364th product will be the amount.



## DECIMAL TABLES of COMPOUND INTEREST,

At the rates of 3, 3½, 4, 4½, and 5 per cent. per annum.

TABLE I.

The amount of one pound for years.

| Years. | 3 per cent. | 3½ p. cent. | 4 per cent. | 4½ p. cent. | 5 per cent. |
|--------|-------------|-------------|-------------|-------------|-------------|
| 1      | 1.0300000   | 1.0350000   | 1.0400000   | 1.0450000   | 1.0500000   |
| 2      | 1.0609000   | 1.0712250   | 1.0816000   | 1.0920250   | 1.1025000   |
| 3      | 1.0927270   | 1.1037178   | 1.1248640   | 1.1411661   | 1.1576250   |
| 4      | 1.1255088   | 1.1375230   | 1.1698586   | 1.1925186   | 1.2255063   |
| 5      | 1.1592740   | 1.1878863   | 1.2166529   | 1.2461819   | 1.2762816   |
| 6      | 1.1940523   | 1.2292553   | 1.2653190   | 1.3022601   | 1.3409956   |
| 7      | 1.2298738   | 1.2722792   | 1.3159318   | 1.3608612   | 1.4071004   |
| 8      | 1.2667700   | 1.3168090   | 1.3685691   | 1.4221006   | 1.4774554   |
| 9      | 1.3047721   | 1.3628973   | 1.4233118   | 1.4860951   | 1.5514282   |
| 10     | 1.3439163   | 1.4105987   | 1.4802443   | 1.5529694   | 1.6288946   |
| 11     | 1.3842338   | 1.4599697   | 1.5394542   | 1.6228550   | 1.7103393   |
| 12     | 1.4257608   | 1.5119686   | 1.6010922   | 1.6958824   | 1.7958563   |
| 13     | 1.4684327   | 1.5639560   | 1.6650735   | 1.7721961   | 1.8856491   |
| 14     | 1.5122587   | 1.6186945   | 1.7316764   | 1.8519449   | 1.9799316   |
| 15     | 1.5572674   | 1.6753488   | 1.8009435   | 1.9352824   | 2.0789282   |
| 16     | 1.6034064   | 1.7339860   | 1.8729812   | 2.0223701   | 2.1828746   |
| 17     | 1.6507476   | 1.7946755   | 1.9479005   | 2.1133768   | 2.2920183   |
| 18     | 1.7002430   | 1.8574892   | 2.0258165   | 2.2084787   | 2.4066192   |
| 19     | 1.7519306   | 1.9225013   | 2.1068492   | 2.3078603   | 2.5269502   |
| 20     | 1.8058112   | 1.9897888   | 2.1911231   | 2.4117140   | 2.6532977   |
| 21     | 1.8609945   | 2.0594314   | 2.2787681   | 2.5202411   | 2.7859626   |
| 22     | 1.9165034   | 2.1315115   | 2.3699188   | 2.6336520   | 2.9252607   |
| 23     | 1.9733585   | 2.2061144   | 2.4647155   | 2.7521663   | 3.0712338   |
| 24     | 2.0327941   | 2.2833284   | 2.5633042   | 2.8760138   | 3.2250000   |
| 25     | 2.0937779   | 2.3632449   | 2.6658303   | 3.0054344   | 3.3863549   |
| 26     | 2.1565912   | 2.4459585   | 2.7724697   | 3.1406790   | 3.5556727   |
| 27     | 2.2212890   | 2.5315671   | 2.8833683   | 3.2820095   | 3.7334563   |
| 28     | 2.2879276   | 2.6201719   | 2.9987033   | 3.4296999   | 3.9212911   |
| 29     | 2.3565655   | 2.7118779   | 3.1186514   | 3.5840364   | 4.1161356   |
| 30     | 2.4272684   | 2.8067937   | 3.2433975   | 3.7453181   | 4.3194224   |
| 31     | 2.5000803   | 2.9050314   | 3.3731334   | 3.91338574  | 4.5380395   |
| 32     | 2.5750827   | 3.0067075   | 3.5080587   | 4.0899810   | 4.7649415   |
| 33     | 2.6522332   | 3.1119423   | 3.6483811   | 4.2740301   | 5.0018885   |
| 34     | 2.7313903   | 3.2208608   | 3.7943163   | 4.4663615   | 5.2533480   |
| 35     | 2.81238624  | 3.3335904   | 3.9460829   | 4.6673478   | 5.5160154   |
| 36     | 2.8952783   | 3.4503661   | 4.1039325   | 4.8773784   | 5.7918161   |
| 37     | 2.9802266   | 3.5713254   | 4.2680893   | 5.0968604   | 6.0814069   |
| 38     | 3.07747834  | 3.69669113  | 4.4388834   | 5.3264182   | 6.3854773   |
| 39     | 3.1670269   | 3.8253717   | 4.6163659   | 5.5638990   | 6.7047511   |
| 40     | 3.2620377   | 3.9592597   | 4.8010206   | 5.8163645   | 7.0399887   |

## DECIMAL TABLES of COMPOUND INTEREST.

TABLE II.

The amount of one pound for days.

|     |           |           |           |           |           |
|-----|-----------|-----------|-----------|-----------|-----------|
| 1   | 1.0000809 | 1.0000942 | 1.0001074 | 1.0001206 | 1.0001336 |
| 2   | 1.0001619 | 1.0001885 | 1.0002149 | 1.0002412 | 1.0002673 |
| 3   | 1.0002428 | 1.0002822 | 1.0003214 | 1.0003602 | 1.0004001 |
| 4   | 1.0003240 | 1.0003770 | 1.0004199 | 1.0004624 | 1.0005048 |
| 5   | 1.0004050 | 1.0004773 | 1.0005174 | 1.0005571 | 1.0005968 |
| 6   | 1.0004860 | 1.0005656 | 1.0006044 | 1.0006432 | 1.0006823 |
| 7   | 1.0005670 | 1.0006600 | 1.0007052 | 1.0007445 | 1.0007836 |
| 8   | 1.0006482 | 1.0007542 | 1.0008000 | 1.0008395 | 1.0008789 |
| 9   | 1.0007291 | 1.0008486 | 1.0008963 | 1.0009359 | 1.0009757 |
| 10  | 1.0008101 | 1.0009442 | 1.0010051 | 1.0010466 | 1.0010876 |
| 20  | 1.0016209 | 1.0018867 | 1.0021513 | 1.0024158 | 1.0026803 |
| 30  | 1.0024324 | 1.0028315 | 1.0032302 | 1.0036284 | 1.0040261 |
| 40  | 1.0032445 | 1.0037771 | 1.0043094 | 1.0048415 | 1.0053731 |
| 50  | 1.0040573 | 1.0047316 | 1.0053871 | 1.0060429 | 1.0066989 |
| 60  | 1.0048703 | 1.0056910 | 1.0064490 | 1.0072668 | 1.0080845 |
| 70  | 1.0056849 | 1.0066193 | 1.0075501 | 1.0084773 | 1.0093809 |
| 80  | 1.0064996 | 1.0075645 | 1.0086333 | 1.0096942 | 1.0107511 |
| 90  | 1.0073151 | 1.0085186 | 1.0097777 | 1.0109315 | 1.0120811 |
| 100 | 1.0081311 | 1.0094696 | 1.0108033 | 1.0121314 | 1.0133459 |
| 110 | 1.0089479 | 1.0104214 | 1.0118900 | 1.0133557 | 1.0145813 |
| 120 | 1.0097653 | 1.0113742 | 1.0128979 | 1.0145765 | 1.0158099 |
| 130 | 1.0105834 | 1.0123279 | 1.0140670 | 1.0158007 | 1.0170391 |
| 140 | 1.0114021 | 1.0132825 | 1.0151572 | 1.0170265 | 1.0182690 |
| 150 | 1.0122215 | 1.0142379 | 1.0162487 | 1.0182537 | 1.0195014 |
| 160 | 1.0130405 | 1.0151943 | 1.0173412 | 1.0194844 | 1.0207358 |
| 170 | 1.0138623 | 1.0161516 | 1.0184350 | 1.0207126 | 1.0219743 |
| 180 | 1.0146837 | 1.0171098 | 1.0195299 | 1.0219442 | 1.0232157 |
| 190 | 1.0155057 | 1.0180689 | 1.0206261 | 1.0231774 | 1.0244598 |
| 200 | 1.0163284 | 1.0190288 | 1.0217233 | 1.0244120 | 1.0257049 |
| 210 | 1.0171518 | 1.0200897 | 1.0228218 | 1.0256481 | 1.0269517 |
| 220 | 1.0179759 | 1.0209515 | 1.0239215 | 1.0268851 | 1.0281994 |
| 230 | 1.0188006 | 1.0219142 | 1.0250223 | 1.0281249 | 1.0294499 |
| 240 | 1.0196260 | 1.0228778 | 1.0261243 | 1.0293655 | 1.0307021 |
| 250 | 1.0204520 | 1.0238424 | 1.0272295 | 1.0306076 | 1.0319568 |
| 260 | 1.0212783 | 1.0248078 | 1.0283319 | 1.0318512 | 1.0332135 |
| 270 | 1.0221062 | 1.0257741 | 1.0294375 | 1.0330963 | 1.0344717 |
| 280 | 1.0229342 | 1.0267414 | 1.0305443 | 1.0343429 | 1.0357313 |
| 290 | 1.0237630 | 1.0277096 | 1.0316522 | 1.0355910 | 1.0369925 |
| 300 | 1.0245924 | 1.0286786 | 1.0327614 | 1.0368406 | 1.0382554 |
| 310 | 1.0254225 | 1.0296486 | 1.0338727 | 1.0380917 | 1.0395207 |
| 320 | 1.0262532 | 1.0306195 | 1.0349832 | 1.0393444 | 1.0407884 |
| 330 | 1.0270847 | 1.0315914 | 1.0360960 | 1.0405985 | 1.0420586 |
| 340 | 1.0279168 | 1.0325641 | 1.0372099 | 1.0418542 | 1.0433313 |
| 350 | 1.0287495 | 1.0335378 | 1.0383250 | 1.0431114 | 1.0446067 |
| 360 | 1.0295830 | 1.3345183 | 1.0394413 | 1.0443700 | 1.0458844 |

The

The use of the foregoing tables.

CASE I.

Principal, rate, and time given, to find the amount.

RULE.

Multiply the amount of 1 l. found in the first table, at the rate and for the time given, by the proposed principal, and the product gives the answer.

5. What will 721 l. amount to in 21 years, at four per cent. per annum?

The tabular number against 21 years at four per cent. 2.2787681. Then  $721 \times 2.2787681 = 1642.9918 = 1642 l. 19 s. 10 d.$  the amount required.

6. What will 358 l. amount to in 40 days, at five per cent. per annum, compound interest?

In the second table, against 40 days, at five per cent. is 1.0053611.

Then  $358 \times 1.0053611 = 359.9192738 = 359 l. 18 s. 4 \frac{1}{2} d.$  the answer.

If the amount be required for any number of years exceeding those in the table, divide the given number of years into two or more such numbers as are in the table, and multiply the amounts answering thereto into one another continually, and the last product by the principal, which will be the amount required.

7. What is the amount of 82 l. 10 s. for 75 years, at five per cent. per annum, compound interest?

First,  $40 + 35 = 75$ .

The amount of 1 l. for 40 years, at 5 per cent. is 7.0399887.

Ditto for 35 years, - - - - - 5.5160154.

Then  $7.0399887 \times 5.5160154 = 38.8326861$ .

Also  $82.5 \times 38.8326861 = 3203.6966$ .

Answer, 3203 l. 13 s. 11 d.

If the amount be required for any number of days which are not in the tables, proceed as with the years in the last example.

8. What is the amount of 523 l. in 275 days, at four per cent. compound interest?

The amount of 1 l. for 270 days, at 4 per cent. is 1.0330963.  
Ditto for 5 days, at 4 per cent. is 1.0006031.

Then  $1.0330963 \times 1.0006031 = 1.0337194$ .

And  $1.0337194 \times 523 = 540.6352462$ .

Answer, 540 l. 12s. 8 d.

To find the amount for years and days, observe the following example.

9. What will 357 l. 15s. amount to in four years and 274 days, at  $3\frac{1}{2}$  per cent. per annum, compound interest?

The amt. of 1 l. for 4 years, at  $3\frac{1}{2}$  per cent. is 1.1475230.

Ditto for 270 days, at  $3\frac{1}{2}$  per cent. is 1.0157741.

Ditto for 4 days, at  $3\frac{1}{2}$  per cent. is 1.0003770.

Then  $1.147523 \times 1.0157741 \times 1.0003770 = 1.1775431$ .

And  $357.75 \times 1.1775431 = 421.266044 = 421 l. 5s. 3 d.$

the answer.

### CASE II.

Amount, rate, and time given, to find the principal.

### RULE.

Divide the amount given by the amount of 1 l. found in the first table, and the quotient will be the answer.

10. What is the present worth of 1642 l. 19s. 10 d. due 21 years hence, at 4 per cent. per annum, compound interest?

The amt. of 1 l. in 21 years, at 4 per cent. is 2.2787681.

Then  $2.2787681 \div 1642.9918 (721, \text{ the answer.})$

11. What is the present worth of 3203 l. 13s. 11 d. due 75 years hence, at 5 per cent. compound interest?

The amt. of 1 l. for 49 years, at 5 per cent. is 7.0399887.

Ditto for 35 years, at 5 per cent. is 5.5160154.

Then  $7.0399887 \times 5.5160154 = 38.8326864$ .

$38.8326864 \div 3203.6966 (82.5 = 82 l. 10s. \text{ the answer.})$

12. What is the present worth of 421 l. 5s. 4 d. due four years and 274 days hence, at  $3\frac{1}{2}$  per cent. per annum?

By

By TABLE first and second.

The amount of 1 l. for four years, is 1.147523.  
 Ditto for 270 days, 1.0257741.  
 Ditto for four days, 1.000377.  
 Then  $1.147523 \times 1.0257741 \times 1.000377 = 1.1775431$ .  
 $1.1775431 \times 421.2668 (357.75 = 357 \text{ l. } 15 \text{ s.})$  the answer.

CASE III.

Any principal, rate, and amount being given, to find the time.

RULE.

Divide the amount by the principal, and the quotient will be the amount of 1 l. at the given rate, which will be found in the first table under that rate, even with the time required.

13. In what time will 721 l. amount to 1642 l. 19 s. 10 d. at 4 per cent. per annum compound interest?

721 1642.9918 (2.278768, the amount of 1 l. for the time; opposite to which, under 4 per cent. in the second table, is 25 years, the answer required.

But if the quotient cannot be truly found in the table, take out the next number, and make it a divisor, by which divide the first quotient, and seek the second quotient in table the second; but if it cannot be truly found in that table, take out the next least number there, and divide the second quotient by it, and then seek again for the third quotient, and the number thus found in the table is the number of days.

14. In what time will 357 l. 15 s. amount to 421 l. 5 s. 4 d. at  $3\frac{1}{2}$  per cent. compound interest?

357.75 421.26 (1.177543, the number next to which, under  $3\frac{1}{2}$  per cent. stands against four years, and is 1.147523.

Then  $1.147523 \times 1.177543 (1.0261608, \text{ the next less number to which, under } 3\frac{1}{2} \text{ per cent. stands against 270 days, and is as follows;}$

viz.

viz. 1.025741) 1.0261608 (1.00377 stands against four days.  
Answer, four years and 274 days.

## C A S E IV.

Principal, time, and amount given, to find the rate of interest.

## R U L E.

Divide the amount by the principal, and the quotient will be the amount of 1l. which being found in the first table, even with the given time, is under the rate required.

15. At what rate per cent. per annum will 721l. become 1642l. 19s. 10d. in 21 years?

721) 1642.9918 (2.278768, the amount of 1l. for 21 years, which will be found under 4 per cent. the answer to the question.



## S E C T. II.

*Purchasing FREEHOLD or REAL ESTATES at COM-  
POUND INTEREST.*

**F**reehold, or real estates, are such as are purchased to continue for ever; questions relating to which (except in reversion) are solved in the most easy manner only by the rule of three.

## C A S E I.

When the yearly income is required.

## R U L E.

As 100l. : is to the proposed rate per cent. : : so is the sum to be laid out : to the yearly income.

1. A person desirous to lay out 1760l. in the purchase of a freehold estate, so as to get  $4\frac{1}{2}$  per cent. for the money, compound interest; what must be the annual income of such an estate?

$$100 : 4.5 :: 1760$$

$$4.5$$

---


$$880$$

---


$$704$$

$$100) 7920.0 (79.2 = 79l. 4s. \text{ the answer.}$$

C A S E

# CASE II.

If the value of the estate is required.

## RULE.

As the rate per cent. is to 100l. :: so is the yearly rent to the value required.

2. An estate brings in yearly 79l. 4s. what would it sell for, allowing the purchaser  $4\frac{1}{2}$  per cent. for his money?

$4.5 : 100 :: 79.2 : 1760$  l. the answer.

# CASE III.

To find the rate per cent. on money laid out on the purchase of freehold estates.

## RULE.

As the money laid out on the purchase is to the yearly rent, :: so is 100l. to the rate per cent.

3. Suppose 1760 l. be paid for a freehold estate, which yields yearly 79l. 4s. what rate of interest hath the purchaser for his money?

$1760 : 79.2 :: 100 : 4.5 = 4\frac{1}{2}$  per cent. answer.

4. Suppose an estate of 79l. 4s. per annum be sold at  $22\frac{2}{5}$  years purchase; how much per cent. hath the purchaser for his money?

$22.4 \times 79.2 = 1760 : 79.2 :: 100 : 4.5$  per cent.

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

# SECT. III.

*Purchasing FREEHOLD ESTATES in REVERSION.*

## CASE I.

THE yearly rent of a freehold estate being known, to find the present worth of the reversion of the said estates after the expiration of a certain number of years.

2.

RULE.

## R U L E.

Find the full value of the estate by the second case of the last section. Then, by case the second of compound interest, find what principal or fund will amount to the full value of the estate at the time and rate given.

Suppose the reversion of a freehold estate 99l. 04 s. per annum, to commence seven years hence, is to be sold; what is its worth in ready money, allowing the purchaser 4½ per cent. for his money?

First, as 4.5 : 100 :: 99.04 : 1760.12

By the first table, against seven years, the amount of 1l. is found 1.3608618.

Then 1.3608618 : 1 :: 1760.12 : 1293.2981147

Answer, 1293l. 5s. 11½ d. nearly.

## C A S E II.

The fund given for the reversion of a freehold estate, to commence after a certain number of years; being known, to find the yearly income, allowing the purchaser so much per cent. for his money.

## R U L E.

Find the amount of the purchase money to the time when the reversion is to commence, by the first case of compound interest, then find the yearly income which that amount will purchase.

Suppose the reversion of a freehold estate, to commence seven years hence, is sold for 1293l. 5s. 11½ d. allowing the purchaser 4½ per cent. compound interest, for his money; what ought the yearly rent to be?

The amount of 1l. for seven years, at 4½ per cent. is 1.3608618.

Then 1293.2981147  $\times$  1.3608618 = 1760.12 amount.

And 100 : 4.5 :: 1760 : 79.2.

Answer, 79l. 4 s. per annum.



SECT. IV.

PURCHASING ANNUITIES.

**A**Nnuities, pensions, salaries, &c. as rents, profits, and payments, made yearly or half-yearly, &c. and they are said to be in arrears when they are due and unpaid for any number of payments.

In order to solve questions in annuities, I have inserted four tables more of compound interest, the construction whereof follow.

*Construction of the first TABLE of ANNUITIES.*

This table shews the present worth or value of  $\text{r} \text{ l.}$  payable at any period, from one to forty years inclusive, and is constructed, by dividing  $\text{r} \text{ l.}$  by its amount found in the second table of compound interest for the time and rate assigned.

Against the first table of compound interest, the present worth of  $\text{r} \text{ l.}$  for three years, at 3 per cent. is  $\text{r} \text{ } 092727$ .  $\text{r} \text{ } 092727$   $\text{r} \text{ } 000000$   $\text{r} \text{ } 9151417$ , for the present worth of  $\text{r} \text{ l.}$  three years hence, compound interest, at 3 per cent.

*Construction of the second TABLE.*

This table shews the amount of  $\text{r} \text{ l.}$  per annum, and is constructed from the first table of compound interest, thus: To  $\text{r} \text{ l.}$  the first year of this table, add the first year of the table for years in compound interest, and the amount will be the second year in this table; to which add the second year in the table of compound interest, and the amount of it will be the third year in this table, &c.

Thus  $\text{r} \text{ } 000000$

add  $\text{r} \text{ } 030000$  the amt. of  $\text{r} \text{ l.}$  for 1 year, at 3 per cent.

$\text{r} \text{ } 030000$  the amount of  $\text{r} \text{ l.}$  for 2 years.

$\text{r} \text{ } 060930$  amount of the second year.

$\text{r} \text{ } 090900$  third year of the second table.

*Construction*

*Construction of the third TABLE.*

The third table shews the present value of 1 l. per annum, and is constructed as follows, viz. the present value of the first year in the first table, is the same as the first year in the third table; the first and second years in the first table, added together, make the second year in the third table; and the third year in the first table, added to the second year in the third table, make the third year in the third.

Thus, 1st year, tables 1st and 3d, at 3 per cent. is .9708738  
The 2d year, in table 1st, - - - - - .9435959

Their sum, 2d year in the third table, is - - 1.9134697  
Third year, in the first table, - - - - - .9131417

Third year, in the third table, - - - - - 2.8286114

*Construction of the fourth TABLE.*

This table shews what annuity 1 l. will purchase, &c. and is constructed, by finding the present worth of 1 l. per annum in the third table at the assigned rate and time, and dividing unity thereby, and the quotient will be the annuity that 1 l. will purchase at the same rate for the same time.

EXAMPLE. What annuity will 1 l. purchase, to continue three years, at 3 per cent.?

In the third table, under 3 per cent. opposite to three years, is 2.8286114

2.8286114) 1.00000000 (.3535304, the annuity for three years.

## DECIMAL TABLE of COMPOUND INTEREST.

TABLE I.

The present worth of one pound for years.

| Years. | 3 percent. | 3½ per ct. | 4 percent. | 4½ per ct. | 5 percent. |
|--------|------------|------------|------------|------------|------------|
| 1      | .9708738   | .9661836   | .9615385   | .9569378   | .9523809   |
| 2      | .9425959   | .9335107   | .9245562   | .9157299   | .9070295   |
| 3      | .9151417   | .9019427   | .8889964   | .8762966   | .8638376   |
| 4      | .8884870   | .8714422   | .8548042   | .8385613   | .8227025   |
| 5      | .8626088   | .8419732   | .8219271   | .8024512   | .7835262   |
| 6      | .8374843   | .8135006   | .7903145   | .7674957   | .7462234   |
| 7      | .8130915   | .7859910   | .7599178   | .7348285   | .7108823   |
| 8      | .7894092   | .7594216   | .7306902   | .7031851   | .6768394   |
| 9      | .7664167   | .7337310   | .7025867   | .6729044   | .6446059   |
| 10     | .7440939   | .7089188   | .6755642   | .6439277   | .6139122   |
| 11     | .7224213   | .6849457   | .6495809   | .6161988   | .5846792   |
| 12     | .7013799   | .6617833   | .6245971   | .5896639   | .5568374   |
| 13     | .6809513   | .6394041   | .6005741   | .5642716   | .5303214   |
| 14     | .6611778   | .6177818   | .5774751   | .5399729   | .5050679   |
| 15     | .6421669   | .5968906   | .5552645   | .5167204   | .4810271   |
| 16     | .6231669   | .5767059   | .5339082   | .4944693   | .4581175   |
| 17     | .6040164   | .5572038   | .5133733   | .4731764   | .4362967   |
| 18     | .5873946   | .5383611   | .4936281   | .4528004   | .4155207   |
| 19     | .5702860   | .5201557   | .4746424   | .4333018   | .3957340   |
| 20     | .5536758   | .5025659   | .4563870   | .4144429   | .3768895   |
| 21     | .5375493   | .4853709   | .4388336   | .3967874   | .3589424   |
| 22     | .5218925   | .4691506   | .4219554   | .3797009   | .3418409   |
| 23     | .5066917   | .4532856   | .4057263   | .3633501   | .3252713   |
| 24     | .4919337   | .4379571   | .3901215   | .3477035   | .3100679   |
| 25     | .4776056   | .4232470   | .3751168   | .3327396   | .2953028   |
| 26     | .4636947   | .4088378   | .3606892   | .3184225   | .2812407   |
| 27     | .4501891   | .3950123   | .3468166   | .3046914   | .2678485   |
| 28     | .4370768   | .3816543   | .3334775   | .2915707   | .2550936   |
| 29     | .4243464   | .3687482   | .3206514   | .2790150   | .2439463   |
| 30     | .4119868   | .3562784   | .3083187   | .2670000   | .2313775   |
| 31     | .3999871   | .3442304   | .2964603   | .2555024   | .2203595   |
| 32     | .3883370   | .3325897   | .2850579   | .2444999   | .2098662   |
| 33     | .3770263   | .3213427   | .2740942   | .2339712   | .1999726   |
| 34     | .3660449   | .3104761   | .2635521   | .2238559   | .1903548   |
| 35     | .3553834   | .2999769   | .2534155   | .2142544   | .18112903  |
| 36     | .3450324   | .2898327   | .2436687   | .2050282   | .17228574  |
| 37     | .3349829   | .2800316   | .2342969   | .1961992   | .1644356   |
| 38     | .3252262   | .2705619   | .2252854   | .1877504   | .1564656   |
| 39     | .3157556   | .2614125   | .2166206   | .1796655   | .1492479   |
| 40     | .3065568   | .2525725   | .2082890   | .1719287   | .1420457   |

## DECIMAL TABLES of COMPOUND INTEREST.

## TABLE II.

The amount of one pound per annum, or annuity for years.

| Year. | 3 per cent. | 3½ per cent. | 4 per cent. | 4½ per cent. | 5 per cent. |
|-------|-------------|--------------|-------------|--------------|-------------|
| 1     | 1.0000000   | 1.0000000    | 1.0000000   | 1.0000000    | 1.0000000   |
| 2     | 2.0300000   | 2.0350000    | 2.0400000   | 2.0450000    | 2.0500000   |
| 3     | 3.0909000   | 3.1062250    | 3.1216000   | 3.1370250    | 3.1525000   |
| 4     | 4.1836270   | 4.2149429    | 4.2464640   | 4.2781911    | 4.3101250   |
| 5     | 5.3091358   | 5.3624659    | 5.4163226   | 5.4707097    | 5.5256312   |
| 6     | 6.4684099   | 6.5501522    | 6.6329755   | 6.7168917    | 6.8019128   |
| 7     | 7.6624622   | 7.7794075    | 7.8982945   | 8.0191518    | 8.1420024   |
| 8     | 8.8923360   | 9.0516866    | 9.2142263   | 9.3800136    | 9.5491029   |
| 9     | 10.1591061  | 10.3684958   | 10.5827953  | 10.8021142   | 11.0265643  |
| 10    | 11.4638793  | 11.7313931   | 12.0061071  | 12.2882094   | 12.5778225  |
| 11    | 12.8077957  | 13.1419919   | 13.4863514  | 13.8411788   | 14.2067871  |
| 12    | 14.1920296  | 14.6019616   | 15.0258055  | 15.4640318   | 15.9171266  |
| 13    | 15.6177904  | 16.1130303   | 16.6268377  | 17.1599133   | 17.7229828  |
| 14    | 17.0863242  | 17.6769864   | 18.2919112  | 18.9221094   | 19.5986320  |
| 15    | 18.5989139  | 19.2956809   | 20.0235876  | 20.7240543   | 21.5785636  |
| 16    | 20.1568813  | 20.9710297   | 21.8245311  | 22.7293367   | 23.6574918  |
| 17    | 21.7615877  | 22.7050158   | 23.6975124  | 24.7417069   | 25.8403664  |
| 18    | 23.4144354  | 24.4996913   | 25.6454129  | 26.8550837   | 28.1323247  |
| 19    | 25.1168684  | 26.3571805   | 27.6712204  | 29.0635625   | 30.5390039  |
| 20    | 26.8703745  | 28.2796818   | 29.7780786  | 31.3714228   | 33.0659541  |
| 21    | 28.6764857  | 30.2694707   | 31.9692017  | 33.7831368   | 35.7192128  |
| 22    | 30.5367803  | 32.3289022   | 34.2479698  | 36.3033779   | 38.5052144  |
| 23    | 32.4528837  | 34.4604137   | 36.6178886  | 38.9370299   | 41.4304751  |
| 24    | 34.4264702  | 36.6665282   | 39.0826041  | 41.6891963   | 44.5019989  |
| 25    | 36.4598643  | 38.9498567   | 41.6459083  | 44.5652101   | 47.7270988  |
| 26    | 38.5530428  | 41.3131017   | 44.3117446  | 47.5706446   | 51.1134538  |
| 27    | 40.7096335  | 43.7590602   | 47.0842144  | 50.7113236   | 54.6691265  |
| 28    | 42.9309225  | 46.2906273   | 49.9675830  | 53.9933332   | 58.4025828  |
| 29    | 45.2183502  | 48.9107993   | 52.9662863  | 57.4230332   | 62.3227119  |
| 30    | 47.5754157  | 51.6226773   | 56.0849377  | 61.0070697   | 66.4328479  |
| 31    | 50.0026782  | 54.4294710   | 59.3283352  | 64.7523878   | 70.7607899  |
| 32    | 52.5027585  | 57.3345025   | 62.7014687  | 68.6662452   | 75.2988294  |
| 33    | 55.0778413  | 60.3412101   | 66.2095274  | 72.7562263   | 80.0637708  |
| 34    | 57.7301765  | 63.4531524   | 69.8579085  | 77.0302565   | 85.0669594  |
| 35    | 60.4680081  | 66.6740127   | 73.6522248  | 81.4966180   | 90.3203073  |
| 36    | 63.2959443  | 70.0076032   | 77.5983138  | 86.1639618   | 95.8361227  |
| 37    | 66.2142226  | 73.4578693   | 81.7022464  | 91.0413443   | 101.6281328 |
| 38    | 69.21594493 | 77.0288947   | 85.9703362  | 96.1332048   | 107.7095458 |
| 39    | 72.2342327  | 80.7249060   | 90.4091497  | 101.4644240  | 114.0950271 |
| 40    | 75.4012597  | 84.5502778   | 95.0255157  | 107.0822231  | 120.7993742 |

## DECIMAL TABLES of COMPOUND INTEREST.

TABLE III.

The present worth of one pound per annum, or annuity  
for years.

| Years. | 3 per cent. | 3½ per cent. | 4 per cent. | 4½ per cent. | 5 per cent. |
|--------|-------------|--------------|-------------|--------------|-------------|
| 1      | 0.9708738   | 0.9661836    | 0.9615535   | 0.9569378    | 0.9523809   |
| 2      | 1.9134697   | 1.8996943    | 1.8860947   | 1.8726678    | 1.8594154   |
| 3      | 2.8286114   | 2.8016370    | 2.7750910   | 2.7489644    | 2.7232480   |
| 4      | 3.7170984   | 3.6730792    | 3.6298952   | 3.5875257    | 3.5459505   |
| 5      | 4.5797072   | 4.5150524    | 4.4518223   | 4.3899767    | 4.3294757   |
| 6      | 5.4171914   | 5.3285530    | 5.2421369   | 5.1578725    | 5.0756921   |
| 7      | 6.2302829   | 6.1145439    | 6.0020547   | 5.8927009    | 5.7863734   |
| 8      | 7.0196922   | 6.8739555    | 6.7327448   | 6.5958861    | 6.4632128   |
| 9      | 7.7861089   | 7.6076865    | 7.4353314   | 7.2687905    | 7.1078217   |
| 10     | 8.5302028   | 8.3166053    | 8.1108955   | 7.9127182    | 7.7217549   |
| 11     | 9.2526241   | 9.0015510    | 8.7604763   | 8.5289169    | 8.3064142   |
| 12     | 9.9540040   | 9.6633343    | 9.3850733   | 9.1185808    | 8.8632526   |
| 13     | 10.6349553  | 10.3027385   | 9.9856473   | 9.6828524    | 9.3935730   |
| 14     | 11.2960731  | 10.9205203   | 10.5631223  | 10.2228253   | 9.8986409   |
| 15     | 11.9379351  | 11.5174169   | 11.1183868  | 10.7395457   | 10.3796580  |
| 16     | 12.5611020  | 12.0941168   | 11.6522649  | 11.2340131   | 10.8377695  |
| 17     | 13.1661185  | 12.6513206   | 12.1656680  | 11.7071914   | 11.2740662  |
| 18     | 13.7535131  | 13.1896817   | 12.6592961  | 12.1599918   | 11.6895869  |
| 19     | 14.3237991  | 13.7098374   | 13.1339385  | 12.5932936   | 12.0853208  |
| 20     | 14.8774748  | 14.2124033   | 13.5903253  | 13.0079365   | 12.4622109  |
| 21     | 15.4150241  | 14.6979742   | 14.0291589  | 13.4047239   | 12.8211527  |
| 22     | 15.9369166  | 15.1671248   | 14.4511142  | 13.7844248   | 13.1630026  |
| 23     | 16.4433608  | 15.6204105   | 14.8568405  | 14.1477749   | 13.4889739  |
| 24     | 16.9355421  | 16.0583076   | 15.2469619  | 14.4954784   | 13.79946418 |
| 25     | 17.4131477  | 16.4815146   | 15.6220787  | 14.8282089   | 14.0939445  |
| 26     | 17.8768424  | 16.8903523   | 15.9827678  | 15.1466115   | 14.3752853  |
| 27     | 18.3270315  | 17.2853645   | 16.3295844  | 15.4513028   | 14.6430336  |
| 28     | 18.7641082  | 17.6670188   | 16.6630618  | 15.7428735   | 14.8981272  |
| 29     | 19.1884546  | 18.0357670   | 16.9837132  | 16.0218885   | 15.141.735  |
| 30     | 19.6004413  | 18.3920454   | 17.2920318  | 16.2888835   | 15.3724510  |
| 31     | 20.0004285  | 18.7362758   | 17.5884921  | 16.5443509   | 15.5928104  |
| 32     | 20.3887655  | 19.0688656   | 17.8735500  | 16.7888909   | 15.8026766  |
| 33     | 20.7657918  | 19.3902082   | 18.1476441  | 17.0223621   | 16.0025491  |
| 34     | 21.1318367  | 19.7006842   | 18.4111062  | 17.2467580   | 16.1929039  |
| 35     | 21.4872200  | 20.0006612   | 18.6646116  | 17.4610124   | 16.3741942  |
| 36     | 21.8328525  | 20.2904938   | 18.90822803 | 17.6660406   | 16.5468416  |
| 37     | 22.1672354  | 20.5705254   | 19.1425771  | 17.8622398   | 16.7118872  |
| 38     | 22.4924616  | 20.8416874   | 19.3678625  | 18.0499902   | 16.8673916  |
| 39     | 22.8082151  | 21.1024999   | 19.5844831  | 18.2296557   | 17.0170006  |
| 40     | 23.1147719  | 21.3550723   | 19.7927721  | 18.4015844   | 17.1590891  |

## DECIMAL TABLES of COMPOUND INTEREST.

TABLE IV.

The annuity which one pound will purchase for any number of years.

| Years. | 3 per cent. | 3½ per ct. | 4 per cent. | 4½ per ct. | 5 per cent. |
|--------|-------------|------------|-------------|------------|-------------|
| 1      | 1.0300000   | 1.350000   | 1.0400000   | 1.0450000  | 1.0500000   |
| 2      | .5226108    | .5264005   | .5301961    | .5339976   | .5378049    |
| 3      | .3535304    | .3569342   | .3603485    | .3637734   | .3672086    |
| 4      | .2690271    | .2722511   | .2754901    | .2787437   | .2820118    |
| 5      | .2183546    | .2214814   | .2246271    | .2277916   | .2309748    |
| 6      | .1845975    | .1876682   | .1907619    | .1938784   | .1970175    |
| 7      | .1605064    | .1635445   | .1666096    | .1697015   | .1728198    |
| 8      | .1424564    | .1454767   | .1485279    | .1516097   | .1547218    |
| 9      | .1284339    | .1314460   | .1344930    | .1375745   | .1406901    |
| 10     | .1172305    | .1202414   | .1232909    | .1263788   | .1295046    |
| 11     | .1080775    | .1110920   | .1141490    | .1172482   | .1203889    |
| 12     | .1004621    | .1034840   | .1065522    | .1096662   | .1128254    |
| 13     | .0940295    | .0970616   | .1001437    | .1032754   | .1064558    |
| 14     | .0885263    | .0915707   | .0946690    | .0978203   | .1010240    |
| 15     | .0837666    | .0868251   | .0899411    | .0931138   | .0963423    |
| 16     | .0796119    | .0826848   | .0858200    | .0890154   | .0922699    |
| 17     | .0759525    | .0790431   | .0821985    | .0854176   | .0886991    |
| 18     | .0727087    | .0758168   | .0789933    | .0822369   | .0855462    |
| 19     | .0698139    | .0729403   | .0761386    | .0794073   | .0827450    |
| 20     | .0672157    | .0703611   | .0735818    | .0768761   | .0802426    |
| 21     | .0648718    | .0680366   | .0712801    | .0746006   | .0779961    |
| 22     | .0627174    | .0659321   | .0691988    | .0725457   | .0759705    |
| 23     | .0608139    | .0640188   | .0673091    | .0706825   | .0741368    |
| 24     | .0590474    | .0622728   | .0655868    | .0689870   | .0724749    |
| 25     | .0574279    | .0606740   | .0640120    | .0674390   | .0709525    |
| 26     | .0559383    | .059254    | .0625674    | .0660214   | .0695643    |
| 27     | .0545642    | .0578524   | .0612385    | .0647195   | .0682919    |
| 28     | .0532932    | .0566027   | .0600130    | .0635208   | .0671225    |
| 29     | .0521147    | .0554454   | .0588799    | .0624146   | .0660455    |
| 30     | .0510193    | .0543713   | .0578301    | .0613915   | .0650518    |
| 31     | .0499989    | .0533724   | .0568554    | .0604435   | .0641321    |
| 32     | .0490466    | .0524415   | .0559486    | .0595632   | .0632804    |
| 33     | .0481551    | .0515724   | .0551036    | .0587445   | .0624900    |
| 34     | .0473220    | .0507597   | .0543148    | .0579819   | .0617554    |
| 35     | .0465193    | .0499984   | .0535773    | .0572705   | .0610717    |
| 36     | .0458038    | .0492842   | .0528869    | .0566058   | .0604345    |
| 37     | .0451116    | .0486133   | .0522396    | .0559840   | .0598398    |
| 38     | .04444593   | .0479821   | .0516319    | .0554017   | .0592842    |
| 39     | .0438439    | .0473878   | .0510608    | .0548557   | .0587646    |
| 40     | .0432624    | .0468273   | .0505235    | .0543431   | .0582782    |

C A S E I.

Principal, rate, and time being given, to find the annuity.

R U L E.

Multiply the annuity which  $\text{rl.}$  will purchase, at the rate and for the time given (found in the fourth table) and the quotient will be the answer.

1. A gentleman hath 1760 l. which he would sell for an annuity, to continue 21 years, at 5 per cent. compound interest; I demand what will be his income per annum?

$$1760 \text{ l.} \times .0779961 = 137.273136.$$

Answer, 137 l. 5 s. 5½ d.

2. A fine for the lease of a tenement is settled at 153 l. under a reserved rent of 16 l. a year: now the tenant cannot conveniently pay more than 50 l. but for twelve years to come of the term is willing rather to pay an adequate rent, computing 5 per cent. compound interest; what ought that rent to be?

$$\text{First, } 153 - 50 = 103.$$

Then, by the fourth table, 1 l. will purchase for 12 years, at 5 per cent. an annuity of .1128254 per annum.

$$\text{Then } 103 \times .1128254 = 11.6210162 = 11 \text{ l. } 12 \text{ s. } 5 \text{ d. advance rent.}$$

$$\therefore 16 \text{ l.} + 11 \text{ l. } 12 \text{ s. } 5 \text{ d.} = 27 \text{ l. } 12 \text{ s. } 5 \text{ d. Q. E. F.}$$

3. A son, previous to his marriage, is minded to have 50 l. a year, freehold estate, settled on his family; and to have immediate possession of it, offers his father in lieu, an annuity for his life, valued at twelve years purchase, discounting 4 per cent. thereon; whereas he is content the estate should be valued at a discount of 3 per cent. which is 33½ years purchase; pray what had the father for his life?

$$\text{First, } 33.8 \times 50 = 1666.8 = 1666 \text{ l. } 13 \text{ s. } 4 \text{ d. value of the annuity.}$$

Then 1 l. by the fourth table, for 12 years, at 4 per cent. will purchase .1065522 per annum.

$$\therefore 1666.8 \times .1065522 = 177.58699 = 177 \text{ l. } 11 \text{ s. } 8 \frac{1}{4} \text{ d.}$$

## C A S E II.

Principal, annuity, and rate given, to find the time.

## R U L E.

Divide the annuity by the principal, and the quotient will be the annuity which 1 l. will purchase at the given rate, which will be found in the fifth table under that rate, and even with the time required.

4. If an annuity of 137 l. 5 s. 5½ d. is purchased for 1760 l. at 5 per cent. compound interest, what time ought it to continue?

1760) 137.273136 (.0779961;  
which under 5 per cent. in the 4th table, is opposite to 21 years.

## C A S E III.

Principal, annuity, and time given, to find the rate.

## R U L E.

Divide the annuity by the principal, and the quotient will be the annuity which 1 l. will purchase for the given time, which will stand even with the time, and under the rate required.

5. If an annuity of 80 l. 4 s. 10½ d. to continue 20 years, be purchased for 1000 l. what rate of interest hath the purchaser for his money?

1000) 80.2425 (.0802425, under 5 per cent. which is the answer.

## C A S E IV.

Annuity, rate, and time given, to find the amount.

## R U L E.

Find the amount of 1 l. per annum, at the rate for the time given, by the first table; by which multiply the annuity, and the product will be the amount required.

6. A minor of 14 had an annuity left him of 70 l. a year, the proceed of which, by will, was to be put out, both principal and interest, yearly, as it fell due, at 5 per cent. till he



he should attain to 21 years of age; the utmost improvement being made of this part of his fortune, what had he then to receive?

The amount of 1 l. annuity, at 5 per cent. forborn seven years, by the second table, is 8.1420084.

$$\text{Then } 8.1420084 \times 70 = 569.940588.$$

Answer, 569 l. 18 s.  $9\frac{1}{4}$  d.

### C A S E V.

Annuity, rate, and amount, being given, to find the time.

#### R U L E.

Divide the amount by the annuity, and the quotient will be the amount of 1 l. at the given rate, which will be found in the second table, under that rate, even with the time required.

7. In what time will an annuity of 70 l. amount to 569 l. 18 s.  $9\frac{1}{4}$  d. compound interest, at 5 per cent.?

70) 569.940588 (8.1420084, even with seven years in the second table, under 5 per cent.

### C A S E VI.

Annuity, time, and amount given, to find the rate.

#### R U L E.

Divide the amount by the annuity, the quotient will be the amount of 1 l. per annum, for the given rate; which will be found in the second table, below the required rate.

8. At what rate per cent. per annum will an annuity of 137 l. 5 s.  $5\frac{1}{4}$  d. amount to 1760 l. in 21 years.

137.273136) 1760.000000 (12.8211523, in the second table, below 5 per cent.

### C A S E VII.

Amount, rate, and time being given, to find the annuity.

#### R U L E.

Divide the amount given, by the amount of 1 l. found in the second table, at the rate and time given, the quotient will be the annuity required.

N n 3

9. What

9. What annuity will amount to 569 l. 18 s.  $9\frac{1}{4}$  d. in seven years, at 5 per cent.?

8.1420084) 569.940588 (70 l. the annuity required.

## C A S E VIII.

Annuity, time in reversion, and rate being given, to find the present worth.

## R U L E.

In the third table find the present value of 1 l. per annum, at the given rate, both for the time being, and also for that and the time in reversion added together, then subtract the time in being from the other, and multiply the remainder by the annuity, the product will answer the question.

10. What ought a man to give down in ready money, for the reversion of 1000 l. a year, to continue 20 years, on a lease which cannot commence till five years are at an end, allowing the purchaser compound interest at 5 per cent.?

The present value of 1 l. per annum, by table 3d, for 25 years, 14.0939445  
for five years, 4.3294767

$$9.7644678 \times 1000 = 9764.4678.$$

Answer, 9764 l. 9 s.  $4\frac{1}{4}$  d.

11. Suppose I would add five years to a running lease of 15 years to come, the improved rent being 186 l. 7 s. 6 d. per annum; what ought I to pay down for this favour, discounting 4 per cent. compound interest?

First,  $15 + 5 = 20$  years, 1 l. is worth - 13.5903253  
Also 15 years is worth - - - - - 11.1183868

2.4719385

$$\text{Then } 186.375 \times 2.4719385 = 460.70753.$$

$\therefore$  460 l. 14 s.  $1\frac{1}{4}$  d. the fine required.

12. Held of a college 486 l. 10 s. a year, on a reserved rent of 94 l. money being at 5 per cent. interest; what fine ought severally to be paid on a 7, a 14, and a 21 years lease?

$$486 \text{ l. } 10 \text{ s.} - 94 = 392 \text{ l. } 10 \text{ s. annuity.}$$

The present worth of 1 l. for the time and rate is 5.7863734.

Then  $392.5 \times 5.7863734 = 2271.15057$ .

$\therefore 2271$  l. 3s. its worth for seven years.

Also the present worth of 1 l. for 14 years, at 5 per cent. is 9.8986409.

Again,  $392.5 \times 9.8986409 = 3885.21655$ .

$\therefore 3885$  l. 4s. 4d. its worth for 14 years.

The present worth of 1 l. for 21 ys. at 5 per ct. is 12.8211527.

Also  $392.5 \times 12.8211527 = 5032.30243$ .

Answer, 5032 l. 6s. for 21 years.

### C A S E IX.

An annuity, several times in reversion, and rate given, to find the present value.

### R U L E.

In the third table find the present value of 1 l. per annum, at the given rate, for the several given times, which being severally multiplied by the annuity, the products will be the several present values of that annuity for the several times given: then subtract the several present values one from another, and the several remainders answer the question.

13. A has a term of seven years in an estate of 50 l. per annum; B hath a term of 14 years in the same estate; and C hath a further term of 10 years after B in the same estate; what is the present value of their several interests in the said estate?

First,  $7 + 14 + 10 = 31$ .

The present worth of 1 l. at 5 per ct. for 31 years, is 15.5928104

For 21 years - - - - - 12.8211527

And 7 years - - - - - 5.7863734

l. s. d.

Then  $50 \times 15.5928104 = 779.64052 = 779$  l. 12 s. 9 $\frac{1}{2}$  d.

Also  $50 \times 12.8211527 = 641.057635 = 641$  l. 1 s. 1 $\frac{1}{2}$  d.

And  $50 \times 5.7863734 = 289.31867 = 289$  l. 6 s. 4 $\frac{1}{2}$  d.

l. s. d. } value of { A's } term.  
 $\therefore 289$  l. 6 s. 4 $\frac{1}{2}$  d. }  
 l. s. d. } value of { B's }  
 Also 641 l. 1 s. 1 $\frac{1}{2}$  d. }  $- 289$  l. 6 s. 4 $\frac{1}{2}$  d. } = 351 l. 14 s. 9 $\frac{1}{2}$  d. }  
 And 779 l. 12 s. 9 $\frac{1}{2}$  d. }  $- 641$  l. 1 s. 1 $\frac{1}{2}$  d. } = 138 l. 11 s. 7 $\frac{1}{2}$  d. }

14. Which is most advantageous, a term of 19 years of an estate of 100 per annum, or the reversion of such an

estate for ever, at the expiration of the said 19 years, computing at the rate of 4 per cent. compound interest?

First,  $4 : 100 :: 100 : 2500$ l. value of the estate for ever.

And by the third table, the present worth of 100*l.* annuity for 19 years, at 4 per cent, is } 1313.3938 = 1313 7 10 $\frac{1}{2}$   
 Value of the reversion is - - 1186.6062 = 1186 12 1 $\frac{1}{2}$

The late Mr. Demoisire and Mr. Simpson have both handled this subject in a very skilful manner; from the latter of which I have extracted the following tables and problems, whereby an annuity on any life or lives may be valued according to the probability of the continuation thereof.

*The use of the Table of Lives.*

If it was required the probability that a person of 36 lives 30 years longer:

Look in the table against 36 years, and opposite thereto is the number 331.

Also against 66 is the number 93, which shews, that out of 331 persons living of 36, only 93 arrived at 66.

∴  $\frac{93}{331}$  is the measure of the probability required.

Let it be required to find the value of an annuity of 100l. for a life of 20, interest at 4 per cent.

By the second table in the foregoing section, the present worth of 100l. discount 4 per cent. due at the expiration of one year, was it sure to be paid, is 96.15385.

But the probability of the continuance of the said life one year, by this table, appears to be only  $\frac{4\frac{1}{2}}{46\frac{1}{2}}$ .

∴  $96.15385 \times \frac{4\frac{1}{2}}{46\frac{1}{2}} = 94.697$ , the value of the first year's rent.

In like manner the value of the second year's rent may be calculated; the probability of his living two years is  $\frac{448}{462} = \frac{32}{33}$ ; and 92.45562, the present worth of 100l. at the end of two years.

∴  $92.45562 \times \frac{32}{33} = 89.65393$ , value of the second year's rent.

And by a like way of proceeding, the values of the third, fourth, fifth, &c. years rents, to the utmost extent of life, may be determined; and the sum of all these will be the required value of the annuity; which will be found to come out 1480l. very near.

A TABLE showing the Probabilities of LIFE, &amp;c.

| Num. of<br>persons. | Ages<br>curr.<br>born | Num. of<br>persons | Ages<br>curr. | Num. of<br>persons. | Ages<br>curr. | Num. of<br>persons. | Ages<br>curr. |
|---------------------|-----------------------|--------------------|---------------|---------------------|---------------|---------------------|---------------|
| 1280                |                       | 462                | 20            | 294                 | 40            | 130                 | 60            |
| — 410               |                       | — 7                |               | — 10                |               | — 7                 |               |
| 870                 | 1                     | 455                | 21            | 284                 | 41            | 123                 | 61            |
| — 170               |                       | — 7                |               | — 10                |               | — 6                 |               |
| 700                 | 2                     | 448                | 22            | 274                 | 42            | 117                 | 62            |
| — 65                |                       | — 7                |               | — 10                |               | — 6                 |               |
| 635                 | 3                     | 441                | 23            | 264                 | 43            | 111                 | 63            |
| — 35                |                       | — 7                |               | — 9                 |               | — 6                 |               |
| 600                 | 4                     | 434                | 24            | 255                 | 44            | 105                 | 64            |
| — 20                |                       | — 8                |               | — 9                 |               | — 6                 |               |
| 580                 | 5                     | 426                | 25            | 246                 | 45            | 99                  | 65            |
| — 16                |                       | — 8                |               | — 9                 |               | — 6                 |               |
| 564                 | 6                     | 418                | 26            | 237                 | 46            | 93                  | 66            |
| — 13                |                       | — 8                |               | — 9                 |               | — 6                 |               |
| 551                 | 7                     | 410                | 27            | 228                 | 47            | 87                  | 67            |
| — 10                |                       | — 8                |               | — 8                 |               | — 6                 |               |
| 541                 | 8                     | 402                | 28            | 220                 | 48            | 81                  | 68            |
| — 9                 |                       | — 8                |               | — 8                 |               | — 6                 |               |
| 532                 | 9                     | 394                | 29            | 212                 | 49            | 75                  | 69            |
| — 8                 |                       | — 9                |               | — 8                 |               | — 6                 |               |
| 524                 | 10                    | 385                | 30            | 204                 | 50            | 69                  | 70            |
| — 7                 |                       | — 9                |               | — 8                 |               | — 5                 |               |
| 517                 | 11                    | 376                | 31            | 196                 | 51            | 64                  | 71            |
| — 7                 |                       | — 9                |               | — 8                 |               | — 5                 |               |
| 510                 | 12                    | 367                | 32            | 188                 | 52            | 59                  | 72            |
| — 6                 |                       | — 9                |               | — 8                 |               | — 5                 |               |
| 504                 | 13                    | 358                | 33            | 180                 | 53            | 54                  | 73            |
| — 6                 |                       | — 9                |               | — 8                 |               | — 5                 |               |
| 498                 | 14                    | 349                | 34            | 172                 | 54            | 49                  | 74            |
| — 6                 |                       | — 9                |               | — 7                 |               | — 4                 |               |
| 492                 | 15                    | 340                | 35            | 165                 | 55            | 45                  | 75            |
| — 6                 |                       | — 9                |               | — 7                 |               | — 4                 |               |
| 486                 | 16                    | 331                | 36            | 158                 | 56            | 41                  | 76            |
| — 6                 |                       | — 9                |               | — 7                 |               | — 3                 |               |
| 480                 | 17                    | 322                | 37            | 151                 | 57            | 38                  | 77            |
| — 6                 |                       | — 9                |               | — 7                 |               | — 3                 |               |
| 474                 | 18                    | 313                | 38            | 144                 | 58            | 35                  | 78            |
| — 6                 |                       | — 9                |               | — 7                 |               | — 3                 |               |
| 468                 | 19                    | 304                | 39            | 137                 | 59            | 32                  | 79            |
| — 6                 |                       | — 10               |               | — 7                 |               | — 3                 |               |
| 462                 | 20                    | 294                | 40            | 130                 | 60            | 29                  | 80            |

N. B. Those marked with the sign — are supposed to die off yearly.

PROBLEM

PROBLEM I.

To find the value of an annuity for an assigned life.

R U L E.

Look for the given age in Table I. and against it, under the assigned rate of interest, will stand the number of years purchase.

1. Suppose one of 18 years of age would sell an annuity of 100l. during his life, what ready money would the annuity be worth, allowing a discount of 4 per cent. compound interest?

First, opposite to 18 years, Table I. under 4 per cent. is 15.2 years purchase.  $\therefore 100 \times 15.2 = 1520$ l. the present worth.

2. A widow lady with 200l. a year jointure, aged 30 years, marries a young merchant, who, to enlarge his capital, proposes to sell the jointure; what ready money should he receive, discounting interest at 5 per cent.?

Opposite to 30, under 5 per cent. Table I. is 11.6.

$\therefore 200 \times 11.6 = 2320$ l. the answer required.

PROBLEM II.

To find the value of an annuity upon two assigned joint lives.

C A S E I.

If the lives are equal.

R U L E.

Against the given age, Table II. under the given rate per cent. will stand the number of years purchase.

3. Let the two given ages be each 18, and the interest 5 l. per cent. and annuity 50 l.

Table II. against 18, under 5 per cent. is 10.5.

$\therefore 50 \times 10.5 = 525$ l. the answer required.

C A S E II.

If the given ages be unequal, but neither of them less than 25, or greater than 50.

R U L E.

## R U L E,

Take half the sum of the two for a mean age, and proceed as in Case I.

4. There are two joint lives upon an annuity of 250l. one of 34, the other of 48; what is the present worth of that annuity, compound interest, at 3 per cent.?

$$\text{First, } \frac{34 + 48}{2} = 41, \text{ half sum of the ages.}$$

Table II. against 41, under 3 per cent. is 8.9 years purchase.

$$\therefore 250l. \times 8.9 = 2225l. \text{ the answer.}$$

## C A S E III.

If one or both ages be within the limits, but so that the difference of the values corresponding to those ages be not more than  $\frac{1}{4}$  of the lesser.

## R U L E.

Add  $\frac{1}{16}$  of that difference to the said lesser value, and the sum will be the value sought.

5. Let one age be 15, and the other 29, annuity 150l. interest 3 per cent. the present value is required.

Against 15, under 3 per cent. per Table II. is 13.9

And against 29 - - - - - 11.0

Difference 2.9

Also  $2.9 \times .4 = 1.16$  and  $11 + 1.16 = 12.16$ , the years purchase.

$$\therefore 12.16 \times 150 = 1824.$$

*A general RULE, be the difference of the values what they will.*

Multiply the difference of the values by half of the lesser of the two values, and divide the product by the greater; then to the lesser add the quotient, which will give the true answer very near.

6. Let one age be 11 years, and the other 68, annuity 160l. and interest at 4 per cent. the present value is required?

Against 11 years, under 4 per cent. is 12.9

Also against 68 - - - - -  $4.6 \div 2 = 2.3$

Difference 8.3

Then  $8.3 \times 2.3 = 19.09$ ; also  $12.9 + 1.48 = 14.8$ .

And  $4.6 + 1.48 = 6.08$  years purchase.

$$\therefore 160 \times 6.08 = 972l. 16s. \text{ the answer required.}$$



PROBLEM III.

To find the value of an annuity upon two lives that is to continue as long as either of them is in being.

CASE I.

If the lives be equal.

RULE.

Find the given age in Table III. and against it, under the proposed rate of interest, is the number of years purchase.

7. Let the given ages be each 50 years, and the rate of interest 4 per cent. required the value of an annuity of 30 l.?

In Table III. against 50, under 4 per cent. is 13.3 years purchase.

$$\therefore 30 \times 13.3 = 399, \text{ the value required.}$$

CASE II.

If both ages be between 25 and 50.

RULE.

Take half their sum for a mean age, which proceed with as in the last case.

8. Suppose one age to be 30 years, and the other 46, rate 3 per cent. and annuity 70 l. required the present value?

Then  $46 + 30 = 76$ , and  $\frac{76}{2} = 38$ , half their sum.

Answering to which, under 3 per cent. stands 17.7 years purchase.

$$\therefore £ 70 \times 17.7 = 1239, \text{ the answer.}$$

CASE III.

If one or both ages be without the limits mentioned in the last case, but the difference of the values corresponding to those ages, as found in Table III. be no more than  $\frac{1}{5}$  of the lesser,

RULE,

## R U L E.

Take half the sum of those values for the value required.

9. If the two proposed ages be 6 and 21 years, the annuity 25 l. and interest 4 per cent. its present value is required?

Against 6 years is - 19.7

And against 21 years is 18.2

2) 37.9 (18.95 years purchase.

$\therefore 25 \times 18.95 = 473.75 = 473 \text{ l. } 15 \text{ s.}$  the answer.

## C A S E IV.

Let the given ages be what they will.

## R U L E.

Find the value of the two joint lives, by Case IV. Prob. II. which subtract from the sum of the values of the two single lives, and the remainder will be the required value upon the longest life.

10. Let the proposed ages be 10 and 66, the rate of interest 4 per cent. and the annuity 70 l. required its present worth?

Table II. against 10 years, under } 13.0  
4 per cent. is - - - - -

Also against 66 - - - - -  $4.9 \div 2 = 2.45$

8.1 difference.

Then  $8.1 \times 2.45 = 19.845$ ; which  $\div 13 = 1.5$ ; also  $1.5 + 4.9 = 6.4$ .

Against the two single lives, per Table I. viz.  $16.4 + 7.3 = 23.7$ .

Lastly,  $23.7 - 6.4 = 17.3$  years purchase.

$\therefore 70 \text{ l.} \times 17.3 = 1211 \text{ l.}$  the answer required.

## P R O B L E M IV.

To find the value of an annuity upon three joint lives.

## C A S E I.

If all the lives be equal.

## R U L E.

R U L E.

Find out the given age in Table IV. and against it, under the proposed rate of interest, will be the number of years purchase.

11. Let each age be 27, the rate of interest 3 per cent. and the annuity 65l. its value is required ?

Table IV. against 27, under 3 per cent. is 8.8 years purchase.

$\therefore 65 \times 8.8 = 572$  l. the answer required.

C A S E II.

If all the three ages be between 15 and 55 years, and the difference between the greatest and least not more than 15 years.

R U L E.

Take  $\frac{1}{3}$  of their sum for the mean age, and proceed as in Case I.

12. Let the proposed ages be 21, 27, and 33, interest 5 per cent. and annuity 50l. its value is required ?

First,  $\frac{21 + 27 + 33}{3} = 27$ , mean age.

Also Table IV. against 27 years, under 5 per cent. is 7.3 years purchase.

$\therefore 50 \times 7.3 = 365$  l. the answer required.

C A S E III.

If one or more of the proposed ages be without the limits mentioned in Case II. but the difference of the values, answering to the greatest and least of them, be not greater than half the least.

R U L E.

To the sum of the two greatest values add twice the least, and take  $\frac{1}{4}$  of the sum for the mean value required.

13. Let the three ages be 7, 15, and 33, the annuity 50l. and interest 3 per cent. the present value is required ?

In Table IV. against  $\left\{ \begin{smallmatrix} 7 \\ 15 \\ 33 \end{smallmatrix} \right\}$  stands  $\left\{ \begin{smallmatrix} 11.9. \\ 11.2. \\ 7.9. \end{smallmatrix} \right\}$

Also  $11.9 + 11.2 = 23.1$ ; and  $79 \times 2 = 158$ ;

Then  $23.1 + 15.8 = 38.9$ ; and  $\frac{38.9}{4} = 9.725$  years val.

$\therefore 50 \times 9.725 = 486.25$ , the value required.

#### C A S E IV.

Let the ages be what they will.

#### R U L E.

Multiply the sum of the three corresponding values by the square of the least of them, reserving the product; multiply the two greater values into each other, and to the double of the product add the square of the lesser value; divide the reserved product by this sum, and subtract the quotient from twice the lesser value, the difference will be the value sought.

14. Let three ages be 13,  $31\frac{1}{2}$ , and 53 years, annuity 60l. and interest 4 per cent.

Against these in Table IV.  $\left\{ \begin{smallmatrix} 13 \\ 31\frac{1}{2} \\ 53 \end{smallmatrix} \right\}$  stand  $\left\{ \begin{smallmatrix} 10.5. \\ 7.3. \\ 5.0. \end{smallmatrix} \right\}$

Then  $10.5 + 7.3 + 5.0 = 22.8$ ; also  $5 \times 5 = 25$ .

Which  $22.8 \times 25 = 570$ , to be reserved.

Again,  $10.5 \times 7.3 = 76.65$ ; which  $\times 2 = 153.3$ .

Also  $153.3 + 25 = 178.3$   $570 \div 3.2$ , nearly.

Then  $5 \times 2 = 10$ , double the least value.

Lastly,  $10 - 3.2 = 6.8$  years purchase.

$\therefore 60 \times 6.8 = 408$ l. the value required.

#### P R O B L E M. V.

To find the value of an annuity upon the longest of three lives.

#### C A S E I.

If the lives be all equal.

#### R U L E.

Seek the common age in Table V. and against it, under the common rate of interest, will be the number of years purchase required.

15. Let

## Chap. VI. COMPOUND INTEREST: 561

15. Let the three ages be each 45 years, the annuity 275 l. and the interest 4 per cent. required its value?

In Table V. against 45, under 4 per cent. stands 15.9 years purchase.

$\therefore 275 \text{ l.} \times 15.9 = 4372 \text{ l.}$  10s. the value required.

### C A S E II.

If none of the ages be less than 10, nor greater than 60 years, and the difference of the greatest and least of them not more than 15 years,

#### R U L E,

To twice the sum of the two least add the greatest, and take  $\frac{1}{5}$  of the sum for a mean age.

16. Let the proposed ages be 16, 24, and 30 years, the annuity 170 l. and interest 4 per cent. the value is required?

First,  $16 + 24 \times 2 = 80$ ; also  $80 + 30 = 110$ .

Then  $\frac{110}{5} = 22$ , mean age, against which, Table V. is 19.4 years purchase.

$\therefore 170 \times 19.4 = 3298 \text{ l.}$  the value sought.

### C A S E III.

If the difference of the greatest and least values found against the proposed ages, in Table V. be no more than  $\frac{1}{4}$  of the least,

#### R U L E,

To twice the sum of the two greatest values add the least, taking  $\frac{1}{5}$  of the sum for a mean value.

17. Suppose the three ages be 28, 35, and 44, the rate 4 per cent. and the annuity 60 l.

By Table V. the value of the three  $\left\{ \begin{array}{l} 28 \\ 35 \\ 44 \end{array} \right\}$  are  $\left\{ \begin{array}{l} 18.3, \\ 17.3, \\ 16. \end{array} \right.$  ages, viz.

Then  $18.3 + 17.3 \times 2 = 71.2$ ; also  $71.2 + 16 = 87.2$ .

And  $5) 87.2$  (17.44 years purchase.

$\therefore 17.44 \times 60 = 1046.4 = 1046 \text{ l.}$  8s. the answer.

## CASE IV.

Let the given ages be what they will.

## RULE.

Find the value answering to the greatest of the given ages in Table III. and the values corresponding to all the three several ages in Table V. and let the difference of the values, answering to the greatest age, be taken and reserved; let the square of the greater of these two be divided by the product of the other two remaining values, and multiply the square of the quotient by the reserved difference; then this last product added to the value of the annuity for the two youngest lives, will be the value required.

18. Suppose the given ages 20, 36, and 60, the interest 4 per cent. and annuity 75*l.* the present value is required?

By Table III. the value found against 60 years is 11.2.

By Table V. those against  $\begin{Bmatrix} 20 \\ 36 \\ 60 \end{Bmatrix}$  are  $\begin{Bmatrix} 19.7, \\ 17.2, \\ 12.7. \end{Bmatrix}$

Then  $12.7 - 11.2 = 1.5$ , the reserved difference.

Again,  $12.7 \times 12.7 = 161.29$ ; also  $19.7 \times 17.2 = 338.84$ .

Then  $338.84 \div 161.290 = .5$  nearly; and  $.5 \times .5 = .25$ .

And the difference reserved  $1.5 \times .25 = .375$ , nearly .4

Also  $\frac{20 + 36}{2} = 28$ , mean age, by Case 4. Problem III.

the value of which, by Table III. is 16.9, or nearly 17 years.

And  $17 + .4 = 17.4$  years purchase.

∴  $17.4 \times 75 = 1305$  *l.* value of the annuity required.

TABLE I. *For the valuation of annuities upon one life.*

| Age. | Years purchaf.<br>at 5 per cent. | Years purchaf.<br>at 4 per cent. | Years purchaf.<br>at 3 per cent. | Age. | Years purchaf.<br>at 5 per cent. | Years purchaf.<br>at 4 per cent. | Years purchaf.<br>at 3 per cent. |
|------|----------------------------------|----------------------------------|----------------------------------|------|----------------------------------|----------------------------------|----------------------------------|
| 6    | 14.1                             | 16.2                             | 18.8                             | 41   | 10.2                             | 11.4                             | 13.0                             |
| 7    | 14.2                             | 16.3                             | 18.9                             | 42   | 10.1                             | 11.2                             | 12.8                             |
| 8    | 14.3                             | 16.4                             | 19.0                             | 43   | 10.0                             | 11.1                             | 12.6                             |
| 9    | 14.3                             | 16.4                             | 19.0                             | 44   | 9.9                              | 11.0                             | 12.5                             |
| 10   | 14.4                             | 16.4                             | 19.0                             | 45   | 9.8                              | 10.8                             | 12.3                             |
| 11   | 14.3                             | 16.4                             | 19.0                             | 46   | 9.7                              | 10.7                             | 12.1                             |
| 12   | 14.2                             | 16.3                             | 18.9                             | 47   | 9.5                              | 10.5                             | 11.9                             |
| 13   | 14.1                             | 16.2                             | 18.7                             | 48   | 9.4                              | 10.4                             | 11.8                             |
| 14   | 14.0                             | 16.0                             | 18.5                             | 49   | 9.3                              | 10.2                             | 11.6                             |
| 15   | 13.9                             | 15.8                             | 18.3                             | 50   | 9.2                              | 10.1                             | 11.4                             |
| 16   | 13.7                             | 15.6                             | 18.1                             | 51   | 9.0                              | 9.9                              | 11.2                             |
| 17   | 13.5                             | 15.4                             | 17.9                             | 52   | 8.9                              | 9.8                              | 11.0                             |
| 18   | 13.4                             | 15.2                             | 17.0                             | 53   | 8.8                              | 9.6                              | 10.7                             |
| 19   | 13.2                             | 15.0                             | 17.4                             | 54   | 8.6                              | 9.4                              | 10.5                             |
| 20   | 13.0                             | 14.8                             | 17.2                             | 55   | 8.5                              | 9.3                              | 10.3                             |
| 21   | 12.9                             | 14.7                             | 17.0                             | 56   | 8.4                              | 9.1                              | 10.1                             |
| 22   | 12.7                             | 14.5                             | 16.8                             | 57   | 8.2                              | 8.9                              | 9.9                              |
| 23   | 12.6                             | 14.3                             | 16.5                             | 58   | 8.1                              | 8.7                              | 9.6                              |
| 24   | 12.4                             | 14.1                             | 16.3                             | 59   | 8.0                              | 8.6                              | 9.4                              |
| 25   | 12.3                             | 14.0                             | 16.1                             | 60   | 7.9                              | 8.4                              | 9.2                              |
| 26   | 12.1                             | 13.8                             | 15.9                             | 61   | 7.7                              | 8.2                              | 8.9                              |
| 27   | 12.0                             | 13.6                             | 15.6                             | 62   | 7.6                              | 8.1                              | 8.7                              |
| 28   | 11.8                             | 13.4                             | 15.4                             | 63   | 7.4                              | 7.9                              | 8.5                              |
| 29   | 11.7                             | 13.2                             | 15.2                             | 64   | 7.3                              | 7.7                              | 8.3                              |
| 30   | 11.6                             | 13.1                             | 15.0                             | 65   | 7.1                              | 7.5                              | 8.0                              |
| 31   | 11.4                             | 12.9                             | 14.8                             | 66   | 6.9                              | 7.3                              | 7.8                              |
| 32   | 11.3                             | 12.7                             | 14.6                             | 67   | 6.7                              | 7.1                              | 7.6                              |
| 33   | 11.2                             | 12.6                             | 14.4                             | 68   | 6.6                              | 6.7                              | 7.4                              |
| 34   | 11.0                             | 12.4                             | 14.2                             | 69   | 6.4                              | 6.7                              | 7.1                              |
| 35   | 10.9                             | 12.3                             | 14.1                             | 70   | 6.2                              | 6.5                              | 6.9                              |
| 36   | 10.8                             | 12.1                             | 13.9                             | 71   | 6.0                              | 6.3                              | 6.7                              |
| 37   | 10.6                             | 11.9                             | 13.7                             | 72   | 5.8                              | 6.1                              | 6.5                              |
| 38   | 10.5                             | 11.8                             | 13.5                             | 73   | 5.6                              | 5.9                              | 6.2                              |
| 39   | 10.4                             | 11.6                             | 13.3                             | 74   | 5.4                              | 5.6                              | 5.9                              |
| 40   | 10.3                             | 11.5                             | 13.2                             | 75   | 5.2                              | 5.4                              | 5.6                              |

TABLE II. For the valuation of annuities upon two joint lives.

| Mean age. | Years purchaf.<br>at 5 per cent. | Years purchaf.<br>at 4 per cent. | Years purchaf.<br>at 3 per cent. | Mean age. | Years purchaf.<br>at 5 per cent. | Years purchaf.<br>at 4 per cent. | Years purchaf.<br>at 3 per cent. |
|-----------|----------------------------------|----------------------------------|----------------------------------|-----------|----------------------------------|----------------------------------|----------------------------------|
| 6         | 11.3                             | 12.7                             | 14.4                             | 41        | 7.2                              | 8.0                              | 8.9                              |
| 7         | 11.5                             | 12.9                             | 14.6                             | 42        | 7.1                              | 7.8                              | 8.7                              |
| 8         | 11.6                             | 13.0                             | 14.7                             | 43        | 7.0                              | 7.7                              | 8.6                              |
| 9         | 11.6                             | 13.0                             | 14.7                             | 44        | 6.9                              | 7.6                              | 8.5                              |
| 10        | 11.6                             | 13.0                             | 14.7                             | 45        | 6.7                              | 7.4                              | 8.3                              |
| 11        | 11.5                             | 12.9                             | 14.6                             | 46        | 6.6                              | 7.3                              | 8.2                              |
| 12        | 11.4                             | 12.8                             | 14.5                             | 47        | 6.5                              | 7.2                              | 8.1                              |
| 13        | 11.3                             | 12.7                             | 14.3                             | 48        | 6.4                              | 7.1                              | 7.9                              |
| 14        | 11.2                             | 12.5                             | 14.1                             | 49        | 6.3                              | 7.0                              | 7.8                              |
| 15        | 11.0                             | 12.3                             | 13.9                             | 50        | 6.2                              | 6.8                              | 7.6                              |
| 16        | 10.8                             | 12.1                             | 13.7                             | 51        | 6.1                              | 6.7                              | 7.4                              |
| 17        | 10.7                             | 11.9                             | 13.5                             | 52        | 6.0                              | 6.6                              | 7.3                              |
| 18        | 10.5                             | 11.7                             | 13.2                             | 53        | 5.9                              | 6.5                              | 7.2                              |
| 19        | 10.3                             | 11.5                             | 13.0                             | 54        | 5.8                              | 6.3                              | 7.0                              |
| 20        | 10.1                             | 11.3                             | 12.8                             | 55        | 5.7                              | 6.2                              | 6.9                              |
| 21        | 10.0                             | 11.2                             | 12.6                             | 56        | 5.6                              | 6.1                              | 6.7                              |
| 22        | 9.8                              | 11.0                             | 12.4                             | 57        | 5.5                              | 6.0                              | 6.6                              |
| 23        | 9.7                              | 10.8                             | 12.2                             | 58        | 5.4                              | 5.8                              | 6.4                              |
| 24        | 9.5                              | 10.6                             | 12.0                             | 59        | 5.3                              | 5.7                              | 6.3                              |
| 25        | 9.4                              | 10.5                             | 11.8                             | 60        | 5.2                              | 5.6                              | 6.1                              |
| 26        | 9.2                              | 10.3                             | 11.6                             | 61        | 5.1                              | 5.5                              | 6.0                              |
| 27        | 9.1                              | 10.1                             | 11.4                             | 62        | 5.0                              | 5.4                              | 5.9                              |
| 28        | 8.9                              | 9.9                              | 11.2                             | 63        | 4.9                              | 5.3                              | 5.7                              |
| 29        | 8.8                              | 9.8                              | 11.0                             | 64        | 4.8                              | 5.2                              | 5.5                              |
| 30        | 8.6                              | 9.6                              | 10.8                             | 65        | 4.7                              | 5.0                              | 5.4                              |
| 31        | 8.5                              | 9.4                              | 10.6                             | 66        | 4.6                              | 4.9                              | 5.3                              |
| 32        | 8.3                              | 9.2                              | 10.4                             | 67        | 4.5                              | 4.8                              | 5.1                              |
| 33        | 8.2                              | 9.1                              | 10.2                             | 68        | 4.4                              | 4.6                              | 4.9                              |
| 34        | 8.1                              | 8.9                              | 10.0                             | 69        | 4.3                              | 4.5                              | 4.8                              |
| 35        | 8.0                              | 8.8                              | 9.9                              | 70        | 4.2                              | 4.4                              | 4.6                              |
| 36        | 7.8                              | 8.6                              | 9.7                              | 71        | 4.1                              | 4.3                              | 4.5                              |
| 37        | 7.6                              | 8.4                              | 9.5                              | 72        | 3.9                              | 4.1                              | 4.3                              |
| 38        | 7.5                              | 8.3                              | 9.3                              | 73        | 3.8                              | 4.0                              | 4.2                              |
| 39        | 7.4                              | 8.2                              | 9.2                              | 74        | 3.7                              | 3.8                              | 4.0                              |
| 40        | 7.3                              | 8.1                              | 9.1                              | 75        | 3.6                              | 3.7                              | 3.8                              |



TABLE III. *For the valuation of annuities upon the longest of two lives.*

| Mean age. | Years purchaf.<br>at 5 per cent. | Years purchaf.<br>at 4 per cent. | Years purchaf.<br>at 3 per cent. | Mean age. | Years purchaf.<br>at 5 per cent. | Years purchaf.<br>at 4 per cent. | Years purchaf.<br>at 3 per cent. |
|-----------|----------------------------------|----------------------------------|----------------------------------|-----------|----------------------------------|----------------------------------|----------------------------------|
| 6         | 16.9                             | 19.7                             | 23.3                             | 41        | 13.2                             | 14.9                             | 17.0                             |
| 7         | 17.0                             | 19.8                             | 23.4                             | 42        | 13.1                             | 14.7                             | 16.8                             |
| 8         | 17.1                             | 19.9                             | 23.5                             | 43        | 13.0                             | 14.5                             | 16.5                             |
| 9         | 17.1                             | 19.9                             | 23.5                             | 44        | 12.9                             | 14.3                             | 16.3                             |
| 10        | 17.1                             | 19.9                             | 23.5                             | 45        | 12.8                             | 14.2                             | 16.1                             |
| 11        | 17.1                             | 19.9                             | 23.5                             | 46        | 12.6                             | 14.0                             | 15.8                             |
| 12        | 17.0                             | 19.8                             | 23.4                             | 47        | 12.5                             | 13.8                             | 15.6                             |
| 13        | 16.9                             | 19.7                             | 23.3                             | 48        | 12.4                             | 13.6                             | 15.3                             |
| 14        | 16.7                             | 19.5                             | 23.1                             | 49        | 12.2                             | 13.4                             | 15.1                             |
| 15        | 16.6                             | 19.3                             | 22.9                             | 50        | 12.1                             | 13.3                             | 14.9                             |
| 16        | 16.4                             | 19.1                             | 22.6                             | 51        | 11.9                             | 13.1                             | 14.6                             |
| 17        | 16.2                             | 18.9                             | 22.4                             | 52        | 11.8                             | 12.9                             | 14.4                             |
| 18        | 16.1                             | 18.7                             | 22.1                             | 53        | 11.6                             | 12.7                             | 14.1                             |
| 19        | 15.9                             | 18.5                             | 21.9                             | 54        | 11.5                             | 12.5                             | 13.9                             |
| 20        | 15.7                             | 18.3                             | 21.6                             | 55        | 11.3                             | 12.3                             | 13.6                             |
| 21        | 15.6                             | 18.2                             | 21.3                             | 56        | 11.2                             | 12.1                             | 13.4                             |
| 22        | 15.4                             | 18.0                             | 21.1                             | 57        | 11.0                             | 11.9                             | 13.1                             |
| 23        | 15.3                             | 17.8                             | 20.8                             | 58        | 10.9                             | 11.7                             | 12.8                             |
| 24        | 15.1                             | 17.6                             | 20.6                             | 59        | 10.7                             | 11.5                             | 12.5                             |
| 25        | 15.0                             | 17.4                             | 20.3                             | 60        | 10.5                             | 11.2                             | 12.2                             |
| 26        | 14.9                             | 17.3                             | 20.1                             | 61        | 10.3                             | 11.0                             | 12.0                             |
| 27        | 14.7                             | 17.1                             | 19.9                             | 62        | 10.1                             | 10.8                             | 11.7                             |
| 28        | 14.6                             | 16.9                             | 19.7                             | 63        | 9.9                              | 10.5                             | 11.4                             |
| 29        | 14.5                             | 16.8                             | 19.5                             | 64        | 9.7                              | 10.3                             | 11.1                             |
| 30        | 14.4                             | 16.6                             | 19.3                             | 65        | 9.4                              | 10.0                             | 10.8                             |
| 31        | 14.2                             | 16.4                             | 19.1                             | 66        | 9.2                              | 9.7                              | 10.5                             |
| 32        | 14.1                             | 16.2                             | 18.9                             | 67        | 8.9                              | 9.4                              | 10.2                             |
| 33        | 14.0                             | 16.1                             | 18.7                             | 68        | 8.7                              | 9.2                              | 9.9                              |
| 34        | 13.9                             | 15.9                             | 18.5                             | 69        | 8.5                              | 8.9                              | 9.5                              |
| 35        | 13.8                             | 15.8                             | 18.3                             | 70        | 8.2                              | 8.6                              | 9.2                              |
| 36        | 13.7                             | 15.6                             | 18.1                             | 71        | 8.0                              | 8.4                              | 8.9                              |
| 37        | 13.6                             | 15.5                             | 17.9                             | 72        | 7.7                              | 8.1                              | 8.6                              |
| 38        | 13.5                             | 15.3                             | 17.7                             | 73        | 7.5                              | 7.8                              | 8.2                              |
| 39        | 13.4                             | 15.2                             | 17.5                             | 74        | 7.2                              | 7.5                              | 7.9                              |
| 40        | 13.3                             | 15.0                             | 17.3                             | 75        | 6.9                              | 7.2                              | 7.5                              |

TABLE IV. *For the valuation of annuities upon three joint lives.*

| Mean age. | Years purchaf.<br>at 5 per cent. | Years purchaf.<br>at 4 per cent. | Years purchaf.<br>at 3 per cent. | Mean age. | Years purchaf.<br>at 5 per cent. | Years purchaf.<br>at 4 per cent. | Years purchaf.<br>at 3 per cent. |
|-----------|----------------------------------|----------------------------------|----------------------------------|-----------|----------------------------------|----------------------------------|----------------------------------|
| 6         | 9.7                              | 10.6                             | 11.7                             | 41        | 5.5                              | 6.1                              | 6.8                              |
| 7         | 9.9                              | 10.8                             | 11.9                             | 42        | 5.4                              | 6.0                              | 6.7                              |
| 8         | 10.0                             | 10.9                             | 12.0                             | 43        | 5.4                              | 5.9                              | 6.5                              |
| 9         | 10.0                             | 10.9                             | 12.0                             | 44        | 5.3                              | 5.8                              | 6.4                              |
| 10        | 10.0                             | 10.9                             | 12.0                             | 45        | 5.2                              | 5.7                              | 6.3                              |
| 11        | 9.9                              | 10.8                             | 11.9                             | 46        | 5.1                              | 5.6                              | 6.2                              |
| 12        | 9.8                              | 10.7                             | 11.8                             | 47        | 5.0                              | 5.5                              | 6.1                              |
| 13        | 9.6                              | 10.5                             | 11.6                             | 48        | 5.0                              | 5.4                              | 5.9                              |
| 14        | 9.5                              | 10.4                             | 11.4                             | 49        | 4.9                              | 5.3                              | 5.8                              |
| 15        | 9.3                              | 10.2                             | 11.2                             | 50        | 4.8                              | 5.2                              | 5.7                              |
| 16        | 9.2                              | 10.0                             | 11.0                             | 51        | 4.7                              | 5.1                              | 5.6                              |
| 17        | 9.0                              | 9.8                              | 10.8                             | 52        | 4.7                              | 5.1                              | 5.5                              |
| 18        | 8.8                              | 9.6                              | 10.6                             | 53        | 4.6                              | 5.0                              | 5.4                              |
| 19        | 8.6                              | 9.4                              | 10.4                             | 54        | 4.5                              | 4.9                              | 5.3                              |
| 20        | 8.4                              | 9.2                              | 10.2                             | 55        | 4.4                              | 4.8                              | 5.2                              |
| 21        | 8.2                              | 9.0                              | 10.0                             | 56        | 4.4                              | 4.7                              | 5.1                              |
| 22        | 8.1                              | 8.9                              | 9.8                              | 57        | 4.3                              | 4.6                              | 5.0                              |
| 23        | 7.9                              | 8.7                              | 9.6                              | 58        | 4.2                              | 4.5                              | 4.9                              |
| 24        | 7.7                              | 8.5                              | 9.4                              | 59        | 4.1                              | 4.4                              | 4.8                              |
| 25        | 7.6                              | 8.3                              | 9.2                              | 60        | 4.0                              | 4.3                              | 4.6                              |
| 26        | 7.4                              | 8.1                              | 9.0                              | 61        | 3.9                              | 4.2                              | 4.5                              |
| 27        | 7.3                              | 8.0                              | 8.8                              | 62        | 3.8                              | 4.1                              | 4.4                              |
| 28        | 7.1                              | 7.8                              | 8.6                              | 63        | 3.7                              | 4.0                              | 4.3                              |
| 29        | 7.0                              | 7.7                              | 8.5                              | 64        | 3.7                              | 3.9                              | 4.2                              |
| 30        | 6.8                              | 7.5                              | 8.3                              | 65        | 3.6                              | 3.8                              | 4.1                              |
| 31        | 6.7                              | 7.4                              | 8.2                              | 66        | 3.5                              | 3.7                              | 3.9                              |
| 32        | 6.5                              | 7.2                              | 8.0                              | 67        | 3.4                              | 3.6                              | 3.8                              |
| 33        | 6.4                              | 7.1                              | 7.9                              | 68        | 3.3                              | 3.5                              | 3.7                              |
| 34        | 6.2                              | 6.9                              | 7.7                              | 69        | 3.2                              | 3.4                              | 3.6                              |
| 35        | 6.1                              | 6.8                              | 7.6                              | 70        | 3.1                              | 3.3                              | 3.4                              |
| 36        | 6.0                              | 6.7                              | 7.4                              | 71        | 3.0                              | 3.1                              | 3.3                              |
| 37        | 5.9                              | 6.5                              | 7.2                              | 72        | 2.9                              | 3.0                              | 3.1                              |
| 38        | 5.8                              | 6.4                              | 7.1                              | 73        | 2.8                              | 2.9                              | 3.0                              |
| 39        | 5.7                              | 6.3                              | 7.0                              | 74        | 2.6                              | 2.7                              | 2.8                              |
| 40        | 5.6                              | 6.2                              | 6.9                              | 75        | 2.5                              | 2.6                              | 2.7                              |

TABLE V. *For the valuation of annuities upon the longest of three lives.*

| Mean age. | Years purchaf.<br>at 5 per cent. | Years purchaf.<br>at 4 per cent. | Years purchaf.<br>at 3 per cent. | Mean age. | Years purchaf.<br>at 5 per cent. | Years purchaf.<br>at 4 per cent. | Years purchaf.<br>at 3 per cent. |
|-----------|----------------------------------|----------------------------------|----------------------------------|-----------|----------------------------------|----------------------------------|----------------------------------|
| 6         | 18.0                             | 21.0                             | 25.0                             | 41        | 14.6                             | 16.4                             | 18.9                             |
| 7         | 18.1                             | 21.1                             | 25.1                             | 42        | 14.5                             | 16.3                             | 18.7                             |
| 8         | 18.2                             | 21.2                             | 25.2                             | 43        | 14.4                             | 16.2                             | 18.5                             |
| 9         | 18.2                             | 21.2                             | 25.2                             | 44        | 14.3                             | 16.0                             | 18.2                             |
| 10        | 18.2                             | 21.2                             | 25.2                             | 45        | 14.2                             | 15.9                             | 18.0                             |
| 11        | 18.2                             | 21.2                             | 25.2                             | 46        | 14.1                             | 15.7                             | 17.7                             |
| 12        | 18.1                             | 21.1                             | 25.1                             | 47        | 13.9                             | 15.5                             | 17.5                             |
| 13        | 18.0                             | 21.0                             | 25.0                             | 48        | 13.8                             | 15.2                             | 17.2                             |
| 14        | 17.9                             | 20.9                             | 24.8                             | 49        | 13.7                             | 15.1                             | 17.0                             |
| 15        | 17.8                             | 20.7                             | 24.6                             | 50        | 13.5                             | 14.9                             | 16.7                             |
| 16        | 17.6                             | 20.5                             | 24.3                             | 51        | 13.4                             | 14.7                             | 16.5                             |
| 17        | 17.5                             | 20.3                             | 24.1                             | 52        | 13.2                             | 14.5                             | 16.2                             |
| 18        | 17.3                             | 20.1                             | 23.8                             | 53        | 13.1                             | 14.3                             | 15.9                             |
| 19        | 17.2                             | 19.9                             | 23.5                             | 54        | 12.9                             | 14.1                             | 15.7                             |
| 20        | 17.0                             | 19.7                             | 23.3                             | 55        | 12.8                             | 13.9                             | 15.4                             |
| 21        | 16.9                             | 19.5                             | 23.1                             | 56        | 12.6                             | 13.7                             | 15.1                             |
| 22        | 16.8                             | 19.4                             | 22.8                             | 57        | 12.5                             | 13.5                             | 14.8                             |
| 23        | 16.6                             | 19.2                             | 22.6                             | 58        | 12.3                             | 13.2                             | 14.5                             |
| 24        | 16.5                             | 19.0                             | 22.3                             | 59        | 12.1                             | 12.9                             | 14.1                             |
| 25        | 16.4                             | 18.8                             | 22.1                             | 60        | 11.9                             | 12.7                             | 13.8                             |
| 26        | 16.3                             | 18.7                             | 21.9                             | 61        | 11.7                             | 12.5                             | 13.5                             |
| 27        | 16.1                             | 18.5                             | 21.6                             | 62        | 11.5                             | 12.2                             | 13.1                             |
| 28        | 16.0                             | 18.3                             | 21.4                             | 63        | 11.3                             | 11.9                             | 12.8                             |
| 29        | 15.9                             | 18.2                             | 21.2                             | 64        | 11.0                             | 11.6                             | 12.5                             |
| 30        | 15.8                             | 18.0                             | 21.0                             | 65        | 11.8                             | 11.4                             | 12.2                             |
| 31        | 15.6                             | 17.8                             | 20.8                             | 66        | 10.5                             | 11.1                             | 11.8                             |
| 32        | 15.5                             | 17.7                             | 20.6                             | 67        | 10.2                             | 10.8                             | 11.5                             |
| 33        | 15.4                             | 17.6                             | 20.4                             | 68        | 9.9                              | 10.5                             | 11.2                             |
| 34        | 15.3                             | 17.4                             | 20.2                             | 69        | 9.6                              | 10.2                             | 10.9                             |
| 35        | 15.2                             | 17.3                             | 20.0                             | 70        | 9.3                              | 9.9                              | 10.5                             |
| 36        | 15.1                             | 17.2                             | 19.9                             | 71        | 9.0                              | 9.6                              | 10.2                             |
| 37        | 15.0                             | 17.0                             | 19.7                             | 72        | 8.7                              | 9.2                              | 9.8                              |
| 38        | 14.9                             | 16.9                             | 19.5                             | 73        | 8.4                              | 8.9                              | 9.5                              |
| 39        | 14.8                             | 16.7                             | 19.3                             | 74        | 8.1                              | 8.6                              | 9.1                              |
| 40        | 14.7                             | 16.6                             | 19.1                             | 75        | 7.8                              | 8.2                              | 8.7                              |

## PROBLEM VI.

To find the value of the reversion of one life after another.

## R U L E.

From the value of the life in expectation take the value of the two joint lives, or from the value of the longest of two lives take the value of the life in possession; the remainder in either case will be the value of the reversion.

19. Suppose the life in possession be 68 years, the life in expectation 11 years, and interest four per cent. and annuity 50*l.* the value of the reversion is required?

Against 11 years, under 4 per cent. 12.9, Table II.

Also against 68 years - - - - 4.6  $\div$  2 = 2.3.

Difference 8.3.

Then  $8.3 \times 2.3 = 19.9$ ; also  $\frac{19.9}{12.9} = 1.48$ .

And  $4.6 + 1.48 = 6.08$ , value of the two joint lives.

Also by Table I. against 11 years, is 16.4.

And  $16.4 - 6.08 = 10.3$  years purchase.

$\therefore 50 \times 10.3 = 515*l.*$  value of the reversion.

But if the youngest life be in possession,

By Table I. against 68 years, at 4 per cent. is 6.9.

And  $6.9 - 60.8 = .8$  years purchase.

$\therefore 50 \times .8 = 40*l.*$  value, if the youngest life be in possession.

## PROBLEM VII.

To find the value of the reversion of two lives after one.

## R U L E.

From the value of the three lives subtract the value of the life in possession, the remainder will be the value of the two lives in reversion.

20. Let the age of the life in possession be 50 years, and those of the two lives in reversion 45 and 56 years, the annuity 75*l.* and interest at 4 per cent. the present value is required?

First,  $50 + 45 \times 2 = 190$ ; also  $190 + 56 = 246$ .

Then 5) 246 (49, mean age, against which, Table V. is 15.1.

Also,

Also, by Table I. the value of the life in possession is 10.1.

Also  $15.1 - 10.1 = 5$  years purchase.

$\therefore 75 \times 5 = 375$  l. value required.

PROBLEM VIII.

To find the value of a reversion of one life after two.

R U L E.

From the value of the three lives take the value of the two lives in possession, the remainder will be the value of the life in reversion.

21. Suppose 18 and 26 be the ages of the two lives in possession; and 32 that of the life in expectation; the annuity 120l. and interest 4 per cent. ?

First,  $18 + 26 \times 2 = 88$ ; also  $88 + 32 = 120$ .

Then 5) 120 (24, against which, under 4 per cent. Table V. is 19 years.

Against  $\left\{ \begin{array}{l} 18 \\ 26 \end{array} \right\}$  Table III. under 4 per ct.  $\left\{ \begin{array}{l} 18.7 \\ 17.3 \end{array} \right\}$

And  $19 - 18 = 1$  year's purchase, or 120l. the answer. 2) 36 (18 years.

What is above observed, hath regard to such annuities as are paid yearly; but if the payments are made half yearly, which is most commonly the case, the above-mentioned Mr. Simpson judiciously observes, that the value at which the annuity is estimated ought to be increased  $\frac{1}{4}$  of a year's purchase; and if quarterly,  $\frac{1}{2}$  of a year's purchase; as the life, upon whose failing the annuity ceases, has nearly the same chance to drop in the second, third, or fourth quarter, as in that foregoing; in which case the purchaser hath a chance to receive  $\frac{1}{4}$ ,  $\frac{1}{2}$ , or  $\frac{3}{4}$  of a year's rent more than the annuity, when the annuity is paid yearly; and intirely loses the last payments, if the death happens but one day before the annuity becomes due.

As my Book is swelled far beyond the limits at first intended, I hope my worthy subscribers will excuse my proceeding in Geometry and Mensuration, as proposed in some of my Advertisements; so shall add a collection of questions, with an Appendix, and conclude,

C H A P.

## CHAPTER VII.

### A COLLECTION of QUESTIONS.

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#### SECT. I.

#### SUPERFICIAL MEASURE.

**SURFACES**, such as land, flooring, painting, tiling, paving, plastering, &c. if it be a four-sided figure, whose opposite sides are equal, by multiplying the length into the perpendicular height, gives the superficial content, and either of the dimensions being given, the other may be found by division.

1. The biggest of the Egyptian pyramids, near Grand Cairo, being square, and measuring, according to Mr. Greaves's account, 693 feet English on a side; how many acres then of ground doth it stand on?

First,  $693 \times 693 = 480249$  square feet.

An acre =  $\square$  feet 43560  $\frac{480249}{43560} (11 \text{ acres.})$

A perch =  $\square$  feet 272.25  $\frac{480249}{272.25} (1089 \text{ perches.})$

2. What difference is there between a floor 28 feet long by 20 broad, and two others that measure 14 feet a-piece by 10; and what do all these come to at 45 s. per square, viz. 10 feet by 10?

First,  $28 \times 20 = 560$ ; also  $14 \times 10 \times 2 = 280$ .

Then  $560 - 280 = 280$ , difference.

Also  $560 + 280 = 840$ ; and  $45 \text{ s.} = 2.25 \text{ l.}$

$10 \times 10 = 100$   $\frac{840}{100} (8.4 \text{ squares.})$

$\therefore 8.4 \times 2.25 = 18.9 = 18 \text{ l. } 18 \text{ s. amount.}$

3. A rectangular four-sided room measures 129 feet 6 inches about, and is to be wainscoted, at 3s. 6d. per yard square: after the due allowance for girt of cornice and members, it is 16 feet 3 inches high; the door is 7 feet by 3 feet 9, the window-shutters, two pair, are 7 feet 3 by 4 feet 6; the check-boards round them come 15 inches be-

# Chap. VII. SUPERFICIAL MEASURE.

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low the shutters, and are 14 inches in breadth; the lining-boards round the door-way are 16 inches broad; the door and window-shutters, being wrought on both sides, are reckoned as work and half, and paid for accordingly; the chimney 3 feet 9 by 3 feet, not being inclosed, is to be deducted from the superficial content of the room; and the estimate of the charge is required?

|                                |             |      |    |               |
|--------------------------------|-------------|------|----|---------------|
| F. I.                          | F. I.       | F.   | I. | P.            |
| First, 129 6' × 16 3' =        | 2104        | 4    | 6, | room.         |
| 7 - × 3 9' =                   | 13          | 1    | 6, | door.         |
| 2                              |             |      |    |               |
| 7 3 × 4 6' =                   | 32          | 7    | 6, | shutters.     |
| 8 6 + 4 6 × 2 = 26 × 1 2 × 2 = | 60          | 8    | →  | cheek-boards. |
| 14 + 3 9' = 17 9' × 1 4' =     | 23          | 8    | →  | door-linings. |
|                                | 2234        | 5    | 6  |               |
| 3 9 × 3, to be deducted =      | 11          | 3    | -  | chimney.      |
|                                | 2223        | 2    | 6  |               |
|                                | Square feet | 2223 | 2  | 6             |

9) 2223 (247 square yards.

|     |    |            |
|-----|----|------------|
| 100 | 1  | 247        |
| 30  | 17 | 6          |
| 12  | 7  | -          |
| £   | 43 | 4 6, cost. |

4. When a roof is of a true pitch, the rafters are  $\frac{1}{2}$  of the breadth of the building; now supposing the eave-boards to project 10 inches on a side, what will the new ridding an out-house cost, that measures 32 feet 9 inches long, by 22 feet 9 inches broad upon the flat, at 15s. per square?

|                                                    |          |                |
|----------------------------------------------------|----------|----------------|
| F. I.                                              | F. I.    | P.             |
| Breadth 22 9' = $\frac{1}{2}$ of which is          | 17       | 9              |
| F. I. P.                                           | F. I. P. | F. I. P.       |
| Also 17 - 9 + 10 = 17 10 9, which × 2 =            | 35       | 9 6            |
| 35 9 6 × 32 9' =                                   | 1172     | feet 2 in. 1 6 |
| 100) 1172 (11.72 squares, and 15s. = .75 l.        |          |                |
| ∴ 11.72 × .75 = 8.79 = 8 l. 15s. 9½ d. the answer. |          |                |

5. If

5. If my court-yard be 47 feet 7 inches square, and I have laid a foot-way of Purbeck-stone, 4 feet wide, along one side of it; what will paving the rest with flints come to, at 6d. per yard square?

First, 47 f. 7 in. — 4 f. = 43 f. 7 in. breadth.

Then  $47\ 7 \times 43\ 7 = 2073\ 10\ 1$ .

And 9) 2073 (230 yards 3 f. 10 inches.

$\frac{1}{2}$  6 d.

$\frac{1}{2}$  230

s. d.

$\frac{1}{2}$  2

$\frac{1}{2}$  2  $\frac{1}{2}$  d. value of the 3 feet 10 inches.

£ 5 15 2  $\frac{1}{2}$ , the answer required.

6. A square cieling contains 114 yards 6 feet of plaistering, and the room 28 feet broad; what was the length of it?

First, 114 yards 6 feet = 1032 square feet.

Then 28) 1032 (36  $\frac{6}{7}$  feet, the answer.

7. An elm plank is 14 feet 3 inches long, and I would have just a yard square slit off; at what distance from the edge must the line be struck?

First, 14 feet 3 inches = 171 inches; also  $36 \times 36 = 1296$  inches in a square yard.  $\therefore 171) 1296 (7\frac{1}{3}$  inches, the answer. (99)

8. Having a rectangular marble slab, 58 inches by 27, I would have a foot square cut off, parallel to the shorter edge; I would then have the like quantity divided from the remainder, parallel to the longer side; and this alternately repeated, till there should not be the quantity of a foot left: what will the dimensions of the remnant be?

First,  $12 \times 12 = 144$ ; also  $\frac{144}{27} = 5.3$ , breadth of the first cut.

Then  $58 - 5.3 = 52.7$ , the remaining length.

Also  $\frac{144}{52.7} = 2.734$ , breadth of the second cut.

Then  $27 - 2.734 = 24.266$ , the remaining breadth.

$\frac{144}{24.266} = 5.934$ , breadth of the third cut.

52.7



$$52.8 - 5.934 = 46.732.$$

$$\frac{144}{46.732} = 3.0814, \text{ breadth of the fourth cut.}$$

$$24.266 - 3.0814 = 21.1846.$$

$$\frac{144}{21.1846} = 6.7974, \text{ breadth of the fifth section,}$$

$$46.732 - 6.7974 = 39.9346.$$

$$\frac{144}{39.9346} = 3.6059, \text{ breadth of the sixth section.}$$

$$21.1846 - 3.6059 = 17.5787.$$

$$\frac{144}{17.5787} = 8.1917, \text{ breadth of the seventh section.}$$

$$39.9346 - 8.1917 = 31.7429.$$

$$\frac{144}{31.7429} = 4.5364, \text{ breadth of the eighth section.}$$

$$17.5787 - 4.5364 = 13.0423.$$

$$\frac{144}{13.0423} = 11.0411, \text{ breadth of the ninth section.}$$

Then  $31.7429 - 11.0411 = 20.7018$ , remaining length at last.

$$\text{Also } \frac{144}{20.7018} = 6.956, \text{ breadth of the tenth section.}$$

$\therefore 13.0423 - 6.956 = 6.0863$ , breadth remaining at the last. Q. E. F.

9. Being about to plant 10584 trees equally distant, the length of the grove must be six times the breadth; how many of the shorter rows will there be?

$$6) 10584 (1764; \text{ then } \sqrt{1764} = 42 \text{ long rows.}$$

$$\therefore 42 \times 6 = 252 \text{ short rows, 42 in a row.}$$

10. A common joist is 7 inches deep, and  $2\frac{1}{2}$  thick; but I want a scantling just as big again, that shall be 3 inches thick: what will the other dimension be?

$$\text{First, } 7 \times 2.5 = 17.5 \text{ inches, area of an end.}$$

$$\text{Also } 17.5 \times 2 = 35, \text{ double area.}$$

$$\therefore 3) 35 (11\frac{2}{3} \text{ inches. Q. E. F.}$$

11. I have a square girder, 19 inches by 11, but one of a quarter of the timber in it, provided it be 9 inches deep, will serve; how broad will it be?

$$\text{First, } 19 \times 11 = 209, \text{ area of an end.}$$

$$\text{Then } 4) 209 (52\frac{1}{4}, \text{ area of an end of the piece wanted.}$$

$$\therefore 9) 52.25 (5.805. \text{ Q. E. F.}$$

12. I have a wooden trough, that, at 6 d. per yard, cost me 3s. 2 d. painting within; the length of it is 102 inches, the depth 21 inches; what is its breadth?

First,  $36 \times 36 = 1296$ , square inches in a yard.

Also  $3s. 2 d. = 38 d.$

d. in. d.

As  $6 : 1296 :: 38 : 8208$ , area of the whole trough.

Then  $102 \times 20 \times 2 = 4284$ , area of the two sides.

3924, area of the bottom and ends.

Then  $102 + 42 = 144$   $3924 (27\frac{1}{2}$  inches. Q. E. F.

13. My plumber has put 28 lb. per foot square into a cistern, 74 inches and twice the thickness of the lead long, 26 inches broad, and 40 deep; he has put three stays within across it, 16 inches deep, of the same strength, and reckons 22 s. per cwt. for work and materials: I being a mason, have paved him a work-shop, 22 feet 10 inches broad, with Purbeck-stone, at 7 d. per foot, and upon the balance I find there is 3s. 6d. due to him; what was the length of his work-shop?

First,  $26 + 40 + 40 = 106$ , breadth of the bottom and sides.

Also  $106 \times 74 = 7844$ , area of the bottom and sides.

Then  $40 \times 26 \times 2 = 2080$ , area of both ends.

And  $26 \times 16 \times 3 = 1248$ , area of the stays.

11172, whole area in inches.

144) 11172 (77.583 square feet.

4) 77.583 (19.39583 cwt.

$\frac{1}{10}$  1.9395833

19.39583

£ 21.335418 = 21 l. 6 s.  $8\frac{1}{2}$  d. value of the cistern.

3 s. 6 d. = .175; also 7 d. = .02918 l.

.02918) 21.160218 (725.5 square feet in his shop.

291) 2.116041

.02625) 19.044375 22 ft. 10 in. = 22.83.

22.83) 725.5 (31.776 = 31 feet  $9\frac{1}{4}$  inches, the answer.

2.28) 72.5

20.55) 653.00 . . .

14. The area of a rectangular powdering-trough of a man of war measures 27 square feet, 112 inches, the depth is 20 inches, the breadth 16; the length is sought?

First, 27 feet 112 inches = 4000 square inches.

Then  $20 \times 16 \times 2 = 640$ , area of both ends.

3360, bottom and both sides.

$\therefore 20 + 20 + 16 = 56$  3360 (60 inches, the answer.

15. In 110 acres of statute-measure, in which the pole is  $16\frac{1}{2}$  feet long, how many Cheshire acres, where the customary pole is 6 yards long; and how many Yorkshire, where the pole in use is 7 yards in length?

sq. yds.

First,  $5.5 \times 5.5 = 30.25$

Also  $6 \times 6 = 36$

And  $7 \times 7 = 49$

$\left. \begin{array}{l} \text{First, } 5.5 \times 5.5 = 30.25 \\ \text{Also } 6 \times 6 = 36 \\ \text{And } 7 \times 7 = 49 \end{array} \right\} = 1 \text{ perch } \left\{ \begin{array}{l} \text{Statute} \\ \text{Cheshire} \\ \text{Yorkshire} \end{array} \right\}$

$\therefore$  Reciprocally,

a. r. p.

As  $30.25 : 110 :: \left\{ \begin{array}{l} 36 : 92.4308 = 92 \text{ } 1 \text{ } 28, \text{ Cheshire} \\ 49 : 67.908 = 67 \text{ } 3 \text{ } 25, \text{ Yorksh.} \end{array} \right\}$

16. I would set 3584 plants in rows, each 4 feet asunder, and the plants 7 feet apart, in a rectangular plot of ground; what land will this take up?

First,  $7 \times 4 = 28$ , square feet (area) between the plants.

Then  $3584 \times 28 = 100352$  square feet.

In an acre are 43560 square feet.

$\therefore 43560 \over 100352$  (2-acres, 1 rod,  $8\frac{1}{2}$  perches, the answer.

10890) 13232

27225) 2342

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A triangle, or three-sided figure, (being the half of a four-sided one of the same height and length) if you multiply the base, or longest side, by the shortest height, you have double the content.

17. A triangular field, 738 links long, and 583 in the perpendicular, brings in 12 l. a year; what is it set at an acre?

First,  $\frac{738}{2} \times 583 = 215127 = 2$  acres, 24 perches.

Also  $2.15127 : 12 :: 1.00000 : 5.5781 = 5 \text{ l. } 11 \text{ s. } 6\frac{1}{2} \text{ d.}$   
the answer. 18. A

18. A piece of garden-box lies in form of a regular pentagon, or figure of five equal sides, each 48 feet; and from the center of the figure to the middle of one of these, it measures 41.57 feet nearly: the area of the figure will be the content of these five triangles; pray what is that?

First,  $\frac{48}{2} \times 41.57 = 997.68$ , area of one of the triangles.

Also  $997.68 \times 5 = 4988.4$  square feet, area of the pentagon.

19. The end-wall of an house is 24 feet 6 inches in breadth, and 40 feet to the roof;  $\frac{1}{3}$  of which is two bricks thick,  $\frac{1}{3}$  more  $1\frac{1}{2}$  brick thick, and the rest one brick thick: now the gable rises 38 course of bricks (four of which usually make a foot in depth) and this is but 4 inches, or half a brick thick; what will this piece of work come to, at 5 l. 10 s. per statute rod, the dimensions of which are given?

4) 38 (9.5, height of the gable.

Also 3) 40 (13.3, height of each floor.

Again,  $24.5 \times 13.3 = 326.6 = 435.6$ , ground-floor.

Also  $326.6$ , first story.

Then  $326.6 = 217.7$ , garret.

And  $\frac{24.5}{2} \times 9.5 = 116.375 = 38.79$ , gable.

1017 feet, statute meas.

272.25) 1017.00 (3.7355 rods, statute measure.

$\therefore 3.7355 \times 5.5 = 20.54525 = 20$  l. 10 s.  $10\frac{1}{2}$  d. the answer.

20. A four-sided figure, whose sides are equal, is called a trapeze: I have an orchard of that form, containing  $3\frac{3}{4}$  acres, which being divided by a diagonal, or a line from corner to corner, the perpendicular of one of the triangles is 430 links, and the other 360: the length of the said diagonal, or common base of those triangles, is required?

First,  $430 + 360 = 790$ ; also  $\frac{790}{2} = 395$ .

And  $3\frac{3}{4}$  acres = 375000 links.

$\therefore 395$ ) 375000 (949 $\frac{3}{4}$  links, the answer.

The

The areas of circles are found, either by multiplying half the circumference by half the diameter, or by multiplying the square of the diameter by .7854, that being the area of the circle whose diameter is 1.

And if the diameter be 1, the circumference will be 3.1416 nearly.

21. Give the area of a circular bowling-green, that is 16 poles a-cross the middle, the circumference being 3.1416 times the diameter of a circle?

$$16 \times 3.1416 = 50.2656 \text{ poles circumference.}$$

$$\frac{16}{2} \times \frac{50.2656}{2} = 201.0625 \text{ square poles.}$$

In an acre are 160) 201 (1 acre, 41 poles, the answer.

Or  $16 \times 16 \times .7854 = 201.0624$ , as before.

22. The surveying wheel is so contrived, as to turn just twice in the length of a pole, or  $16\frac{1}{2}$  feet; what then is its diameter?

One round, per question, is  $8\frac{1}{2}$  feet.

$$3.1416) 8 \text{ } 2500 \text{ (2.626 feet} = 2 \text{ ft. } 7\frac{1}{2} \text{ in. the answer.}$$

23. I would turf a round plat, measuring 130 feet about, and would know the charge at 4 d. per yard square?

$$3.1416) 130.0000 \text{ (41.38, diameter.}$$

$$65 \times 20.69 = 1344.85 \text{ square feet.}$$

$$9) 1344.85 \text{ (149.428 square yards.}$$

$$85) 149.428 \text{ (2.4908} = 2 \text{ l. 9 s. } 9\frac{1}{2} \text{ d. the answer.}$$

In an  $\frac{1}{4}$  acre are 2420 square yards.

24. I want the length of a line, by which my garden may strike a round orangery, that shall contain just half an acre of land?

$$\text{First, } .7854) 2420.0000 \text{ (3081.23}$$

$$\text{And } \sqrt{3081.23} \text{ (55.5, diameter.}$$

$$\therefore 2) 55.5 \text{ (27.75} = 27\frac{3}{4} \text{ yards, the answer.}$$

25. Agreed for an oaken kerb to a round well, at 8 d. per foot square; it is exactly 42 inches in diameter, with in the brick-work, and the breadth of the kerb is to be  $14\frac{1}{2}$  inches; what will it cost to?

$$\text{First, } 14.5 + 42 + 14.5 = 71, \text{ greater diameter.}$$

P p

Then

Then  $71 \times 71 = 5041$ ; also  $5041 \times .7854 = 3959.2014$   
 Then  $42 \times 42 = 1764$ ; also  $1764 \times .7854 = 1385.4456$

Difference of the areas are - - - - - 2573.7558

Then 144) 2573.7558 (17.8733, area of the kerb.

Also 8 d. = .8 shilling.

$17.8733 \times .8 = 14.3$  s. 11 d. nearly, the answer.

26. It is observed, that the extreme end of the minute-hand of a public dial moves just 5 inches in the space of  $3\frac{1}{4}$  minutes; the question is, what is the length of that index?

As  $3.25 : 5 :: 60 : 92.307$ , circumference.

Also  $3.1416 : 1 :: 92.307 : 29.38$ , diameter.

$\therefore 2) 29.38$  (14.69 inches, the answer.

27. A, B, C join for a grind-stone 36 inches over, value 20 s. towards which A paid 7 s. B 8 s. and C 5 s. the waste-hole, through which the spindle passed, was 5 inches square; to what diameter ought the stone to be worn, when B and C begin severally to work with it?

Begin your calculations from the center.

First,  $36 \times 36 = 1296$ , which  $\times .7854 = 1017.8784$ , the area of the whole stone.

$5 \times 5 = 25 + 25 = 50$ ; also  $\sqrt{50} = 7.071068$ , the diameter of the circle circumscribing the spindle-hole.

Then  $50 \times .7854 = 39.27$ , area of the circle circumscribing the spindle-hole.

Also  $1017.8784 - 39.27 = 978.6084$ , area to be divided.

As  $20 : 978.6084 :: \begin{cases} 7 \text{ s.} : 324.5129 = \text{A's} \\ 8 : 391.4433 = \text{B's} \\ 5 : 244.6521 = \text{C's} \end{cases} \text{ area.}$

Then  $244.6521 + 39.27 = 283.9221$ .

$\cdot 7854) 283.9221 (361.5$ .

Also  $\sqrt{361.5} = 19.03$ , diameter where C begins to grind.

And  $391.4433 + 283.9221 = 675.3654$ .

$\cdot 7854) 675.3654 (859.9$ .

$\therefore \sqrt{859.9} = 29.324$  inches diameter, where B begins to grind. Q. E. F.

28. I demand what difference there is in the area of the section of a round tree, 20 inches over, and its inscribed and circumscribed squares?

First,  $10 \times 10 = 100$ ; also  $100 + 100 = 200$ .

Then  $\sqrt{200} = 14.142135$ , side of the inscribed square,

Also  $14.142135 \times 14.142135 = 200$ , its area.

Again,  $20 \times 20 = 400$ , area of the circumscribed square.

Lastly,  $400 \times .7854 = 314.16$ , area of the circular section.

Hence the inscribed square is 114.16 } inches { too little.

And the circumscribed - - 85.84 } inches { too much.

29. Having paved a semicircular alcove with black and white marble, at 2s. 4d. per foot, the mason's bill was just 10l. what then was this arch in front, considering that as .7854, the area of the circle, the square of whose diameter is 1, so is the area of any other circle to the square of its diameter?

First, 2s. 4d. = .116l. : 1 :: 10l. : 85.7143 feet area.

Then  $85.7143 \times 2 = 171.4286$ .

Also .7854) 171.4286 (218.269.

$\therefore \sqrt{218.269} = 14.7739 = 14$  ft.  $9\frac{1}{2}$  in. the answer.

30. What proportion is there between the arpent of France, which contains 100 square poles, of 18 feet each, and the English acre, containing 160 square poles, of  $16\frac{1}{2}$  feet each; considering that the length of the French foot is to that of the English, as 16 to 15?

First,  $18 \times 18 \times 100 = 32400$  French feet, the arpent.

Then  $16\frac{1}{2} \times 16\frac{1}{2} \times 160 = 43560$  English feet in an acre.

Also  $16 \times 16 = 256$ ; and  $15 \times 15 = 225$ .

Recip.  $256 : 32400 :: 225 : 36864$  English feet, an arpent,

So that the English acre is to the arpent of France, as 605 to 512, or nearly as 13 to 11; or as 1 to .84628, the answer.

31. In turning a one-horse chair within a ring of a certain diameter, it was observed, that the outer wheel made two turns, while the inner made but one; the wheels were equally high, and supposing them fixed at the statutable distance, or

5 feet asunder on the axletree; pray what was the circumference of the track described by the outer wheel?

$$3.1416 \times 4 = 12.5664, \text{ the circumference of the wheel.}$$

$$\therefore 12.5664 \times 5 = 62.832 \text{ by the greater.}$$

$$\text{And } 31.416 \text{ by the lesser.}$$

Multiply half the arch by half the diameters; also find the area of a sector; that is, any part of a circle cut through, from the center to the circumference.

32. The area of a sector (suppose one of the divisions of a wilderness) which being struck from a center with a line 30 yards long, makes the sweep, or circular part, 63 feet, is required?

$$63 \text{ feet} = 21 \text{ yards is half, being } 10.5 \text{ yards.}$$

$$\text{Then } 10.5 \times 30 = 315 \text{ yards, the answer.}$$

33. If the chord or line drawn through the two ends of the curve be 15 inches shorter than the arch line, I demand the segment?

$$\text{First, } 15 \text{ inches} = .41\bar{6} \text{ yards.}$$

$$\text{Then } 21 - .41\bar{6} = 20.58\bar{3}, \text{ which } \div 2 = 10.291\bar{6}.$$

$$30 \times 30 = 900.$$

$$\text{The } \square 10.291\bar{6} = 105.9184$$

$$\sqrt{794.0816} = 28.18, \text{ per pendicular.}$$

$$\text{Then } 10.291\bar{6} \times 28.18 = 290.0191\bar{6}, \text{ area of the triangle.}$$

$$\therefore 315 - 290.0191\bar{6} = 24.98, \text{ the answer.}$$

An ellipse, or oval, is measured, by multiplying the product of the long and short diameters by .7854, as in the circle, and this will give the superficial content.

34. The ellipse in Grosvenor-square measures 840 links the longest way, and 612 across, within the rails; the walls are 14 inches thick; what ground do they stand upon?

$$\text{First, } 8.40 \times 66 = 554.4$$

$$\text{Also } 6.12 \times 66 = 403.92$$

$$\text{And } 12) 28.0 = 2.3$$

$$\text{Then } 554.4 + 2.3 = 556.73 \text{ } \left. \begin{array}{l} \text{Also } 403.92 + 2.3 = 406.253 \end{array} \right\} \text{diameters of the greater.}$$

$$\text{Then } 556.73 \times 406.253 \times .7854 = 177637.66$$

$$\text{And } 554.4 \times 403.92 \times .7854 = 175877.17$$

$$\text{Area covered by the wall } - - = 1760.49 \text{ square ft.}$$

$$\therefore 4840$$



$\therefore 4840 \times 9 = 43560$  175877.17 (4 acres, 6 perches, its area. Q. E. F.

The dimensions of all similar figures are in proportion to their areas, as the squares of their respective sides; *et contra*.

35. If a round pillar, 7 inches over, has 4 feet of stone in it; of what diameter is the column, of equal length, that measures ten times as much?

$$4 \times 10 = 40 \text{ feet.}$$

$$4 \text{ feet} : 49 :: 40 \text{ feet} ; 490.$$

$$\sqrt{490} = 22.136 \text{ inches. Q. E. F.}$$

36. A pipe of six inches bore will be 3 hours in running off a certain quantity of water; in what time will 4 pipes, each 3 inches bore, be in discharging double the quantity?

$$6 \times 6 = 36; \text{ also } 3 \times 3 \times 4 \times 2 = 72.$$

$$\therefore 36 : 3 :: 72 : 6. \text{ Q. E. F.}$$

37. A yard of rope 9 inches round weighs, suppose 22 lb. what will a fathom of that weigh, which measures a foot round?

$$9 \times 9 = 81; \text{ also } 12 \times 12 \times 2 = 288.$$

$$81 : 22 :: 288 : 78\frac{2}{3}. \text{ Q. E. F.}$$

38. If 20 feet of iron-railing shall weigh half a ton, when the bars are an inch and quarter square; what will 50 feet of ditto come to, at  $3\frac{1}{2}$ d. per pound, the bars being but  $\frac{3}{4}$  of an inch square?

$$1.25 \times 1.25 \times 20 = 31.25.$$

$$\frac{9}{16} = .875 \times .875 \times 50 = 38.28125.$$

$$\text{As } 31.25 : 1120 :: 38.28125 : 1372.$$

$$\begin{array}{r} \frac{1}{10} \overline{) 1372} \\ \underline{17 \quad 3} \\ 2 \quad 17 \quad 2 \end{array}$$

$$\text{£ } 20 - 2, \text{ the answer.}$$

39. A looking-glass is 16 inches by 9, and contains a foot of glass; what will the content of the plate be, that has twice the length, and three times the breadth?

$$2 \times 16 = 32; \text{ and } 3 \times 9 = 27.$$

$$\text{Then } 32 \times 27 = 864 \text{ square inches.}$$

$$\therefore 144) 864 (6 \text{ square feet, the answer.}$$

40. A sack that holds three bushels of corn is  $22\frac{1}{2}$  inches broad, when empty; what would the sack contain, that, being of the same length, had twice its circumference, or twice its breadth?

$$22.5 \times 22.5 = 506.25; \text{ also } 45 \times 45 = 2025.$$

$$506.25 : 3 \text{ bush.} :: 2025 : 12 \text{ bush. } Q. E. F.$$

41. My plumber has set me up a cistern, and his shop-book being burnt, he has no means of bringing in the charge, and I do not chuse to take it down to have it weighed; but by measure he finds it contains 64 square feet  $\frac{1}{16}$ , and that it was  $\frac{3}{4}$  of an inch precisely in thickness. Lead was then wrought at 2 s. l. per fodder. Let the accountant, from these items, make out the poor man's bill; considering farther, that 4 oz.  $\frac{1}{4}$  is the weight of a cubic inch of lead.

$$\text{First, } 64.3 \times 144 = 9259.2 \text{ square inches.}$$

$$\text{Also } 9259.2 \times .375 = 3472.2 \text{ solid inches.}$$

$$\text{And } 3472.2 \times 4.28 = 15151.41 \text{ ounces.}$$

$$\text{Likewise } 15151.41 \text{ oz.} = 8.455 \text{ cwt.}$$

$$\therefore 19.5 \text{ cwt.} : 2 \text{ s. l.} :: 8.455 : 9.10538.$$

$$\text{Answer, } 9 \text{ l. } 2 \text{ s. } 1\frac{1}{4} \text{ d.}$$

## S E C T. II, M E A S U R E M E N T *of* S O L I D S.

**M**ULTIPLY the area by the depth, to find the solidity of uniform bodies, or such as are equal from top to bottom.

1. What is the difference of a solid half foot, and half a foot solid?

$$\text{First, } 6 \times 6 \times 6 = 216, \text{ solid inches in } \frac{1}{2} \text{ foot solid.}$$

$$\text{And } 2) 1728 (864, \text{ solid inches in } \frac{1}{2} \text{ a solid foot.}$$

$$\therefore 216) 864 (4 \text{ times as much as the first.}$$

2. What

2. What is the proportion, in point of space, between a room  $25\frac{1}{2}$  feet long, 20 feet 12 inches broad, 14 feet high, and two others of just half the dimensions?

F. I.      F. I.                      F. I.  
 First,  $25\ 6 \times 20\ 2 \times 14 = 7199\ 6$ .  
 Also  $12\ 9 \times 10\ 1 \times 7 \times 2 = 1799\ 10\ 6$ , which  
 is evidently just  $\frac{1}{2}$  of the first.

3. Another room is 17 feet 7 inches long within, 13 feet 10 inches broad, and 9 feet 6 inches high; it has a chimney carried up straight in the angle, the plan whereof is just half of 5 feet 6 inches, by 4 feet 2: the question is, how many cubic feet of air the same will contain, allowing the content of the fire-place and windows at four solid yards?

F. I.      F. I.      F. I.              F. I.  
 First,  $17\ 7 \times 13\ 10 \times 9\ 6 = 2310\ 8\ 11$   
 Then  $5\ 6 \times 2\ 1 \times 9\ 6 = 108\ 10\ 3$

Rein.  $2201\ 10\ 8$   
 And  $4 \times 27 - - - = 108 - -$

Answer, feet  $2309\ 10\frac{2}{3}$  inches.

4. A ship's hold is 112 feet 6 inches long, 32 broad, and 5 feet 6 inches deep; how many bales of goods, 3 feet 4 inches long, 2 feet 4 inches broad, and 3 feet deep, may be stowed therein, leaving a gang-way the whole length of 4 feet and  $\frac{1}{2}$  broad?

First,  $112.5 \times 32 \times 5.5 = 19800$   
 Gang-way  $112.5 \times 4.5 \times 5.5 = 2784.375$   
 Remaining capacity  $17015.625$

Also  $3.8 \times 2.8 \times 3 = 23\frac{1}{3} = \frac{70}{3}$ .

And  $17015.625 = 17015\frac{1}{2} = \frac{136125}{8}$ .

$\therefore \frac{70}{3} \times \frac{136125}{8} \left( \frac{408375}{560} \right) = 729\frac{27}{112}$ , the answer.

5. I want a rectangular cistern, that, at 16 lb. to the foot square, shall weigh just a fodder of lead; it must be 8 feet long, and  $4\frac{1}{2}$  over; how many hogheads, wine measure, will this contain, taking it at  $\frac{1}{4}$  of an inch from the top?

A fodder of lead weighs  $19\frac{1}{4}$  cwt. = 2184 lb.

16) 2184 (136.5 square feet.

Then  $8 \times 4.25 = 34$ , area of the bottom,

Also  $136.5 - 34 = 102.5$ , sides and ends.

$8 + 8 + 4.25 = 4.25$ , round.

24.5) 102.5 (4.183673 feet = 50.204 inches deep.

8 feet = 96 inches, and  $4\frac{1}{4}$  feet = 51 inches.

Also  $50.204 \times .75 = 49.454$ .

Then  $96 \times 51 \times 49.454 = 242126.784$  cubic inches,

282) 242126 (858 gal. = 16 hds. 42 gal. the answer.

6. A log of timber is 18 feet 6 inches long, 28 inches broad, and 14 thick, *die* square all through; now, if 2 solid feet and  $\frac{1}{4}$  be sawed off the end, how long will the piece then be?

First, 18 inches = 1.5; also 14 inches = 1.16.

$1.5 \times 1.16 = 1.75$  2.5 (1.42857, length of the piece cut off,

Then  $18.5 - 1.42857 = 17.07143$  feet, the answer.

7. The solid content of a square stone is found to be 126 $\frac{1}{2}$  feet, its length is 8 feet 6 inches; what is the area of one end, and what the depth, if the breadth assigned be 38 $\frac{1}{2}$  inches?

Length 8.5) 126.25 (14.853 feet = 2138.8234 inches, area of an end,

38.5) 2138.8234 (55.55 inches deep, the answer.

8. The dimensions of the circular Winchester bushel are 18 $\frac{1}{2}$  inches over, and 8 inches deep; how many quarters of grain then will the square bin hold, that measures 7 feet 10 long, 3 feet 10 broad, and 4 feet 2 deep within?

First,  $18.5 \times 18.5 \times .7854 = 268.80315$ .

Then  $268.80315 \times 8 = 2150.4$ , cubic inches in a bushel.

Also 7 ft. 10 in. = 94 in., 3 ft. 10 in. = 46 in., and 4 ft. 2 in. = 50 inches.

Then  $94 \times 46 \times 50 = 216200$  cubic inches, content of the bin.

2150.4) 216200.0 (100 bush. 2 pecks = 12 qts. 4 bush. 2 pecks, Q. E. F.

9. Taking

9. Taking the dimensions of the bushel, as above, what must the diameter of the circular measure be, which at 12 inches deep will hold 9 bushels of sea-coal struck?

First,  $2150.4 \times 9 = 19353.6$  inches, the content.

Then 12) 19353.6 (1612.8, area of the circle.

Also .7854) 1612.8000 (2053.47.

$\therefore \sqrt{2053.47} = 45.3$  inches. Q. E. F.

10. A prism of two equal bases, and six equal sides, that measures 28 inches across the center, from corner to corner; the superficial and the solid content is required, taking the length at 134 inches?

Radius  $14 \times 14 = 196$ ; also  $7 \times 7 = 49$ .

Then  $196 - 49 = 147$ .

Also  $\sqrt{147} = 12.1243557$ , perpendicular.

Then  $12.1243557 \times 7 = 84.8705$ , area of one triangle.

Also  $84.8705 \times 6 = 509.223$ , area of the base.

And  $14 \times 6 \times 134 = 11256$ .

Other base  $= 509.223$

Area of the prism  $12274.446$  inches.

$\therefore 1296$ )  $12274$  (9 yards, 4 feet, 34 inches, its area. Q. E. F.

Again,  $509.223 \times 134 = 68235.88$  solid inches.

$\therefore 1728$ )  $68235$  (39 solid feet, 843 cubic inches. Q. E. F.

11. I have a rolling-stone, 44 inches in circumference, and am to cut off three cubic feet from one end; whereabouts must the section be made?

First, 3 cubic feet = 5184 cubic inches.

If the circumference be 44, the diameter is 14,

Then  $22 \times 7 = 154$ , area of an end.

$\therefore 154$ ) 5184 (33.66 inches, the answer.

12. I would have a syringe, an inch and  $\frac{1}{4}$  in the bore, to hold a pint, wine measure, of any fluid; what must the length of the piston sufficient to make an injection with it, be?

First,  $1.25 \times 1.25 \times .7854 = 1.2271875$ , area of the circle.

In a pint are 28.875 cubic inches.

$\therefore 1.2271875$ ) 28.8750000 (23.5294 inches, the answer.

13. I would have a cubic bin made capable of receiving just  $13\frac{1}{2}$  quarters of wheat, Winchester measure; what will be the length of one of its sides?

In a bushel are 2150.4 cubic inches.

Then  $2150.4 \times 8 \times 13.5 = 232248.2$  cubic in.  $13\frac{1}{2}$  qrs.

$\therefore \sqrt[3]{232248.2} = 61.4678$  inches. Q. E. F.

14. A Bath-stone, 20 inches long, 15 over, and 8 deep, weighs 220lb. how many cubic feet thereof will freight a ship of 290 tons?

First,  $20 \times 15 \times 8 = 2400$  cubic inches.

Also 2400 inches : 220lb.  $:: 1728 : 158.4$ .

Also 290 tons = 649600 pounds.

$\therefore 158.4 \times 649600.0 (4101$  feet, the answer.

15. The common way of measuring timber being to girt a round straight tree in the middle, and to take  $\frac{1}{4}$  of the girt for the side of a square, equal to the area of the section there; if this be not considered in the price appointed, pray on which side lies the advantage?

A piece of timber 2 foot long, and 4 feet round, is a foot customary measure.

Also if a circle be 4 feet round,  $3.1416 \times 4 (1.2732$  diameter.

And a circular piece of timber 1 foot in length will contain 1.2732 feet:

$\therefore 1.2732 \times 50$ , the feet in a load, is 63.66.

So that, in a load of timber, there is gained by the buyer nearly  $13\frac{1}{2}$  feet.

The circumscribing cylinder is in proportion to its greatest inscribed globe, and cone of the same base, and perpendicular altitude, as 3, 2, and 1.

Therefore the cube of the diameter of any cylinder, of equal height and breadth, multiplied by .7854, the area of a circle, whose diameter is 1, will be the solidity.

The cube of the diameter of a globe, multiplied by  $\frac{2}{3}$  of .7854, viz. .5236, gives its solid content.

And the said cube, multiplied by  $\frac{1}{3}$  of .5236, or .2618, gives the solidity of any cone, whose breadth and height are equal.

Also

Also their superficial content may be found, by considering the cylinder as a square surface, multiplying the height by the circumference, and adding a double area for the two bases; the globe, as a rectangle of the diameter and circumference; and the cone as a triangle, whose base is the circuit, and perpendicular the slope height, adding the area of the base.

16. The solid content of a globe 20 inches in diameter; a cylinder of the same diameter, 20 inches long; and a cone 20 inches diameter at the base, and 20 inches high, are severally required; and also what they will cost painting, at 8d. a yard?

$$20 \times 20 \times 20 \times \left\{ \begin{array}{l} .7854 = 6283.2, \text{ cylinder's} \\ .5236 = 4188.8, \text{ globe's} \\ .2618 = 2094.4, \text{ cone's} \end{array} \right\} \text{ solidity.}$$

$$\begin{aligned} \text{Also } 20 \times 3.1416 &= 62.832, \text{ circumference.} \\ \text{Then } 62.832 \times 10 &= 628.32, \text{ area of the two bases.} \\ \text{And } 62.832 \times 20 &= 1256.64 \end{aligned}$$

$$\begin{aligned} \text{Cylinders } 1884.96, & \text{ superficial content.} \\ \text{Again, } 62.832 \times 20 &= 1256.64, \text{ ditto of the globe.} \\ & 1015.24, \text{ ditto of the cone.} \\ \hline & 4156.84, \text{ sum of their areas.} \end{aligned}$$

$$\begin{aligned} \text{Also } 20 \times 20 &= 400 \\ \text{And } 10 \times 10 &= 100 \end{aligned}$$

$$\begin{aligned} \sqrt{500} &= 22.31614, \text{ slope height of the} \\ 31.416 \times 22.31614 &= 701.08385 \text{ conc.} \\ \text{Area of the base } \frac{628.32}{2} &= 314.16 \end{aligned}$$

$$\text{Area of the cone, as above } 1015.24385$$

$$\text{As } 1296 : 8d. :: 4156.84 : 25.6 = 2s. 1\frac{1}{2}d. \text{ the ansr.}$$

17. Our satellite, the moon, is a globe in diameter 2170 miles; I require how many quarters of wheat she would contain, if hollow, 2150 $\frac{1}{10}$  solid inches being the bushel; and how much yard-wide stuff would make her a waistcoat, was she to be clothed?

$$\text{First, } 2170 \times 2170 \times 2170 \times .5236 = 5350308686.8, \text{ solid miles in the moon.}$$

Then

Then  $1760 \times 1760 \times 1760 = 545177600$ , solid yards in a mile.

Also  $5350308686.8 \times 5451776000 = 2916868449128-7756800$ , solid yards in the moon.

In a solid yard are 46656 cubic inches.

$29168684491287756800 \times 46656 = 1360894143625521-581260800$  solid inches.

$17203.2$   $1360894143625521581260800.0$   $(7910703194-8470144000$  quarters of wheat the moon would hold, if hollow. Q. E. F.

Again,  $2170 \times 3.1416 = 6817.272$ , circumference of the moon.

Also  $6817.272 \times 2170 = 14793480.25$  square miles.

$1760 \times 1760 = 3097600$  square yards in a square mile.

Then  $14793480.24 \times 3097600 = 45824284391424$  square yards, Q. E. F.

18. Supposing the atmosphere, or body of the air and vapours, furrounds the globe of the earth and sea to 60 miles above the surface, and the earth is 7970 miles in diameter; how many cubic yards of air then hang about and revolve along with this planet?

First,  $7970 + 120 = 8090$ , diameter of the earth and atmosphere.

Then  $7970 \times 7970 \times 7970 \times .5236 = 265078559622.8$ , solid miles in the globe of the earth.

Also  $8090 \times 8090 \times 8090 \times .5236 = 377233177544.4$  miles solid in the earth and atmosphere.

And  $277233177544.4 - 265078559622.8 = 12154617-921.6$ , solid miles in the atmosphere.

Also  $12154617921.6 \times 545177600 = 6626425427414-8761600$ , solid yards in the atmosphere. Q. E. F.

19. A gentleman bargaineth with a mason for a piece of marble in the form of a tetraedron, on which he intends to have four sun-dials; the side of each triangle is  $2\frac{1}{2}$  feet, or 30 inches; I demand its value, at 2d. a solid inch, and what it will cost polishing, at 1s. 3d. per foot superficial?

First,  $30 \times 30 = 900$ ; also  $15 \times 15 = 225$ .

Then  $900 - 225 = 675$ ; also  $\sqrt{675} = 25.98$ , perpendicular of each triangle.



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Again,  $25.98 \times 25.98 = 674.9604$ ; also  $12.99 \times 12.99 = 168.7401$ .

Then  $674.9604 - 168.7401 = 506.2203$ .

$\sqrt{506.2203} = 22.4993$ , perpendicular of the tetraedron.

Then  $25.98 \times \frac{30}{2} = 389.7$ , area of a triangle.

And  $389.7 \times 4 = 1558.8$  inches  $= 10.825$  square feet.

Then  $389.7 \times \frac{22.4993}{3} = 2922.659108$  inches.

|                             |       |       |                |
|-----------------------------|-------|-------|----------------|
|                             | l.    | s.    | d.             |
| Answer, the marble comes to | 24    | 7     | $1\frac{1}{4}$ |
| And polishing to            | - - - | 13    | $6\frac{1}{4}$ |
|                             |       | <hr/> |                |

In all, £ 25 -  $7\frac{1}{2}$

To find the solidity of a pyramid, or cone: multiply the area of the base by  $\frac{1}{3}$  of its perpendicular altitude.

20. A square pyramid, whose sides at the base measure 30 inches a-piece, and is 21 feet high by the slope in the middle of each side of the base, is to be sold at 7 s. per solid foot; and if the polishing the surface of the sides will be 8 d. per foot more, I would know the cost of this stone when finished?

First,  $21 \times 21 = 441$ ; also  $1.25 \times 1.25 = 1.5625$ .

Then  $441 - 1.5625 = 439.4375$ .

$\sqrt{439.4375} = 20.9627$ , perpendicular height.

Also  $2.5 \times 2.5 = 6.25$ , area of the base.

Then  $3) 20.96276$  ( $6.9876$ , nearly).

And  $6.25 \times 6.9876 = 43.6725$  solid feet.

Then  $21 \times 1.25 = 26.25$ , area of one triangular side.

Also  $26.25 \times 4 = 105$ , area of the sides.

|                                  |    |       |                |
|----------------------------------|----|-------|----------------|
|                                  | l. | s.    | d.             |
| Answer, at 7 s. per solid foot,  | 15 | 5     | $8\frac{1}{4}$ |
| And polishing, at 8 d. per foot, | 3  | 10    | -              |
|                                  |    | <hr/> |                |

£ 18 15  $8\frac{1}{4}$

When figures run uniformly taper, but not to a point, they are to be considered as frustums, or portions of the cone or pyramid; by supposing, therefore, what is wanting to make the figure entire, and then deducting the part cut off, we find the solidity of the part proposed.

In

In order to complete the cone, use this analogy; as half the difference of the top and bottom are to the depth, so is half the greater diameter to the altitude of the whole cone.

Or else, to the areas of the top and bottom add the square roots of the products of those areas, and this multiply by  $\frac{1}{3}$  of the height of the frustum for the solidity.

21. A round maff-vat meafures at the top 72 inches over, within, at the bottom 54, the perpendicular depth being 42 inches, the content in ale-gallons is required?

$$\text{As } 9 : 42 :: 36 : 168.$$

$$72 \times 72 \times .7854 = 4071.5136, \text{ area of the top of the tun.}$$

$$\frac{168}{3} = 56 = \frac{1}{3}, \text{ altitude of the cone.}$$

$$\text{Also } 54 \times 54 \times .7854 = 2290.2264, \text{ area of its bottom.}$$

$$168 - 42 = 126, \text{ which } \frac{126}{3} = 42, \text{ altitude of the piece}$$

wanting.

$$\text{Then } 4071.5136 \times 56 = 2288004.7616, \text{ the whole pyramid.}$$

$$\text{Also } 2290.2264 \times 42 = 96189.5088, \text{ piece wanting.}$$

---


$$131815.2528 \text{ cubic inches.}$$

$$\therefore 282) 131815. (467 \text{ gallons } 3\frac{61}{71} \text{ pints, the answer.}$$

Or,

$$4071.5136 + 2290.2264 = 6361.74, \text{ sum of the areas.}$$

$$\text{Also } 4071.5136 \times 2290.2264 = 9324687.9346.$$

$$\text{And } \sqrt{9324687.9346} = 3053.6352.$$

$$\text{Then } 6361.74 + 3053.6352 = 9415.3752.$$

$$\therefore 9415.3752 \times \frac{42}{3} = 131815.2528, \text{ cub. in. as before.}$$

22. The shaft of a round pillar, 16 inches in diameter at the top, is about eight of the bottom diameters in height,  $\frac{1}{4}$  whereof is truly cylindrical, and the other  $\frac{3}{4}$  swelling; but we will suppose it tapers straight; and that it is  $\frac{1}{8}$  less at top than at bottom; the price of the stone and workmanship is sought, at 3s. 6d. per cubic foot; and farther, the superficial content, including both ends?

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5) 16, top diameter.

$$\begin{array}{r} \hline + 3.2 \\ \hline \end{array}$$

19.2, bottom diameter.

$$\begin{array}{r} \hline \times 8 \\ \hline \end{array}$$

3) 153.6, height.

51.2, cylindrical.

102.4, a conical frustum.

First,  $19.2 \times 19.2 \times .7854 = 289.525085$ , area of the greater.

Also  $16 \times 16 \times .7854 = 201.0624$ , area of the lesser base.

Sum of the areas  $490.588385$

Then  $289.526 \times 201.0624 = 58212.7924$ .

$$\sqrt{58212.7924} = 241.273$$

Then  $490.588 + 241.273 = 731.861$ .

$\therefore 731.861 \times \frac{102.4}{3} = 24980.84546$ , conical frust. } solidity.

And  $201.0624 \times 51.2 = 10294.39488$ , cylinder

Solid content of pillar =  $35275.25034$  inches.

Then 1728)  $35275.25$  (20.414 solid feet.

Answer, 31. 11 a.  $5\frac{1}{4}$  d. cost, at 3s. 6d. per foot.

$16 \times 3.1416 = 502656$ , circum. of the cylinder.

$19.2 \times 3.1416 = 60.3187$ , circum. of the base.

2) 110.5843 (55.29215.

Then  $\frac{19.2}{2} - \frac{16}{2} = 1.6$ ; also  $1.6 \times 1.6 = 2.56$

Also  $102.4 \times 102.4 = 10485.76$

$$\begin{array}{r} \hline 10488.32 \\ \hline \end{array}$$

$\sqrt{10488.32} = 102.4124125$ , slope height.

Then  $102.4125 \times 55.2926 = 5662.6534$ , conical. superf.

Also  $51.2 \times 50.2656 = 2573.5987$ , cylinder.

289.526, bottom area.

201.0624, top area.

$$\begin{array}{r} \hline 8726.8405 \\ \hline \end{array}$$

Superficial content  $8726.84$  inches = 60.8 feet. Q. E. F.

23. A

23. A stick of square timber tapers straight; the side at the greater end is  $19\frac{1}{2}$  inches, at the less  $13\frac{1}{2}$  inches; the length 16 feet 6 inches; the value, at 2 s. 6 d. per foot solid, is demanded?

First,  $19\frac{1}{2} - 13\frac{1}{2} = 6$ , difference of the sides.

3) 36 (12, the third part of the square of that difference,

Also 16 feet 6 inches = 198 inches, the length.

Then  $19.5 \times 13.5 + 12 \times 198 = 54499.5$  cubic inches = 31.539 feet.

$\therefore 31.539 \times 1.25 = 39.42375 = 3\text{l. } 18\text{s. } 10\text{d.}$  the answer.

To measure a common cask: find the areas at head and bung; add  $\frac{1}{3}$  of the less, and  $\frac{2}{3}$  of the greater, for a mean area; this multiplied by the length of the cask is its solidity in inches; which reduce. Or, to double the square of the bung diameter, add that of the head; then multiply by the length of the cask, and divide by 1077.24 for beer, or by 882.42 for wine gallons.

24. What quantity of brandy will the distillers tun contain, that measures 40 inches within at the head, 52 at bung, and is 100 inches long; and how many barrels of London ale would fill it?

First,  $40 \times 40 \times .7854 = 1256.64$ .

Also  $52 \times 52 \times .7854 = 2123.7216$ .

Then  $\frac{1256.64}{3} = 418.88$ ; also  $2123.7216 \times \frac{2}{3} = 1415.8144$ .

Then  $1415.8144 + 418.88 = 1834.6944$ .

$\therefore 1834.6944 \times 100 = 183469.44$  cubic inches; the content.

Also 231) 183469.44 (= 794 gallons of brandy.

And 282) 183469.44 (= 650 $\frac{1}{2}$  gallons = 20 barrels, 10 $\frac{1}{2}$  gallons of London ale. Q. E. F.

25. The famous tun of Heidelberg, that being heretofore annually replenished with Rhenish, had in it some wine that was many ages old, before the French demolished it in the late war: it was 31 feet in length, and 21 feet in diameter, and pretty nearly cylindrical; pray how many tuns of wine would the same contain?

First,  $21 \times 21 \times .7854 = 346.3614$ , area of one end.

Then  $346.3614 \times 31 = 10737.2634$  solid feet.

Also

# Chap. VII. MISCELLANEOUS QUESTIONS. 593

Also  $10737.2034 \times 1728 = 18553887.4752$  cubic inches.  
 $231 \mid 18553887 = 80319.8$  gallons = 318 tuns, 183.8  
gallons, the answer.



## S E C T. III.

### MISCELLANEOUS QUESTIONS.

1. **A** Detachment of four regiments consisted of 4600 men; Col. A's regiment exceeded Col. B's by 33, Col. C's by 95 men, and Col. D's by 200 men, how many men were in each regiment ?

$$\begin{array}{r}
 4600 \\
 33 \\
 95 \\
 200 \\
 \hline
 4) 4928 (1232 \\
 1232 - 33 = 1199 \\
 1232 - 95 = 1137 \\
 1232 - 200 = 1032
 \end{array}
 \left. \vphantom{\begin{array}{r} 4600 \\ 33 \\ 95 \\ 200 \\ \hline 4) 4928 (1232 \\ 1232 - 33 = 1199 \\ 1232 - 95 = 1137 \\ 1232 - 200 = 1032 \end{array}} \right\} \text{Colonel } \left\{ \begin{array}{l} \text{A's.} \\ \text{B's.} \\ \text{C's.} \\ \text{D's.} \end{array} \right.$$

2. There are 8000 men in garrison besieged, whose daily allowance is 24 ounces of bread for 7 weeks ; but the governor finding the siege is likely to continue a longer time, who can hold out 14 weeks at least, though he has by this time lost 1500 of his men, whereby he finds himself obliged to shorten that allowance of provisions ; how much bread must each man's daily allowance be reduced to ?

Recip. 7 weeks : 24 ounces :: 14 weeks : 12 ounces.

Then  $8000 - 1500 = 6500$  men left.

Recip. 8000 men : 12 oz. ::  $6500 : 14\frac{10}{17}$ . Q. E. F.

3. Required to find the least three whole numbers, so that  $\frac{1}{3}$  of one,  $\frac{1}{4}$  of another, and  $\frac{1}{5}$  of a third, shall be equal ?

First, taking  $\frac{1}{3}$  and  $\frac{1}{4}$ .

Then  $3 \times 14 = 42$  ; also  $5 \times 8 = 40$ .

And  $\frac{1}{3} : \frac{1}{4} :: 42 : 40$ .

Q q

Then

Then taking  $\frac{1}{12}$  and  $\frac{7}{10}$ ; then  $5 \times 20 = 100$ ; and  $7 \times 14 = 98$ .

Also  $\frac{1}{12} : \frac{7}{10} :: 100 : 98$ .

And  $98 : 42 :: 100 : 42\frac{5}{7}$ .

$\therefore 40.42$  and  $42\frac{5}{7}$  are numbers in the same ratio, which  $\times 7$  gives 280, 294, and 300, whole numbers; these numbers  $\div 2$ , gives 140, 147, and 150, the least whole numbers.

Q. E. F.

$$\left. \begin{array}{l} \text{For } 280 \times \frac{1}{4} \\ 294 \times \frac{5}{14} \\ 300 \times \frac{7}{10} \end{array} \right\} = 105. \quad \text{Also } \left. \begin{array}{l} 140 \times \frac{1}{4} \\ 147 \times \frac{5}{14} \\ 150 \times \frac{7}{10} \end{array} \right\} = 52\frac{1}{2}.$$

4. An usurer dying, had left the whole sum of his fortune to be disposed of in the following manner: To A  $\frac{2}{5}$ , to B  $\frac{1}{10}$ , to C  $\frac{1}{5}$ , to D  $\frac{1}{10}$ , to E  $\frac{1}{10}$ , and to F  $\frac{1}{10}$ ; which sums being all paid, the remainder he ordered to be paid to G, which was 800l. Quere the usurer's whole sum, and what each had to their share?

$$\frac{2}{5} = \frac{80}{200}, \quad \frac{1}{10} = \frac{20}{200}, \quad \frac{1}{8} = \frac{25}{200}, \quad \frac{1}{20} = \frac{10}{200}, \quad \frac{1}{40} = \frac{5}{200}, \quad \text{and } \frac{1}{50} = \frac{4}{200} \text{ their sum being } \frac{184}{200} = \frac{23}{25}.$$

$$\therefore \frac{25}{25} - \frac{23}{25} = \frac{2}{25} = 800 \text{ l.}$$

$$2 : 800 :: 25 : 10000 \text{ l. whole estate.}$$

$$\left\{ \begin{array}{l} 80 : 4000, \text{ A's.} \\ 60 : 3000, \text{ B's.} \\ 25 : 1250, \text{ C's} + 800 = 2050 \\ 10 : 500, \text{ D's.} \\ 5 : 250, \text{ E's.} \\ 4 : 200, \text{ F's.} \end{array} \right.$$

5. A worthless miser, as I'm told,  
Had boarded up vast store of gold;  
Large sums put out to usury,  
Till aged fourscore years and three,  
When Death depriv'd him of his self,  
And took him from his second self:  
Of wives it happen'd he had three,  
Three sons, and daughters two had he;  
His third wife did survive him still,  
But mark the tenor of his will:  
Of rusty gold, ten thousand pound  
Was in this miser's coffers found;

Each

Each son must be paid down in store,  
 Each daughter's fortune three times o'er ;  
 Each daughter's, as the will was made,  
 Must twice the widow's part be paid :  
 Now the old miser's in his grave,  
 Tell me the fortune each must have !

1 widow 1 share, 2 daughters 4 shares, and 3 sons  
 18 shares ;  $1 + 4 + 18 = 23$ , divisor for the widow's part.

l. s. d. qrs.  
 23) 10000 (434 15 7  $\frac{37}{23}$ , widow's part.  
 $\times 2$

869 11 3  $\frac{214}{23}$ , each daughter's part.  
 $\times 3$

4608 13 10  $\frac{319}{23}$ , each son's part.

6. A stone, weighing 40 pounds, is by accident broke into four pieces, by which may be weighed any quantity or number of pounds, from 1 to 40 : Quere, the weight of each piece ?

*A general RULE for the solution of QUESTIONS of this nature.*

To double the first or least weight, which always contains one pound ; add 1, and it gives the second weight : again, to double the sum of these two weights, add 1, it produces the third weight ; and again, to double the sum of these three weights, add 1, and we shall have the fourth weight.

Thus 1 lb. = first or least weight.

Then will  $2 + 1 = 3 =$  next least weight.

Also  $3 + 1 \times 2 + 1 = 9 =$  third.

And  $9 + 3 + 1 \times 2 + 1 = 27 =$  the fourth.  
 The sum of which, viz.  $1 + 3 + 9 + 27 = 40$ .

7. A lovely pair, delight of human race,  
 Collateral thus their sprightly lineage trace ;  
 A thousand years are since their ancient stem,  
 Which branching forth, supply'd the branch to them ;  
 Each male and female, as by what appears,  
 Liv'd to the age of threescore and ten years ;  
 And each fair female brought forth children seven,  
 In seven successive years ; the gifts of heav'n ;

Q q 2

From

From twenty-one to twenty-seven of age,  
 A boy, and girl each year, by turns engage;  
 The teeming mother views them with a smile,  
 Their pleasing innocence her cares beguile:  
 No jealousies the parents joys molest,  
 But each fond couple is with virtue blest.  
 Happy for those, who, to no vices blind,  
 Can virtue choose, and such relations find.  
 To what amount did all this kindred thrive,  
 How many dead, of each sex what alive?  
 And of the living, let it next be told,  
 How many virgins but just twenty old?

First, 70) 1000 (14 generations.

Then if in 70 years 1 woman be increased to 3, in 70 more (viz. 140) 3 will be increased to 9; also, in 70 more (viz. 210) 9 will become 27.

∴ The number of women after 1000 years will be  $3^{14} = 4782969$ .

And the number of persons, men and women, then living, will be  $\frac{7}{2} \times 3^{14}$ , or  $7 \times 3^{13} = 11160261$ .

Lastly, supposing an equal number of all ages to be living at that time, then  $\frac{7}{8} \times 3^{14} = 1366562$  women living under 20.

And  $\frac{7}{8} \times 3^{14} = 1434890$ , women living under 21.

∴  $\frac{21-20}{70}$ , or  $\frac{1}{70} \times 3^{14} = 68328$  women living between 20 and 21.

8. A petticoat of silk, 3 yards, 2 feet, and 1 inch long, and half a yard and 10 inches wide, is sent me to be quilted in equal squares of four-tenths of an inch to each square side, and 58 stitches to be taken in 9 inches length; it is required to find the exact number of stitches the petticoat will take, and what the work will come to, at 5 s. per thousand stitches?

First, 3 yards, 2 feet, 1 inch = 133 } inches.  
 Also  $\frac{1}{2}$  yard, and 10 inches = 28 }

Then  $133 \times 28 = 3724$  square inches.

And  $.4 \times .4 = .16$ , area of one quilted square.

$.16$ )  $3724.00$  (23275 quilted squares in all.

Now  $.4 + .4 = .8$ , inches of work in every square.

Also  $23275 \times .8 = 18620$ , inches of work in all the squares, besides  $133 + 28 = 161$  inches for the half border; viz.  $18620 + 161 = 18781$  wrought inches.

As 9 inches : 58 stitches ::  $18781 : 121033\frac{1}{9}$  stitches.

1000 stitch. : .25l. ::  $121033\frac{1}{9} : 30l. 5s. 2d.$  nearly.

Q. E. F.

APPEN-



# A P P E N D I X;

*Containing the Method of finding the Sums of certain Progressions, some Problems in Maxima and Minima; and the Investigation of the Sums of certain Infinite Series.*

## SECTION I.

*Of finding the sums of any number of terms in certain progressions.*

### PROPOSITION I.

TO find the sum of any given number of terms of the series

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} \text{ \&c. or } \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} \text{ \&c.}$$

Divide the given number of terms by the same more 1, the quotient will be the sum required.

The sum of three terms will therefore be  $= \frac{3}{4}$ , that of five  $= \frac{5}{6}$ , and that of ten  $= \frac{10}{11}$ , &c. &c.

### PROPOSITION II.

To find the sum of any given number of terms of the series

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{4.5.6}, \text{ \&c.}$$

Multiply the number of terms more 1 by the same more 2; divide unity, or 1, by twice that product, and subtract the quotient from  $\frac{1}{2}$ , the remainder will be the sum required.

### EXAMPLE I.

Let the sum of five terms, viz.  $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{4.5.6} + \frac{1}{5.6.7}$  be sought.

Then  $6 \times 7 = 42$ , and  $\frac{1}{4} - \frac{1}{84} = \frac{80}{336} = \frac{5}{21}$ , the sum required.

## E X A M P L E II.

Let the sum of eight terms of the above series be required.

Here  $9 \times 10 = 90$ , and  $\frac{1}{4} - \frac{1}{180} = \frac{176}{720} = \frac{11}{45}$ , the sum required.

## P R O P O S I T I O N III.

To find the sum of any given number of terms of the series  $\frac{1}{1.2.3.4} + \frac{1}{2.3.4.5} + \frac{1}{3.4.5.6} + \frac{1}{4.5.6.7}$ , &c.

Let the number of terms, added to 1, 2 and 3, respectively, be continually multiplied together; divide unity by 3 times that product, and subtract the quotient from  $\frac{1}{1.2.3}$ , the remainder will be the sum of the terms required.

## E X A M P L E.

Let the sum of 20 terms of the above series be sought.

Then  $21 \times 22 \times 23 = 10626$ , and  $\frac{1}{18} + \frac{1}{3 \times 10626} = \frac{31860}{573804} = \frac{295}{5313}$ , equal the sum required.

## P R O P O S I T I O N IV.

To find the sum of any given number of terms of the series  $\frac{1}{1.2.3.4.5} + \frac{1}{2.3.4.5.6} + \frac{1}{3.4.5.6.7} + \frac{1}{4.5.6.7.8}$ , &c.

Let the number of terms be increased by 1, 2, 3 and 4, respectively, multiply those sums continually together, divide unity by four times their product, and subtract the quotient from  $\frac{1}{2.3.4.5}$ , the remainder will be the sum required.

## E X A M P L E.

Let the value of 96 terms of this series be sought.

Then  $97 \times 98 \times 99 \times 100 \times 4 = 3764376$ , and  $\frac{1}{96} - \frac{1}{3764376} = \frac{3764280}{361380096} = \frac{15845}{15057504}$ , equal the sum required.

It may not be improper to observe, that the sum of the series's

$$\frac{1}{1.2}$$

$$\begin{array}{rcl}
 \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} & \&c. & \left. \begin{array}{l} \text{will be} \\ \text{ad infinitum,} \end{array} \right\} \begin{array}{l} 1 \\ \frac{1}{2} \\ \frac{1}{6} \\ \frac{1}{24} \end{array} \\
 \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{4.5.6} & \&c. & \\
 \frac{1}{1.2.3.4} + \frac{1}{2.3.4.5} + \frac{1}{3.4.5.6} + \frac{1}{4.5.6.7} & \&c. & \\
 \frac{1}{1.2.3.4.5} + \frac{1}{2.3.4.5.6} + \frac{1}{3.4.5.6.7} + \frac{1}{4.5.6.7.8} & \&c. &
 \end{array}$$

P R O P O S I T I O N V.

To find the sum of any number of terms of the series  $1^2 + 2^2 + 3^2 + 4^2 + 5^2 \&c.$  or,  $1 + 4 + 9 + 16 + 25 \&c.$

Let  $\frac{1}{2}$ , half the number of terms, and  $\frac{1}{3}$  of the square of the said number, be collected into one sum, multiply that sum by the number of terms, and the product will be the aggregate of the terms required.

E X A M P L E I.

Let the sum of six terms of the abovesaid series be required.

Then  $\frac{1}{6} + 3 + \frac{36}{3} \times 6 = 91$ , the sum required.

E X A M P L E II.

Conceive a pyramid to be constituted of geometrical square slabs, each a foot thick, and suppose the base, or greatest slab, to be 20 feet square, the next 19, the next 18, the next 17, and so on, it is required to find the solid content of such a pyramid?

Here  $\frac{1}{6} + 10 + \frac{400}{3} \times 20 = \frac{861}{6} \times 20 = 2870$  feet, the solidity required.

P R O P O S I T I O N VI.

To find the sum of any number of terms of the series  $1^3 + 2^3 + 3^3 + 4^3 + 5^3 \&c.$  or,  $1 + 8 + 27 + 64 + 125 \&c.$

Let the number of terms more 1 be squared, and multiplied by the square of the number of terms;  $\frac{1}{4}$  of this product will be the sum required.

## E X A M P L E.

Let the value of 8 terms of this series be required.

Then  $9 \times 9 \times 64 = 5184$ , and  $\frac{5184}{4} = 1296$ , the value sought.

## P R O P O S I T I O N VII.

To find the sum of any number of terms of the series  $1^4 + 2^4 + 3^4 + 4^4 + 5^4$  &c. or,  $1 + 32 + 81 + 256 + 625$  &c.

Let  $\frac{1}{2}$  of the biquadrate,  $\frac{1}{2}$  of the cube, and  $\frac{1}{2}$  of the square of the number of terms, be collected into one sum, from which subtract  $\frac{1}{30}$ ; multiply the remainder by the number of terms, and the product will be the sum required.

## E X A M P L E.

Let the value of eight terms be required: then  $\frac{8^4}{2} + \frac{8^3}{3} = \frac{32806}{30}$ ; and  $\frac{32806}{30} - \frac{1}{30} \times 8 = \frac{261160}{30} = 8772$ , the sum required.

## P R O P O S I T I O N VIII.

To find the sum of any number of terms of the series  $1^5 + 2^5 + 3^5 + 4^5 + 5^5$  &c. or,  $1 + 64 + 162 + 512 + 1250$  &c.

Let  $\frac{1}{2}$  of the biquadrate,  $\frac{1}{2}$  the cube, and  $\frac{1}{2}$  of the square of the number of terms, be collected into one sum, from which subtract  $\frac{1}{12}$ ; multiply the remainder by the square of the number of terms, and the product will be the sum of the terms required.

## E X A M P L E.

Let the sum of 10 terms be required: then  $\frac{10^4}{6} + \frac{10^3}{2} + \frac{5 \times 10^2}{12} = \frac{159000}{12}$ ; and  $\frac{159000}{12} - \frac{6}{12} \times 100 = \frac{1589400}{12} = 220825$ , the value required.

## S E C T I O N

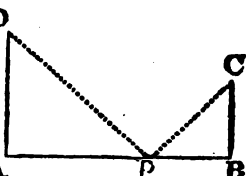
## S E C T. II.

*A Collection of PROBLEMS concerning the Maxima and Minima of Quantities.*

## P R O B L E M I.

**G**IVEN the position of the points D and C, in respect to the given right line AB, to find the point P, so that DP + PC shall be a minimum.

**THEOREM.**  $\frac{BC \times AB}{AD + BC} = BP$ ,  
when DP + PC is least possible.



## E X A M P L E.

Let AB = 50, AD = 40, and A P B  
BC = 30, required PB in the above circumstances ?

Then  $\frac{30 \times 50}{40 + 30} = \frac{1500}{70} = 21.42857 = PB$ , and therefore  
AP = 28.57143.

## P R O B L E M II.

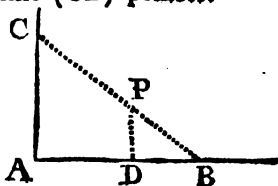
Through the given Point P, placed within the right angle CAB, to draw the shortest line (CB) possible.

**THEOREM.**  $\overline{AD \times PD}^{\frac{1}{2}} = C$   
DB, where DP is perpendicular to AB.

## E X A M P L E.

Suppose AD = 10, DP = 8, A  
required the position of the line  
CB when a minimum ?

Then  $10 \times 64 = 640$ , whose cube root is 8.618 nearly =  
DB, from which the position of the line is determined.



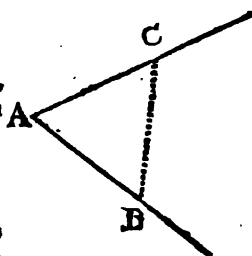
## P R O B L E M III.

Two right lines AC and AB, making the given angle A, it is required to cut off a given area ACB with the shortest line (CB) possible ?

**THEOREM.**

**THEOREM.**  $1.4142 \times \sqrt{\frac{a}{s}} =$

$AB = AC$ , where  $a$  = the given area,  
and  $s$  = the natural sine of the given  
angle  $A$ .



### EXAMPLE.

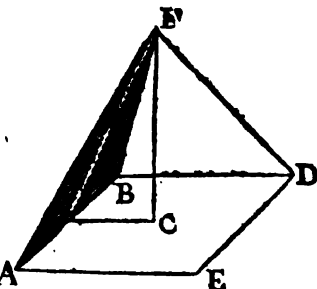
Let the angle  $A$  be  $= 54^\circ 20'$ , to  
find the length of the shortest fence  
 $BC$ , so as to inclose just 50 acres.

Then will  $s = .812423$  to radius 1, and  $a = 500$  square  
chains. Therefore  $\sqrt{\frac{500}{.812423}} \times 1.4142 = 35.07216 =$   
 $AB = AC$ ; wherefore, by Trigonometry,  
As  $\sin \angle B$  - - -  $62^\circ 50'$  Co-ar. 0.050765  
To  $\log$  of  $AC = BC$  35 07 1.544934  
So is  $\sin \angle A$  - 54 20 9.909782  
To  $\log$  of  $BC$  31 02 chains 1.505481 required.

### PROBLEM IV.

Of all the pyramids  $AFBDE$  of a given solidity, to find  
that of the least superficies, excluding the square base  $ABDE$ .

**THEOREM.** Height  $CF$   
 $= \frac{3 \times \text{solidity}}{AB^2}$ . And  $AB$   
 $= BD = DE = EA =$   
 $\sqrt{\frac{3 \times \text{solidity}}{CF}}$ .



### EXAMPLE.

To find the dimensions of a square pyramid, made with the least surface, to contain just  
one malt bushel, or 2150.4 cubic inches.

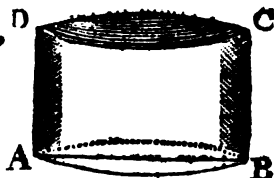
Then  $\frac{3 \times 2150.4}{2} = 3225.6$ , whose cube root is 14.775  
inches, the depth  $CF$ ; and therefore  $\frac{3 \times 2150.4}{14.775} = 436.629$ ,  
whose square root, is  $= 20.89 = AB$ , each side of the  
square base.

### PROBLEM

P R O B L E M V.

To determine the dimensions of a cylinder ABCD, open at the top, so as to contain any quantity of liquor, grain, &c. and to have the least internal superficies, or, which is the same thing, to be made of the least metal of a given thickness.

THEOREM.  $AB = 2 \times \sqrt[3]{\frac{\text{Solidity}}{3.1416}}$ , D  
and  $AD (= BC) = \frac{1}{3}AB$ .



E X A M P L E.

To find the dimensions of a cylindric bushel, made of the least quantity of metal, of a given thickness.

Here  $\frac{2150.4}{3.1416} = 684.492$ , and  $\sqrt[3]{684.492} \times 2 = 17.626$  inches = AB the diameter;  $\therefore$  the depth DA = 8.813 inches.

P R O B L E M VI.

To find that frustum of a cone, of a given base and altitude, which moving in direction of its axis, with its lesser end against the parts of an homogeneous fluid, shall suffer the least resistance possible from it.

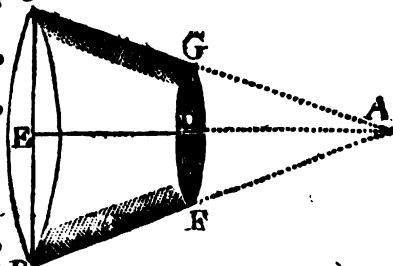
THEOREM.  $EA = \frac{ED + \sqrt{ED^2 + CB^2}}{2}$ , C

and, by similar triangles,  
 $AE : AD :: BC : FG$ .

E X A M P L E.

Let the base CB = 6,  
and altitude ED = 8; B

then, per Theorem,  $\sqrt{\frac{64 + 36}{2}} + 8 = 9 = EA$ ;  $\therefore 9$   
(= EA)  $\rightarrow 8$  (= ED) = 1 = DA. Therefore, 9 : 1 :: 6 :  
 $\frac{2}{3} = \frac{2}{3} = GF$ . Whence the frustum CGFB is determined.



P R O.

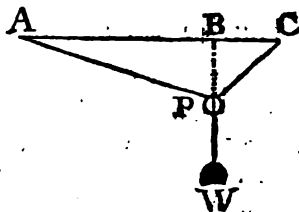
## P R O B L E M VII.

Let  $P$  be a pulley hanging freely at the end of a cord  $AP$ , fastened at  $A$ ; and let  $W$  be a weight connected to the cord  $CPW$ , put over the pulley  $P$ , which cord is fastened at  $C$ , so that the points  $A$  and  $C$  lie in the same horizontal line  $AC$ . Now, if the pulley and cords be supposed to have no weight, it is required to find in what place the pulley will settle or come to rest?

## T H E O R E M.

$$\sqrt{\frac{8AC^2 + AP^2 \times AP + AP^2}{4AC}} =$$

$AB$ ; from which point  $B$ , if a perpendicular be let fall, will be a tangent to the pulley  $P$ , and pass through the center of gravity of the weight  $W$ , when that and the pulley come to rest.



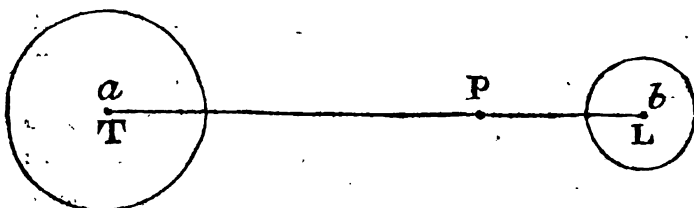
## E X A M P L E.

Let  $AC = 10$ ,  $AP = 8$ , required  $AB$ .

Then  $\sqrt{\frac{8 \times 100 + 64 \times 8 + 64}{40}} = 7.478 = AB$   
required.

## P R O B L E M VIII.

To find a point  $P$ , in a right line connecting the centers  $a$  and  $b$  of two spherical bodies  $T$  and  $L$ , of given diameters and densities; at which, if a third be placed, it shall be the least subject to their joint attraction.



**T H E O R E M.** Let the quantity of matter in the body  $T$ , be to that in the body  $L$ , (which will always be found by their given diameters and densities) as  $M$  to  $1$ ; when will



will  $\frac{ab \times M^{\frac{1}{2}}}{1 + M^{\frac{1}{2}}} = aP$ , the distance of the point P from the center  $a$  of the greater body T.

E X A M P L E.

Suppose the mean distance of the Moon and Earth to be equal to 240000 miles, and the quantity of matter in the Earth to that in the Moon, as 40 to 1; required to find where a body must be placed in a right line connecting their centers, so as to be the least attracted by these two planets?

First,  $40^{\frac{1}{2}} = 3.42$  very near; and by the Theorem,  $\frac{240000 \times 3.42}{4.42} = 185701.3$  miles, the distance of the required point from the Earth's center; and, consequently, 54298.7 miles, equal its distance from the Moon's.

P R O B L E M IX.

The latitude of the place and sun's declination being given, to find what time of the day the shadows of perpendicular objects move the slowest.

T H E O R E M.

From the natural sine of the given latitude subtract the square root of the difference of the squares of the natural sines of the latitude and declination; divide that remainder by the natural sine of the declination, and the quotient will be the natural sine of the sun's altitude at the time required; from whence the time itself will be readily found.

E X A M P L E.

At what time, on June 10th, 1765, will the shadow of a perpendicular object move the slowest at Spalding \* in Lincolnshire?

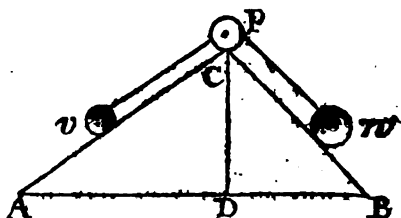
\* Latitude  $52^{\circ} 46' N.$

The sun's declination, on June 10, is  $23^{\circ} 5' N.$  its nat. sine = .39207, to rad. 1. The nat. sine of  $52^{\circ} 46'$ , the given lat. is = .796178. Then by the Theorem  $796178^2 - .39207^2 = .480180522784$ , whose square root is = .69295 nearly, and  $\frac{796178 - .69295}{.39207} = 2632897 =$  the natural sine of  $15^{\circ} 16'$  nearly, the Sun's altitude at the required time, whence the time itself is found to be 40' past five in the morning, or 20' past six in the evening, very near.

## PROBLEM X.

Being the PRIZE-QUESTION in the MATHEMATICAL MAGAZINE, Numb. IV.

Let AC and CB be two given inclin'd planes, and let the given weight  $w$  be supposed to descend along CB, whilst  $v$  (being connected by a string moving parallel to the planes over the pulley P) ascends along AC; it is required to determine the weights of  $v$ , so that its momentum, in these circumstances, may be the greatest possible?

*Method of Solution.*

Put  $CB = a$ ,  $CA = b$ , and  $CD = c$ ; then, per Mechanics,  $(a : c :: w : ) \frac{cw}{a}$  will express the force with which the weight  $w$  tends to descend along the plane CB; and  $(b : c :: v : ) \frac{cv}{b}$ , that of  $v$  along the plane CA; therefore  $\frac{cw}{a} - \frac{cv}{b}$  will be as the efficacious force wherewith the weights are accelerated: this divided by  $w + v$ , their quantity of matter, and there arises  $\frac{bcw - acv}{ba \times w + v}$  for their common velocity; which, multiplied by  $v$ , gives  $\frac{bcwv - acv^2}{ba \times w + v}$ , a maximum.

In Fluxions it will be  $av^2 + 2avv\dot{v} - bw^2\dot{v} = 0$ , which equation solved, gives  $v = \sqrt{\frac{b+a}{a}} - 1 \times w$ .

## COROLLARY.

If the inclinations of the places be equal, then will  $v = \sqrt{2-1} \times w = .4142 w$ .

SCHOLIUM.

If  $w$  = the weight of the atmosphere, pressing upon the piston in the cylinder of an engine for raising water by fire, and  $v$  = the quantity of water raised at one stroke of the great beam; then will  $v = \sqrt{2-1} \times w$ , likewise; or  $w : v :: 1 : .4142$ , when the engine produces the greatest effect in a given time.

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

S E C T. III.

*Of the Investigation of the Sums of certain infinite Series.*

I.  
**PUT**  $\frac{x}{1-x} = x + x^2 + x^3 + x^4 \&c.$  where,

taking the fluents  $v = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} \&c.$  multiply

each side of the last equation by  $x^n$ , and there arises

$vx^n = x^{n+1} + \frac{x^{n+2}}{2} + \frac{x^{n+3}}{3} + \frac{x^{n+4}}{4} \&c.$  whose

fluents are  $\frac{vx^{n+1}}{n+1}$  — the fluent of  $\frac{x^{n+2}}{n+1}$ , or  $\frac{vx^{n+1}}{n+1}$

the fluent of  $\frac{x^{n+1}}{n+1} \times \frac{x}{1-x}$ , or  $\frac{vx^{n+1}}{n+1}$  + the fluent of

$\frac{x^{n+1}}{n+1} \times \frac{x}{1-x} = \frac{x^{n+2}}{n+2} + \frac{x^{n+3}}{2n+3} + \frac{x^{n+4}}{3n+4} +$

$\frac{x^{n+5}}{4n+5} \&c.$  But  $\frac{x^{n+1}}{n+1} \times \frac{x}{x-1} = \frac{1}{n+1} \times x^n +$

$\frac{x^{n-1}}{n-1} + \frac{x^{n-2}}{n-2} + \frac{x^{n-3}}{n-3} \dots$  to  $x = v$ . Therefore

$\frac{vx - v}{n+1} + \frac{1}{n+1} \times \frac{x^{n+1}}{n+1} + \frac{x^n}{n} + \frac{x^{n-1}}{n-1} + \frac{x^{n-2}}{n-2} \dots$

to

to  $x$ , is = the sum of the infinite series  $\frac{x^{n+2}}{n+2} + \frac{x^{n+3}}{2.n+3} + \frac{x^{n+4}}{3.n+4} + \frac{x^{n+5}}{4.n+5} \&c.$  Suppose, now,  $x = 1$ , and the above series's will become  $\frac{1}{n+1} \times \frac{1}{n+1} + \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} \dots$  to 1, equal to  $\frac{1}{n+2} + \frac{1}{2.n+3} + \frac{1}{3.n+4} + \frac{1}{4.n+5} \&c. ad infinitum.$

Therefore, taking  $n = 0$ , or any positive integer, we can obtain the sums of as many infinite series's of this kind as we please. Thus,

Suppose  $n = 0$ , then the sum of the infinite series

$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \frac{1}{5.6} \&c.$  will be equal 1, as observed in Chap I.

Taking  $n = 1$ , then the sum of the infinite series

$\frac{1}{1.2} + \frac{1}{2.4} + \frac{1}{3.5} + \frac{1}{4.6} + \frac{1}{5.7} \&c.$  will be equal  $\frac{3}{4}$ .

Taking  $n = 2$ , then the sum of the infinite series

$\frac{1}{1.4} + \frac{1}{2.5} + \frac{1}{3.6} + \frac{1}{4.7} + \frac{1}{5.8} \&c.$  will be  $= \frac{11}{16}$ .

Again, taking  $n = 3$ , then the sum of the infinite series

$\frac{1}{1.5} + \frac{1}{2.6} + \frac{1}{3.7} + \frac{1}{4.8} + \frac{1}{5.9} \&c.$  will be  $= \frac{25}{48}$ .

2.

Assume  $v = \frac{x}{1-x^2} = x + x^3 + x^5 \&c.$  then  $v = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} \&c.$  Multiplying each side of the last equation by  $x^n$ , we shall have  $vx^n = x^{n+1} + \frac{x^{n+3}}{3} + \frac{x^{n+5}}{5} + \frac{x^{n+7}}{7} \&c.$  whose fluents are  $\frac{vx^{n+1}}{n+1} =$  the fluent

ent

ent of  $\frac{x^n + 1}{n+1}$ , or  $\frac{vx^n + 1}{n+1}$  + the fluent of  $\frac{x^n + 1}{n+1} \times \frac{x}{x^2 - 1} = \frac{x^n + 2}{n+2} + \frac{x^n + 4}{3n+4} + \frac{x^n + 6}{5n+6} + \frac{x^n + 8}{7n+8}$  &c.

But  $\frac{x^n + 1}{n+1} \times \frac{x}{x^2 - 1} = \frac{1}{n+1} \times \frac{1}{x^{n-1}x + x^{n-3}x + x^{n-5}x \dots \dots \dots \text{to } x - v}$ . Therefore the fluent of

$\frac{x^n + 1}{n+1} \times \frac{x}{x^2 - 1}$  is  $= \frac{1}{n+1} \times \frac{x}{n} + \frac{x^{n-2}}{n-2} + \frac{x^{n-4}}{n-4} \dots$

$\dots \text{to } x - v$ ; consequently,  $\frac{vx^n + 1}{n+1}v + \frac{1}{n+1} \times \frac{x^n}{n} +$

$\frac{x^{n-2}}{n-2} + \frac{x^{n-4}}{n-4} \dots \dots \text{to } x$  is = the sum of the infinite

series  $\frac{x^n + 2}{n+2} + \frac{x^n + 4}{3n+4} + \frac{x^n + 6}{5n+6} + \frac{x^n + 8}{7n+8}$ , &c.

Taking  $x = 1$ ; then will  $\frac{1}{n+1} + \frac{1}{n} + \frac{1}{n-2} +$

$\frac{1}{n-4} \dots \dots \text{to } 1$  = the sum of the infinite series  $\frac{1}{1 \cdot n + 2}$

$+ \frac{1}{3 \cdot n + 4} + \frac{1}{5 \cdot n + 8}$ , &c. where  $n$  may be any number

in this progression, 1, 3, 5, 7, 9, &c.

Suppose  $n = 1$ , then the sum of the infinite series

$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + \frac{1}{9 \cdot 11}$ , &c. will be  $= \frac{1}{2}$ .

Taking  $n = 3$ , then the sum of the infinite series

$\frac{1}{1 \cdot 5} + \frac{1}{3 \cdot 7} + \frac{1}{5 \cdot 9} + \frac{1}{7 \cdot 11} + \frac{1}{9 \cdot 13}$ , &c. will be  $= \frac{1}{4}$ .

Taking  $n = 5$ , then the sum of the infinite series

$\frac{1}{1 \cdot 7} + \frac{1}{3 \cdot 9} + \frac{1}{5 \cdot 11} + \frac{1}{7 \cdot 13} + \frac{1}{9 \cdot 15}$ , &c. will be  $= \frac{1}{8}$ .

Again taking  $n = 7$ , then the sum of the infinite series

$\frac{1}{1 \cdot 9} + \frac{1}{3 \cdot 11} + \frac{1}{5 \cdot 13} + \frac{1}{7 \cdot 15} + \frac{1}{9 \cdot 17}$ , &c. will be  $= \frac{1}{16}$ .

And proceeding thus, by taking  $n = 9, 11, 13$ , &c. agreeable to what is above specified, the reader may sum as many series's of this kind as he pleases. Moreover, if we assume other values of  $v$ , we can, with equal facility tend this method much further.